Dynamical Chiral Symmetry Breaking and Phase Transition of Cuprate Superconductors in QED_3*

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Abstract With the coupled Dyson-Schwinger equations in the framework of the unified QED_3 theory, we study the phase transition between the antiferromagnet(AF) and the *d*-wave superconductor (*d*SC) of planar cuprates at T=0. By solving the coupled Dyson-Schwinger equations both analytically and numerically in rainbow approximation in Landau gauge and comparing the obtained results with that given in the $1/N$ expansion, we find that there exists a chiral symmetry breaking from *d*SC phase to AF phase when the quasi-fermion flavors $N \lesssim 4$ in half-filling and the AF phase can possibly coexist with the *d*SC phase in the underdoped region. By comparing the pressure between the coexistent AF-*d*SC phase and *d*SC phase, we find that AF-*d*SC coexisting phase is the stable phase, the AF phase can then coexist with the *d*SC phase.

Key words dynamical chiral symmetry breaking, QED_3, Dyson-Schwinger equation, phase structure and phase transition of cuprate superconductor, *d*-wave superconductivity, antiferromahnet

1 Introduction

Recently, the interplay between the antiferromagnet(AF) phase and superconductivity remains one of the central themes in the physics of high-T_c cuprates[1—3]. An appealing connection between AF and *d*-wave superconducting (*d*SC) phases has been discussed, based on the ideas originally articulated by Emery and Kivelson[4]. When long range *d*SC phase is destroyed by thermal or quantum vortex-antivortex fluctuations[5, 6], the resulting state can be either a symmetric algebraic Fermi liquid (AFL phase, i.e., pseudogap phase)[7, 8] or, if the fluctuations are sufficiently strong, an incommensurate antiferromagnet[9—11]. In the latter case, AF phase(spin density wave phase) arises through an inherent dynamical instability of the underlying effective low energy theory of a phase fluctuating *d*-wave superconductor, a (2+1) dimensional quantum electrodynamics, QED_3[7, 8]. This instability is known as the spontaneous chiral symmetry breaking. Experiments have found tantalizing hints of such coexistence in zero applied magnetic field in Y and La based cuprates[12—15]. It has been shown previously[16] that, in the framework of QED_3, such a coexistence can occur locally in the vicinity of fluctuating field-induced vortices[17], as found in the experiments of neutron scattering[13], muon-spin resonance[18] and scanning tunneling microscopy[19].

It has been known that Dyson-Schwinger (D-S) equation approach provides a nonperturbative framework which admits simultaneous study of chiral

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symmetry breaking and confinement[20, 21]. With the chiral symmetry preserved nonperturbative truncation, this model has been widely used to study the properties of strong interaction vacuum and hadrons in free space and in medium (in the framework of QCD)[22, 23] as well as those of electrodynamic interaction (in the framework of QED)[24—27], and to simulate the chiral symmetry restoration and deconfinement in the system with finite temperature and/or finite chemical potential[23]. By implementing the $1/N$ expansion technique, Franz and collaborators have solved the D-S equation in the framework of QED_3 under leading order approximation of $1/N$ expansion, i.e., taking the inverse of the fermion propagator in the form $S^{-1}(p)=\mathrm{i}\gamma\cdot p+\Sigma(p)$, $\Pi(p)=\alpha/p$, and shown that the chiral symmetry breaking could take place even the gauge field has a small mass m[17]. It opened up a possibility for the coexistence of bulk AF and dSC phases in the QED_3 theory of underdoped cuprates. However, it is important to take into account the wave function renormalization, the self-energy, and the full gauge boson polarizaion. Furthermore, the inverse of the fermion propagator is usually decomposed as $S^{-1}(p)=\mathrm{i}\gamma\cdot pA(p)+B(p)$. It leaves a room to investigate the phase structure and phase transition of the underdoped cuprates by solving the D-S equations with the general expression of the fermion propagator. Moreover, the stability of the coexistent phase of the AF and the dSC has not yet been analyzed in the view of thermodynamics. We then try to shed light on these problems in this paper.

The paper is organized as follows: In Sec. 2 we describe briefly the unified QED_3 theory of cuprate superconductors. In Sec. 3 we investigate phase structure and the phase transition from dSC order to AF order of cuprate superconductors by solving the coupled Dyson-Schwinger equations. In Sec. 4 we discuss the coexistence of AF order and dSC order through the chiral symmetry breaking point. In Sec. 5, we analyze the stability of the coexistent phase of AF order and dSC order in the point of thermodynamics. Finally, we give a summary and brief discussion on our results in Sec. 6.

