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Impurity Entanglement in Spin-1 Heisenberg XX Chain*

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Abstract We investigate the impurity entanglement of the ground states and the thermal states at finite temperature for a three-qutrit system of the Heisenberg XX model in the presence of a uniform magnetic field. As a measure of the entanglement, negativity of the state is analyzed in detail as a function of the impurity parameter, temperature and magnetic fields. Magnetic fields are mainly to reduce entanglement. Impurity parameter plays an important role in enhancing the entanglement and improving the critical temperature. We can use them to control entanglement.

Key words Heisenberg XX chain, impurity, negativity

1 Introduction

Quantum entanglement plays a central role in quantum information processing (QIP). Apart from the conceptual significance in quantum mechanics, it also can be exploited to accomplish many physical tasks such as superdense coding[1, 2], quantum teleportation[3], and quantum cryptographic key distribution[4, 5]. So, to some extent, it can be regarded as a resource for QIP.

The entanglement of the quantum spin system in the field of condensed-matter physics has been studied extensively. The entanglement of formation and relative entropy of entanglement are the basic measures for the bipartite systems. Using these measures, the ground-state entanglement and thermal entanglement have been revealed for the class of spin-1/2 model[6—12] in the solid-state systems, such as the anisotropy effect, high dimensions, multiple qubits, and spin-s model were considered. And concurrence[13], a rigorous computable measure of entanglement of the mixed states, has so far been obtained just for the case of spin-1/2 system and it can't be used in high spin systems. Owing to many meaningful applications of high spin quantum systems, the entanglement in a quantum Heisenberg system with spin-1 needs to be studied.

Recently, Vidal et al. raised negativity[14] as a measure of bipartite entanglement, which has drawn much attention. Negativity is a measure for the degree of violation of the criterion of positive partial transpose (PPT) in entangled states, and there has been some work on it[15—21].

Impurity plays an important role in practical systems, which may change the whole entanglement properties of the quantum systems. Besides these unforeseen effects, the presence of impurity can also be used to control the amount of bipartite entanglement, so the entanglement in a spin-1 Heisenberg chain with impurity deserves investigation.

The XX model was intensively investigated in 1960 by Lieb et al[22]. More recently the XX model has been realized in the quantum-Hall system[23], the cavity QED[24] system and quantum dot spins for a computer.

In this paper, the entanglement in a spin-1 Heisen-

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berg XX chain with impurity is investigated. In Section 2, we briefly give the model Hamiltonian and the definition of the negativity. In Section 3, we investigate the properties of the ground states and the thermal states. Finally, Section 4 contains the concluding remarks.

2 Formalism

For an isotropic spin-1 three-qutrit Heisenberg chain with a uniform magnetic field B along z axes, the Hamiltonian is given by

$$H = J_1(S_1S_2 + S_3S_1) + JS_2S_3 + B\sum\nolimits_{i=1}^{3} S_i \ , \qquad (1)$$

where J_n is the coupling constant between the lattice n and $n+1$. The periodic boundary condition is imposed. In this paper we assume the impurity spin to locate at the first site, i.e. $J_1 = J_3 \equiv J_1$, $J_2 = J$. Where $S_i = 1$ $(i = 1,\ 2,\ 3)$ is the total spin for each site and its components take the form:

$$S_i^x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$S_i^y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix},$$

$$S_i^z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Let us study the entanglement of thermal equilibrium states described by the density operator

$$\rho(T) = \exp(-\beta H)/Z \ , \qquad (2)$$

where $\beta = 1/k_{\rm B}T$, $k_{\rm B}$ is the Boltzmann's constant, which is assumed to be 1. $Z = {\rm Tr}[\exp(-\beta H)]$, is the partition function. The entanglement in the thermal states is referred to as the thermal entanglement.

We will choose the negativity as the entanglement measure

$$N(\rho) = \frac{||\rho^{\rm T_A}||_1 - 1}{2} \ , \qquad (3)$$

where $||\rho^{\rm T_A}||_1$ denotes the trace norm of the partial transpose of $\rho^{\rm T_A}$.