2 The framework of the QED_3 theory of a cuprate superconductor

It has been known that the d-wave superconducting state can be taken as the starting point to construct the general feature of the phase diagram of high-T_c cuprate superconductor[8, 9]. Amongst these, none is more prominent than the Neel antiferromagnetic insulator very near half-filling. A d-wave superconductor whose phase coherence has been destroyed by unbinding of quantum vortex-antivortex pairs indeed becomes an antiferromagnet. The antiferromagnetism arises naturally through an inherent dynamical instability of QED_3, known as the spontaneous chiral symmetry breaking (CSB)[28], and most typically takes the form of an incommensurate spin-density-wave (SDW), whose periodicity is tied to the Fermi surface.

Within the QED_3 theory of the superconductivity[8, 9], the dynamical agent responsible for the emergence of AF order is a noncompact $U(1)$ Berry gauge field A_μ which encodes the topological frustration encountered by the nodal fermions as they propagate in the background of the fluctuating vortex-antivortex plasma. In the non-superconducting phase, the Berry gauge field is massless[7—9]. Its quanta, “Berryons”, have the same effect as photons in ordinary QED_3: they mediate long range interactions between fermions and lead to the chiral instability if the number of quasi-fermion species N (pairs of Dirac nodes per unit cell in a dSC, generally $N=2$) is less than a critical value, In the dSC phase, as the vortices bind to finite pairs or loops, the Berry gauge field becomes massive[7—9].

This theory has been used as a continuum description of the competition between long range antiferromagnetic phase and superconducting phase in planar cuprate system. In Euclidean space, the Lagrangian of the general QED_3 for the massless fermion with N flavors in a general covariant gauge can be written as[29]

$$\mathscr{L} = \mathscr{L}_{\mathrm{QED}_3} + \frac{1}{2} D_\mu \Phi^* D^\mu \Phi - \lambda(\Phi^* \Phi - Nv)^2, \quad (1)$$

with

$$\mathscr{L}_{\mathrm{QED}_3} = \frac{1}{4} F_{\mu\nu}^2 + \sum_{l=1}^{N} \bar{\Psi}_l (\mathrm{i}\,\partial\!\!\!/ - e A\!\!\!/) \Psi_l \ , \quad (2)$$

where the 4-component spinor Ψ_l is the fermion field and $l = 1, \cdots, N$ are indices of the species of the fermion field. Φ is a complex scalar field with vacuum expectation value v. The addition of the scalar field does not change the global symmetry structure of the theory. We remove the Higgs boson from the spectrum by taking the limit $\lambda \to \infty$ with v fixed. This leaves the theory ultraviolet complete. The Berry gauge boson mass is $m = e\sqrt{Nv}$.

3 Phase transition from *d*SC order to AF order at half-filling

Starting from a d-wave superconducting phase, as one moves closer to half-filling at T=0, true phase coherence is lost. Strong vortex-antivortex pair fluctuations, acting under the protective umbrella of a d-wave particle-particle (p-p) pseudogap, spontaneously induce formation of particle-hole (p-h) "pairs". The glue that binds these p-h "pairs" and plays the role of "phonons" in this pairing analogy is provided by the massless Berry gauge field A_μ. Remarkably, the antiferromagnetic insulator is spontaneously generated in the form of the incommensurate SDW. It seems therefore reasonable to argue that this SDW must be considered as the progenitor of the Neel-Mott-Hubbard insulating antiferromagnet at half-filling[10].The low-energy theory for the coupled system of d-wave quasiparticles and fluctuating vortices can take the forms of the (2+1)-dimensional quantum electrodynamics (QED_3) for two flavor Dirac four-component spinors. In order to investigate the dynamical chiral symmetry breaking of the dSC order, we firstly study the general property of QED_3 with N flavor Dirac fermions and massless gauge bosons, then compare the critical fermion flavors N_c with N=2 to determine the arise of chiral phase transition of the dSC phase. The Lagrangian for massless A_μ with N Dirac spinors in the three dimensional Euclidean space is just the one in Eq. (2).

The Dyson-Schwinger (D-S) Equation for the fermion propagator is given by

$$S^{-1}(p) = S_0^{-1}(p) + e^2 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \gamma_\mu S(k) \Gamma_\nu(p,k) D_{\mu\nu}(q)\,. \quad (3)$$

For the Berryon propagator we also have a D-S equation, namely

$$D_{\mu\nu}^{-1}(q) = {D_0^{-1}}_{\mu\nu}(q) - \Pi_{\mu\nu}(q)\,. \quad (4)$$

Without introducing unknown functions, it is more convenient to write the Berryon propagator in terms of the vacuum polarization tensor $\Pi_{\mu\nu}(q)$ as

$$\Pi_{\mu\nu}(q) \ = \ e^2 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \mathrm{tr}\left[\gamma_\mu S(k) \Gamma_\nu(k,p) S(p)\right]. \quad (5)$$

The full fermion propagator is usually written as

$$S^{-1}(p) = \mathrm{i}\gamma\cdot p A(p) + B(p)\,, \quad (6)$$

and the full Berryon gauge boson propagator is given by

$$D_{\mu\nu}(q) = \frac{\delta_{\mu\nu} - q_\mu q_\nu}{q^2[1+\Pi(q)]} - \xi \frac{q_\mu q_\nu}{q^4}\,, \quad (7)$$

where ξ is the gauge parameter, and $\Pi(q)$ is the vacuum polarization for Berryon, defined by

$$\Pi_{\mu\nu} = (q^2 \delta_{\mu\nu} - q_\mu q_\nu)\Pi(q)\,. \quad (8)$$

QED_3 is a super-renormlizable theory and it does not suffer from the ultraviolet divergence that emerges in four-dimensional QED(QED_4). It has an intrinsic mass scale given by the dimensional gauge coupling $\alpha = e^2 N/8$, which plays a role similar to the scale parameter Λ_{QCD} in QCD. In the leading order of $1/N$ expansion, it was found that, at large momenta ($p \gg \alpha$), the effective coupling between fermions and gauge bosons vanishes (asymptotic freedom), whereas it has a finite value or an infrared stable fixed point at $p \ll \alpha$[28]. Early investigations indicate that the chiral symmetry can be broken only if the number of fermion flavors N (i.e., the pairs of fermion nodes in planar cuprates) is smaller than a critical value, $N_\mathrm{c} = 32/\pi^2 \approx 3.2$, in the leading order in Laudau gauge[26] and $N_\mathrm{c} = 4/3(32/\pi^2)$ in the next to leading order corrections in a nonlinear gauge[30]. These results have been questioned in Refs. [31, 32], where it was argued that the $1/N$ expansion method is not appropriate for studying the nonperturbative phenomena, and it was found that chiral symmetry is broken for all values of N, although the generated mass scale is exponentially decreasing with increasing N. More detailed investigations with the coupled set of D-S equations for the fermion and photon propagators being solved show that there exists a critical number of flavors $N_c \approx 4$ for the phase transition to take place[27, 33].

We will take a method similar to that of Maris and collaborators[27, 33] to solve the D-S equations here. For simplicity, we consider the bare vertex, and make use of Landau gauge. From the coupled integral of Eqs. (3) and (4), we obtain

$$A(p) = 1 + \frac{2e^2}{p^2}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{A(k)(p\cdot q)(k\cdot q)}{A^2(k)k^2+B(k)^2}\frac{1}{q^4(1+\Pi(q))}\,, \tag{9}$$

$$B(p) = 2e^2\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{B(k)}{A^2(k)k^2+B(k)^2}\frac{1}{q^2(1+\Pi(q))}\,, \tag{10}$$

$$\Pi(q) = \frac{2Ne^2}{q^2}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left(2k^2-4k\cdot q-\frac{6(k\cdot q)^2}{q^2}\right)\times \frac{A(k)}{A^2(k)k^2+B^2(k)}\frac{A(p)}{A^2(p)p^2+B^2(p)}\,, \tag{11}$$

with $q_\mu = p_\mu - k_\mu$.

It has been shown that the infrared behavior of QED$_3$, at least in Landau gauge, is dominated by power laws for the chiral symmetric phase[33]. Here we will investigate the chiral symmetry broken phase with fermion flavors N close to the critical value N_c of the phase transition (Assuming that there is a chiral symmetry broken phase, and a corresponding N_c). In this region the dynamically generated fermion mass will be extremely small compared to α. The momentum range $B(0) \ll p \ll \alpha$ dominates the integral on the right-hand side of the D-S equations for the $A(p)$, $B(p)$ and $\Pi(q)$. We then start from a power law ansatz for the vector dressing function $A(p) = cp^{2a}$, where a is the anomalous dimension of fermion vector dressing function to discuss the solution of the D-S equation.