As the negativity is a computable measure of entanglement for a bipartite system of any dimension, here we choose it to measure the thermal entanglement. In multi-particle system at finite temperature, it can then be used to detect the entanglement between the components of any subsystem, as well as of any bipartition $\{m\}$ and $\{n{-}m\}$ of the whole system.

The negativity $N(\rho)$ is equivalent to the absolute value of the sum of the negative eigenvalues of $\rho^{\rm T_A}$. $N(\rho) = \sum_i |\mu_i|$, where μ_i is the negative eigenvalues of $\rho^{\rm T_A}$.

3 Results and discussion

As the density matrix is twenty-seven dimensions, it is very tedious to write the eigenvalues of $\rho^{\rm T_A}$. To see the effects of impurity and magnetic field B on the entanglement, we will discuss the dependence of N on the impurity parameter J_1, temperature T and magnetic field B in detail as follows.

The entanglements between Sites $\{1\}$ and $\{2,3\}$ and the one between Sites $\{1,2\}$ and $\{3\}$ can be measured by means of the negativity $N_{1\text{-}23}$ and $N_{12\text{-}3}$.

3.1 Entanglement of the ground states

Entanglement in the ground states is plotted in Fig. 1 under two circumstances of B=0 and B=1. It is evident that the negativity shows a kind of symmetry with respect to the point of J_1=0 for both $N_{1\text{-}23}$ and $N_{12\text{-}3}$. In Fig. 1(a) the negativity is plotted as a function of J_1 when J=1 and B=0. As the impurity parameter increases in absolute values of J_1, $N_{1\text{-}23}$ increases from 0 to a maximum 1 rapidly, and then decreases to a stable value.

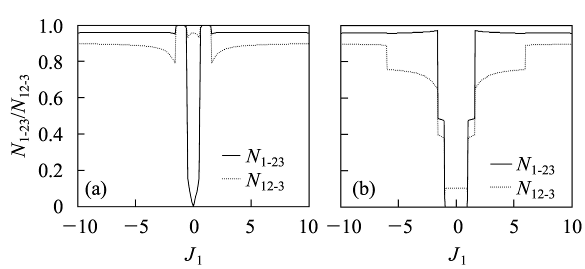

Fig. 1. The negativity $N_{1\text{-}23}$ and $N_{12\text{-}3}$ as a function of the impurity parameter J_1 when J=1. (a) B=0; (b) B=1.

The negativity $N_{12\text{-}3}$ exists even for J_1=0, which indicates that there still exists entanglement between Sites {1,2} and {3} in such cases. As J_1 increases, the negativity $N_{12\text{-}3}$ also becomes a stable curve after some oscillations. In Fig. 1(b) the negativity is plotted as a function of J_1 when J=1 and B=1. The magnetic field suppresses entanglement and influences the critical impurity parameter J_{1c}, J_{1c} means the value of impurity parameter J_1 when the entanglement exits. I.e. In Fig. 1(b) entanglement exits only when $J_{1c} \geqslant 1.8$ for $N_{1\text{-}23}$, but in Fig. 1(a) $N_{1\text{-}23}$ and $N_{12\text{-}3}$ exit only when the value of J_{1c} is over zero.

3.2 Thermal entanglement

Next let's divert our attention to the more realistic case of nonzero temperatures, i.e., the entanglement of thermal states. We first consider the case of B=0.

In Fig. 2 we present the change of the negativity $N_{1\text{-}23}$ and $N_{12\text{-}3}$ as a function of both temperature T and impurity parameter J_1. We only consider the case of J=1 because we find the entanglement is invariant under the substitution $J \rightarrow -J$ in our study. One can see from the 3D figures that the negativity decreases monotonously with the temperature. We also find the negativity increases monotonously with the impurity parameter J_1 and reaches a stable value when J_1 is large enough. In general, the figures look similar. However, the details are quite different.