Near the phase transition point, $B(0) \to 0$, there exists infrared divergences in Eqs. (9), (10) and (11), we introduce then a dimensional regularization to avoid this problem. Firstly we derive the power law of Berryon polarization, after substituting $A(p)$ into Eq. (11), with the help of dimensional regularization, we arrive at

$$\Pi(q) = \frac{8\alpha}{\pi^{3/2}c^2}\frac{\Gamma(3/2-a)^2\Gamma(2a+1/2)}{\Gamma(a+1)^2\Gamma(3-2a)}q^{-(1+4a)}\,. \tag{12}$$

To analyze the D-S equation of the fermion, we assume $a > -1/4$ (for $\Pi(q^2 \to 0)$ is infinte). The above discussion once shows that there holds $p \ll \alpha$ for the Berryon dressing, we have then

$$\frac{1}{1+\Pi(p)} \approx \Pi(p)^{-1}\,. \tag{13}$$

Together with Eq. (9), we obtain

$$cp^{2a} = 1 + f(a)p^{2a}\,, \tag{14}$$

where

$$f(a) = \frac{c(a+1)}{4N}\times \frac{\Gamma(a+1)\Gamma(3-2a)\Gamma(2a)\Gamma(1-a)}{\Gamma(3/2-a)\Gamma(2a+1/2)\Gamma(3/2-2a)\Gamma(5/2+a)}\,, \tag{15}$$

where the "1" at the right hand side of Eq. (14) corresponds to a renormalization constant, and has to be canceled by a cut-off in infrared region or by dimensional regularization. Comparing with $A(p) = cp^{2a}$, we obtain

$$N = \frac{(a+1)}{4}\times \frac{\Gamma(a+1)\Gamma(3-2a)\Gamma(2a)\Gamma(1-a)}{\Gamma(3/2-a)\Gamma(2a+1/2)\Gamma(3/2-2a)\Gamma(5/2+a)}\,. \tag{16}$$

Next we investigate the chiral symmetry broken phase close to the critical value N_c of the phase transition. Assuming that the vector dressing function $A(p)$ of the fermion and the vacuum polatization function $\Pi(p)$ of the Berryon are continuous from the phase transition point and the dynamically generated fermion mass is extremely small compared to α, following the method of Appelquist[26], i.e., setting the dominant integral range as $B(0) \ll p \ll \alpha$, we have

$$B(p) = g(a)\left(\int_{B(0)}^{p}\frac{B(k)p^{4a-1}}{k^{4a}}\mathrm{d}k + \int_{p}^{\alpha}\frac{B(k)}{k}\mathrm{d}k\right), \tag{17}$$

where

$$g(a) = \frac{\Gamma^2(a+1)\Gamma(3-2a)}{N\pi^{1/2}\Gamma^2(3/2-a)\Gamma(2a+1/2)}\,. \tag{18}$$

This integral equation is equivalent to a differential equation

$$\frac{\mathrm{d}}{\mathrm{d}p}\left(p^{2-4a}\frac{\mathrm{d}B(p)}{\mathrm{d}p}\right) - g(a)(4a-1)p^{-4a}B(p) = 0\,, \tag{19}$$

with boundary condition

$$B(0)\geqslant 0\,, \tag{20}$$

$$\left[p\frac{\mathrm{d}B(p)}{\mathrm{d}p}+B(p)\right]\bigg|_{p=\alpha}=0\,. \tag{21}$$

In the linear regime $B(p)\ll\alpha$, from the integral Eq. (17), we obtain both the chiral symmetric solution $B(p)\equiv 0$ and the nontrivial solution as

$$B(p)\sim p^{b}\,, \tag{22}$$

where

$$b=\frac{-1+4a\pm\sqrt{(1-4a)^2+4(4a-1)g(a)}}{2}\,. \tag{23}$$

It is apparent that the value of the square root in Eq. (23) can be taken as a criterion of phase transition. Letting this value be zero and using Eqs. (16) and (18), we reproduce the critical number of fermion flavors (pairs of fermion nodes) $N_{\rm c}$=3.962 given in Ref. [33].

An explicit numerical solution of Eq. (16) is shown in the left panel of Fig. 1. For the sake of comparison we also display the anomalous dimension for the case of leading approximation of $1/N$ expansion. It has been argued in Refs. [31, 34] that $A(p)\simeq(p/\alpha)^a$ with $a=\dfrac{8}{3N\pi^2}$. The left panel of Fig. 1 indicates evidently that the two curves fit very well when N is very large. It is not surprising that the two curves deviate from each other in the range of small N since the $1/N$ expansion is a perturbative method.

To check the above analysis, we solved the coupled Eqs. (9), (10) and (11) numerically. The obtained vector dressing functions $A(p)$ of the fermion at several fermion flavors N of the chiral symmetric phase (i.e., with $B(p)\equiv 0$) are illustrated in the right panel of Fig. 1, The figure shows evidently that the numerically obtained vector dressing function $A(p)$ of the fermion in the chiral symmetric phase is in good agreement with the power law ansatz in the infrared region. It is also apparent that the power law behavior is not valid in the ultraviolet region. Furthermore, we have also solved the coupled D-S equations in the chiral symmetry broken phase numerically. The obtained variation behavior of the dynamical mass of $M(p^2)=B(p^2)/A(p^2)$ at $p^2=0$ with respect to the flavors of the fermion is shown in Fig. 2. From Fig. 2, one can easily recognize that the critical fermion flavors $N_{\rm c}$=3.6 and the above analytical result agrees with that given by solving the coupled D-S equations. Meanwhile, the dynamical mass for the same N obtained in our present calculation is larger than the one given in the $1/N$ expansion, and so does the critical value $N_{\rm c}$. In addition, we display the solutions $A(p)$ and $B(p)$ of the chiral symmetry broken phase in the upper row of Fig. 3. It is obvious that, in the chiral symmetry broken phase, $A(p)$ is of the order 1, and varies slowly with the growth of momentum. Such a behavior is definitely distinct from that of the chiral symmetric phase. Besides, in the ultraviolet range all curves follow their respective asymptotic limits. On the other hand, we can clearly notice two distinct mass scales where the fermion dressing function $A(p)$ has a kink near $p=e^2$ and the $B(p)$ holds also a kink at $p\sim M(0)$.

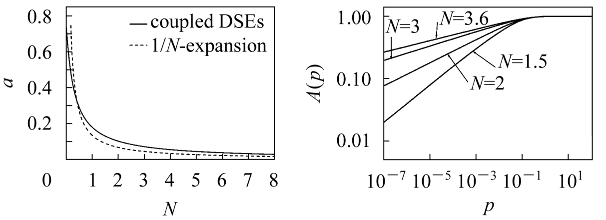

Fig. 1. Left-panel: The anomalous dimension of the vector dressing function of the fermion from our infrared analysis, compared with that under the leading order approximation of $1/N$ expansion. Right-panel: The numerically obtained dressing function $A(p)$ of the fermion in the chiral symmetry phase for $N=1.5, 2, 3,$ and 3.6, (the scale is set by choosing e^2=1).

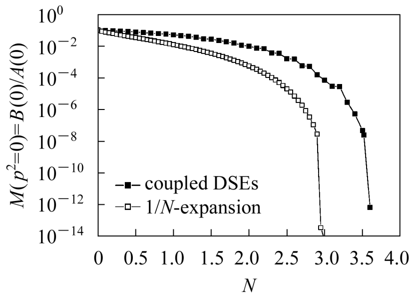

Fig. 2. Numerical result of the dynamical fermion mass $M(p^2=0)=B(p^2=0)/A(p^2=0)$ as a function of the fermion flavor N and the comparison with those under leading order approximation of $1/N$ expansion (the scale is set by choosing e^2=1).

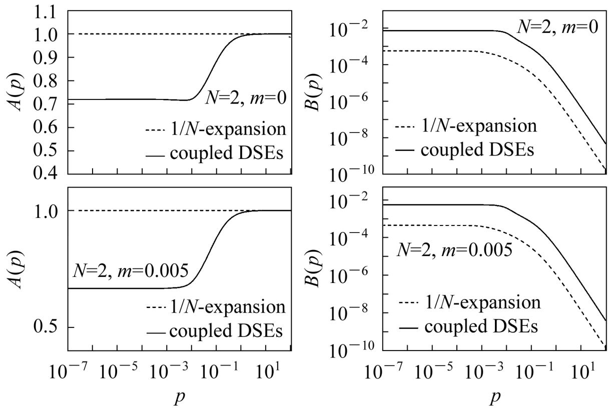

Fig. 3. Numerical solutions of the self-energy functions $A(p)$ and $B(p)$ and the comparison with those under leading order approximation $1/N$ expansion in the chirally broken phase. The upper two are for those with N=2, m=0, and the lower two for those with N=2, m=0.005 (the scale is set by choosing e^2=1).

The above discussion shows obviously that the D-S equation of the quasi-fermion has two solutions. One is the Wigner solution, $B(p)\equiv 0$, which describes the chiral symmetry phase and the quasi-fermions are massless and not confined. The corresponding phase can then be the d-wave superconductive. The other is the Nambu solution, which depicts the chiral symmetry breaking phase and the quasi-fermions are massive and confined. Such a phase corresponds to, in turn, the antiferromagnetic insulator which is confined by the gauge field in the presence of chiral symmetry breaking. The possibility of the chiral phase transition for integer fermion flavor number N=2 displayed in Fig. 2 indicates that the dSC phase can shift to the AF phase of cuprate superconductors by a spontaneous chiral symmetry transition at half-filling at T=0. The transition from the dSC phase to the AF phase may be understood as an instability of the gapless nodal fermionic excitation in the presence of free topological defects towards the formation of bound states[17].