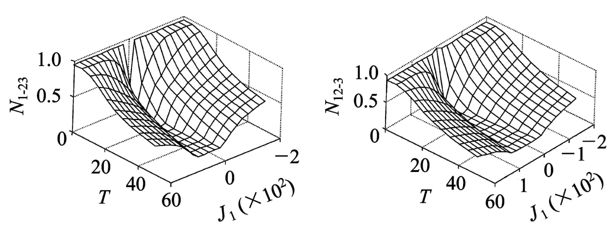

Fig. 2. The negativity $N_{1\text{-}23}$ and $N_{12\text{-}3}$ as a function of the impurity parameter J_1 and the temperature T with a specific J=1 and B=0.

Also, one can observe that the threshold temperature $T_{\rm th}$ will change with the variation of the impurity parameter J_1. There exists a threshold temperature $T_{\rm th}$ and a critical impurity parameter J_{1c} for a chosen set of parameters.

Then we see the case of $B \neq 0$. We find that the negativity shows a symmetry with respect to the point of J_1=0 and B=0 in Fig. 3. So we only give the parts of $J_1>0$ and $B>0$ for clarity. In Fig. 3 we present the changes of the global thermal negativity $N_{1\text{-}23}$ and $N_{12\text{-}3}$ as a function of both magnetic field B and impurity parameter J_1.

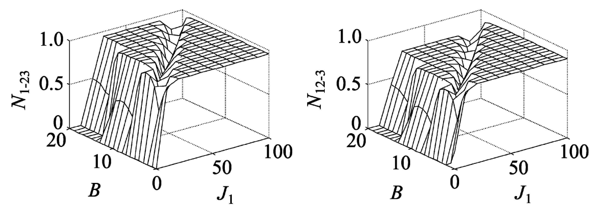

Fig. 3. The negativity $N_{1\text{-}23}$ and $N_{12\text{-}3}$ as a function of the impurity J_1 and the magnetic fields B with a specific J=1 and T=0.5.

The role of the magnetic fields is mainly to reduce the entanglement and we can use it to control the entanglement. And for a given T, the critical impurity parameter J_{1c} increases with the increase of B. We also find that the entanglement $N_{1\text{-}23}$ is larger than $N_{12\text{-}3}$ in general. This implies the impurity plays an important role in enhancing the entanglement of the 3-qutrit Heisenberg XX system. For a fixed T, one can obtain a robust entanglement by controlling B and J_1.

We also compute the negativity $N_{1\text{-}23}$ and $N_{12\text{-}3}$ as a function of the impurity J_1 and the magnetic field B with a specific J=1 and T=5. There is no distinct difference compared with Fig. 3.

4 Conclusion

In this paper, we investigate the impurity entanglement for a three-qutrit system of the Heisenberg XX chain in the presence of a uniform magnetic field. Through the calculation, we give the numerical solution of $N_{1\text{-}23}$ and $N_{12\text{-}3}$ at absolute zero temperature and the one at finite temperature with magnetic field. It is clear that the magnetic fields suppresses the entanglement. At finite temperatures we have also presented much intriguing features of this model by varying the controllable parameters. We find that the threshold temperature $T_{\rm th}$ will change with the variation of the impurity parameter J_1. We also find that the negativity decreases monotonously with the temperature. One can obtain a robust entanglement by controlling magnetic field B and impurity parameter J_1.

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自旋为1的海森堡XX链的杂质纠缠*

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摘要　采用Negativity研究了匀强磁场下自旋为1的3-qutrit海森堡XX模型的基态纠缠和热纠缠. 分别探讨了纠缠伴随杂质, 温度、磁场的变化情况. 研究表明磁场的作用主要是降低纠缠, 磁场并不改变临界温度. 杂质的加入有利于增加纠缠, 临界温度的改变来自杂质参数J_1的变化. 可以通过调节温度T, 杂质参数J_1和磁场B来控制纠缠.

关键词　海森堡XX链　杂质　负值度

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