4 Possibility of coexistence of AF phase and *d*SC phase

In the last section, we discuss the solutions of the D-S equations and the dependence of the quasi-fermion mass on the fermion flavor in the case of massless Berryon (i.e., gauge boson). Next we analyze the effect of the Berryon mass on the generation of the quasi-fermion mass. In the dSC phase, the vortex-antivortex pairs are not fully destroyed, they bind to finite pairs of loops, then the Berry gauge field becomes massive. Based on the full coupled D-S equations, we find the solutions with dynamical chiral symmetry breaking and finite fermion mass even as the gauge boson acquires a small mass m, which means that the AF phase can possibly coexistent with dSC phase. It has been known that the massive Berryon propagator is generally a little different from the massless one (in Eq. (7)) and can be written as

$$D_{\mu\nu}(q)=\frac{\delta_{\mu\nu}-q_\mu q_\nu}{q^2+q\Pi(q)+m^2}-\xi\frac{q_\mu q_\nu}{q^4}\,. \tag{24}$$

After some derivation in Landau gauge with ξ=0, we have a set of coupled equations for the self-energy functions $A(p)$ and $B(p)$ in the fermion propagator and the vacuum polarization $\Pi(q)$ of the gauge Berryon as

$$A(p) = 1+\frac{2e^2}{p^2}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{A(k)(p\cdot q)(k\cdot q)}{A^2(k)k^2+B(k)^2}\times \frac{1}{q^2(q^2+q^2\Pi(q)+m^2)}\,, \tag{25}$$

$$B(p) = 2e^2\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{B(k)}{A^2(k)k^2+B(k)^2}\times \frac{1}{(q^2+q^2\Pi(q)+m^2)}\,, \tag{26}$$

$$\Pi(q) = \frac{Ne^2}{q^2}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left(2k^2-4k\cdot q-\frac{6(k\cdot q)^2}{q^2}\right)\times \frac{A(k)}{A^2(k)k^2+B^2(k)}\frac{A(p)}{A^2(p)p^2+B^2(p)}\,, \tag{27}$$

where $q_\mu=p_\mu-k_\mu$. It is evident that, with a massive gauge field, the simple power-law ansatz is no longer appropriate to solve the coupled Eqs. (25), (26) and (27). The coupled equations also have a chiral symmetric solution $B(p)\equiv 0$ for all N. We then solve the coupled equations numerically to find chiral symmetry broken solutions with an upper momentum cutoff $\Lambda\geqslant\alpha$. Practical calculation indicates that the obtained results have no dependence on the cutoff Λ. The obtained results of the $A(p)$ and $B(p)$ in the case of gauge boson possessing a small mass are displayed in the lower row of Fig. 3. It is evident that the behavior of the solutions $A(p)$ and $B(p)$ at massive gauge boson is quite similar to that of the solution when the gauge boson is massless, though their values are comparatively smaller. The obtained dependence of the critical fermion flavors on the Berryon mass and the dynamical mass of the fermion $M(p^2=0)=B(p^2=0)/A(p^2=0)$ as a function of the Berryon mass in the case of fermion flavors N=2, which corresponds to the fermion species in cuprate superconductor, are illustrated in Fig. 4, Fig. 5, respectively.

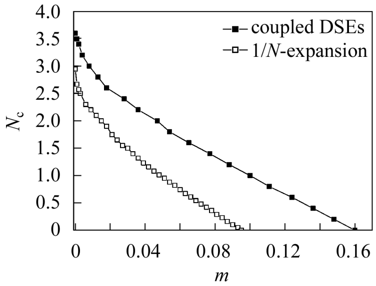

Fig. 4. Numerical result of the critical flavor of fermion as a function of the Berryon mass m (the scale is set by choosing e^2=1) and the comparison with that under the leading order approximation of $1/N$ expansion.

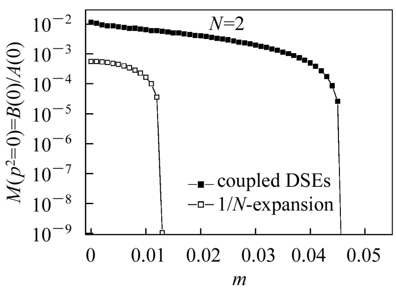

Fig. 5. Numerical solution of the dynamical fermion mass $M(p^2=0)=B(0)/A(0)$ for fermion flavors N=2 as a function of the Berryon mass m (the scale is set by choosing e^2=1) and the comparison with that under the leading order approximation $1/N$ expansion.

From Fig. 4, one can recognize evidently that there exists merely a chiral symmetric solution for all N when $m \geqslant 0.160e^2$. It means that, for a given Berryon mass m, if N is less than the corresponding $N_{\rm c}$, there exists a dynamical mass $M=B/A$, i.e., the chiral symmetry is broken. Whereas, there is no dynamical mass for any N if $m > 0.160e^2$. We also found that for all Berryon mass the critical number of fermions N is larger than that given in the $1/N$ expansion, which means that the $A(p)$ and $\Pi(p)$ play important roles in the formation of the phase structure. More concretely, from the relation between the dynamical mass at zero momentum $M(0)=B(0)/A(0)$ and the Berryon mass m in the cases of fermion number N=2, which we really care about for the cuprates, shown in Fig. 5, one can notice that the "topological" fermion can gain a mass through spontaneous chiral symmetry breaking in the presence of a small gauge boson mass. With the increasing of the Berryon mass, the topological fermion mass decreases rapidly. When $m \to 0.046e^2$, $M(0)$ decreases by eight orders compared with the neighboring value, i.e., an extremely rapid variation emerges. We can then regard it as the critical Berryon mass for the phase transition to take place. Furthermore, when $m > 0.046e^2$, there is merely a chiral symmetric solution $M(p) \equiv 0$.

Looking over Fig. 5, one may infer that the mass of the quasi-fermion is not a constant but varies with the mass of the gauge boson. Since the dynamical mass of a fermion results from the chiral symmetry breaking, the variance of the fermion mass indicates that the breaking of the chiral symmetry is not at the same level. It has been well known that the condensate of the fermion is a order parameter to identify the chiral symmetry breaking. We then evaluate the fermion condensate, which reads

$$\langle\bar{\psi}\psi\rangle = -{\rm Tr}S(p) = -4\int\frac{{\rm d}^3p}{(2\pi)^3}\frac{B(p)}{A^2(p)p^2+B^2(p)}\,. \quad (28)$$

The obtained ratio of the fermion condensate at finite gauge boson mass to that with massless gauge boson is illustrated in Fig. 6. Fig. 6 shows apparently that the ratio of the fermion condensates decreases monotonously from 1 to 0 as the mass of the gauge boson increases from 0 to $0.046e^2$. The deviation of the fermion condensates' ratio from 1 at nonzero gauge boson mass manifests the chiral symmetry is broken partially, or the chiral symmetry is partially restored. One can then infer that there may exist a phase for the AF phase and dSC phase to coexist.

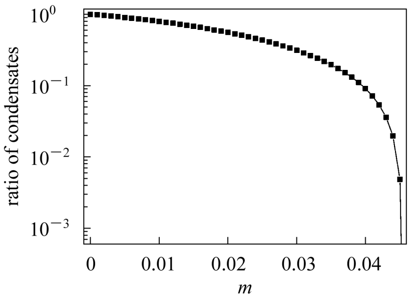

Fig. 6. The ratio of chiral condensate of coexistent AF-dSC phase as a function of the Berryon mass m(the scale is set by choosing e^2=1).

5 Stability of the AF and *d*SC coexisting phase

Recalling the above discussion, one can recognize that, when the gauge boson (or the Berryon) is massive, there may exist a dSC phase and a dSC-AF coexisting phase. To determine which is the practically appearing phase, we should explore the stability of the two phases. We implement then the equilibrium statistical field theory. It has been shown that such a theory can yield the D-S equations in a straightforward manner[35]. More than that, all the thermodynamic functions can be obtained from the partition function[23]. The thermodynamic potential and pressure densities are

$$-\omega(T)=p(T)=\frac{1}{\beta V}\ln\mathcal{Z}_T\,, \tag{29}$$

where βV is the four-volume normalising factor. The stable phase of the system is that in which the pressure is maximal or equivalently the thermodynamic potential energy is minimised.

A simple estimate of the pressure due to the dressed-fermions can be obtained via the "steepest descent" approximation[36, 37], which yields

$$p_\Sigma(\mu,T)=\frac{1}{\beta V}\left\{\mathrm{TrLn}\left[\beta S^{-1}\right]-\frac{1}{2}\mathrm{Tr}\left[\Sigma\, S\right]\right\}, \tag{30}$$

where S is the solution of the gap equation with Σ the associated self energy, and "Tr" and "Ln" are extensions of "tr" and "ln" to matrix-valued functions. Eq. (30) is just the auxiliary field effective action[36] or CJT effective action[38], which yields the free fermion pressure in the absence of interactions when $\Sigma\equiv 0$. At this level of truncation, the total pressure receives an additive contribution from dressed-gauge bosons:

$$p_\Delta(\mu,T)=-\frac{1}{\beta V}\,\frac{1}{2}\mathrm{TrLn}\left[\beta^2 D_{\mu\nu}^{-1}\right]\,, \tag{31}$$

where $D_{\mu\nu}$ is the dressed $T\neq 0$ gauge boson 2-point function, and this yields the free- gauge boson pressure in the absence of interactions.

In our study, we consider the situation of temperature $T\to 0$. The total pressure of the system is the sum of the fermion pressure and the Berryon pressure. The obtained result of the pressure difference between the AF-dSC coexisting phase and the dSC phase as a function of the gauge boson mass m is illustrated in Fig. 7. From Fig. 7, one can realize clearly that the pressure of the coexistence of AF-dSC phase is larger than that of the dSC phase and their differences become less with the increase of gauge boson mass. Therefore, the AF phase can coexist with dSC phase in view of thermodynamics. Moreover, the coexistent AF-dSC phase is a mixed phase of the chiral symmetric and the chiral symmetry breaking. When the gauge boson mass is zero, the chiral symmetry is broken, and the system is in AF phase. With the increase of the gauge boson mass, the effect of chiral symmetry breaking becomes weaker, so that the AF phase coexists with the dSC phase. In other word, the AF-dSC coexisting phase is a chiral symmetry partially restored phase. As the gauge boson mass approaches to the critical mass $m_c=0.046e^2$, the chiral symmetry can be restored completely, then the system is in the dSC phase.

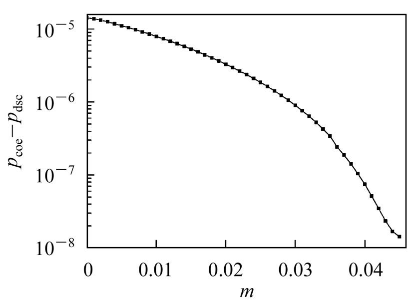

Fig. 7. The difference of pressure $p_{\mathrm{coe}}-p_{\mathrm{dsc}}$ as a function of the Berryon mass m between coexistent AF-dSC phase and dSC phase(N=2, and the scale is set by choosing e^2=1).

6 Summary and discussion

In the framework of a unified QED_3 theory at low energy, we investigate the dynamical chiral symmetry breaking of cuprate superconductors by solving the Dyson-Schwinger equations thoroughly. We then explain the phase transitions between the antiferromagnetic(AF) phase and the *d*-wave superconducting(*d*SC) phase based on the solutions of the Dyson-Schwinger equations. The calculated results show that the AF order arises from phase fluctuating *d*-wave superconductor via spontaneous chiral symmetry breaking, provided that the fluctuations are strong enough(the gauge boson mass $m=0$). With the increase of gauge boson mass from zero to $m<0.046e^2$, the AF phase and the *d*SC phase coexists in the point of view of thermodynamics. Such a coexistent AF-*d*SC phase is the chiral symmetry partially restored phase, where the AF phase is a chiral symmetry broken phase and the *d*SC phase is a chiral symmetric phase. In this region the system remains superconducting while fermionic excitations become fully gapped. The small gap should be observable in thermodynamic and transport measurements.

It should be mentioned that the QED_3 theory we implemented in the present work does not take into account the Dirac cone anisotropy $\alpha_D = v_F/v_\Delta$ of cuprate superconductors. Intuitively one would expect that N_c decreases with increasing anisotropy because the phase space for the interactions that ultimately drive the chiral symmetry breaking is reduced as the overlap between the two pairs of Dirac cones with opposite anisotropy diminishes. Another problem is the deficiency of rainbow approximation in our calculation. The rainbow approximation violates the Ward-Takahashi identity and therefore loses gauge covariance. In order to ensure gauge covariance, transverse structure of the fermion-Berryon vertex is needed. All of our discussions are the applications of D-S equations at zero temperature, we need to consider the effect of finite temperature for the practical system. The related investigations are now under progress.

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手征对称性动力学破缺和铜氧化物超导体相变的三维量子电动力学研究*

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摘要 利用三维量子电动力学理论中的Dyson-Schwinger方程方法, 研究了零温情况下平面铜氧化合物超导体的反铁磁相和d波超导相之间的相变. 通过在朗道规范下近似解析求解和数值求解完全耦合的Dyson-Schwinger方程、并将所得结果与$1/N$展开方法的结果相比较, 发现在半填充准费密子味道数约小于等于4的情况下, 通过手征对称性自发破缺, d波超导相可以演化到反铁磁相, 并且反铁磁相有可能与d波超导相共存. 通过进一步比较不同相的压强, 还说明反铁磁与d波超导共存相为稳定相, 从而反铁磁相确实可以与d波超导相共存.

关键词 手征对称性动力学破缺 三维量子电动力学 戴森–施温格方程 铜氧化合物超导体的相结构和相变 d 波超导 反磁铁

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