Horizontal Synchro-Betatron Resonances due to Beam-Beam Interaction with Horizontal Crossing*

ZHANG Yuan1)

(Institute of High Energy Physics, CAS, Beijing 100049, China)

Abstract The synchro-betatron resonances (SBRs) can be excited by beam-beam kick when there is a crossing angle during collision, which are studied by numerical simulations in the paper. The studies indicate that the SBRs would be excited when $\nu_{\rm x,\pi} \gtrsim (p+r\nu_{\rm s})/q$, where p, q and r are integers, $\nu_{\rm x,\pi}$ is the horizontal π-mode tune and $\nu_{\rm s}$ is the synchrotron tune, respectively.

Key words synchro-betatron resonance, beam-beam interaction, crossing angle

1 Introduction

The synchro-betatron resonances (SBRs) are excited when the synchrotron and betatron tunes satisfy the relation $k\nu_{\rm x}+l\nu_{\rm y}+m\nu_{\rm s}=n$, where k, l, m and n are integers, $\nu_{\rm x}$, $\nu_{\rm y}$ and $\nu_{\rm s}$ are the betatron and synchrotron tunes, respectively. There are three most important mechanisms for exciting a coupling between the transverse betatron oscillations and longitudinal synchrotron oscillations in a synchrotron. The first one occurs if there is a dispersion in an accelerating cavity. The second is given by transverse fields which vary in the longitudinal direction over the bunch. Finally the third arises from the beam-beam interaction (BBI) when there is a crossing angle[1].

The luminosity of the storage ring DORIS is limited by the SBRs which are excited by the BBI. The resonance frequencies are given by $\nu_\beta=(p+r\nu_{\rm s})/q$, where p, q, r are integers, ν_β and $\nu_{\rm s}$ are the betatron and synchrotron tunes, respectively. It is proved that these resonances are caused by the crossing angle[2].

We study the horizontal SBRs due to the BBI using the code SBBE[3—5], and the simulation is done using the design parameters of BEPCⅡ[6].

2 Approximations in the code

The SBBE code is a three-dimensional particle-in-cell code, and parallelized with the message passing interface. The details of the code are presented in Refs. [4, 5]. We focus on the approximations in the code here: (1) Linear betatron oscillations are assumed, where the radiation damping effect and the excitation random effect are included. (2) The sinusoidal synchrotron oscillation is assumed here, while nonlinear synchrotron oscillations are important for higher sidebands and for large synchrotron oscillation amplitude[7]. (3) The sudden energy change at the RF cavity is omitted. (4) Though the code is termed a three-dimensional one, the actual potential calculation is only two-dimensional. The longitudinal field is not taken into account.

3 Simulation results

The corresponding design parameters of BEPCⅡ are shown in Table 1. The following results are partly introduced in Ref. [5].

---

*Supported by National Natural Science Foundation of China (10275079), JSPS Core University Program and Supercomputing Center, CNIC, CAS

1) E-mail: zhangy@mail.ihep.ac.cn

Table 1. Design parameters of BEPCⅡ.

E	1.89GeV	$\varepsilon_x/\varepsilon_y$	144/2.2nm
C	237.53m	σ_z	1.5cm
N_b	93	σ_e	5.16×10^{-4}
I_b	9.8mA	ν_x/ν_y	6.53/7.58
ξ_y	0.04	ν_s	0.034
θ_c	2×11mrad	τ_x/τ_y	31553/31553 turn
β_x^*/β_y^*	1m/1.5cm	τ_s	15777 turn

3.1 SBR versus tune

During the tune survey in the area ($0.505\leqslant\nu_x\leqslant0.545$, $0.545\leqslant\nu_y\leqslant0.595$), we find that there exist horizontal coherent oscillations somewhere. These oscillations are mainly decided by the horizontal tunes, which are shown in Fig. 8 in Ref. [5]. There exist clear coherent motions at $\nu_x=0.505$, 0.515, and 0.520. It's strange that the resonances are damped at $\nu_x=0.510$ after several ten-thousand turns. No clear coherent motions are found at other working points. The data of the dipole amplitudes are analyzed by performing a fast Fourier transform. Fig. 1(a) shows the π-mode tune versus the σ-mode one. It should be mentioned that $2\times0.517-\nu_s=1$ and $2\times0.534-2\nu_s=1$ satisfy the requirement for the SBRs. We find that there exist strong horizontal dipole oscillations when $\nu_{x,\pi}\gtrsim0.517$ or 0.534.

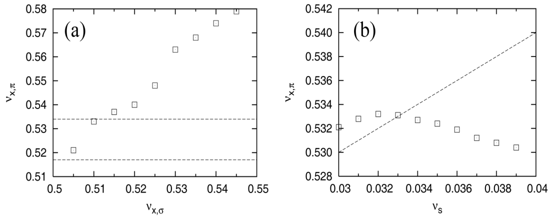

Fig. 1. (a) The horizontal π-mode tune versus the σ-mode tune at $\nu_y=0.57$. The two straight lines correspond to $\nu_{x,\pi}=0.517$ and 0.534, respectively; (b) The horizontal π-mode tune versus the synchrotron tune at $(\nu_x,\nu_y)=(0.51,0.57)$. The straight line corresponds to $2\nu_{x,\pi}-2\nu_s=1$.

There also exist strong coherent oscillations at $(\nu_x,\nu_y)=(0.517,0.57)$, where $\nu_{x,\pi}$ is near 0.538. It's hard to distinguish if the resonance be excited by $\nu_{x,\sigma}$ or by $\nu_{x,\pi}$ in the case. While for $I_b=4.9$mA at the working point, no clear oscillations are found. Another fact is that the resonances are not excited at $(\nu_x,\nu_y)=(0.534,0.57)$ either. It seems that the SBRs due to the BBI would be excited when $\nu_{x,\pi}$ is on some resonance line.

The resonances versus the synchrotron tunes are also studied at $(\nu_x,\nu_y)=(0.51,0.57)$. Fig. 2 shows the evolution of the horizontal dipole motions at various synchrotron tunes, and Fig. 1(b) shows the corresponding horizontal π-mode tunes. There exist strong coherent oscillations at $\nu_s=0.030$ and 0.031, where the corresponding $\nu_{x,\pi}\gtrsim(1+2\nu_s)/2$. The initial coherent motions are damped at $\nu_s\in[0.032,0.037]$, where the corresponding $\nu_{x,\pi}\lesssim(1+2\nu_s)/2$. There exist no clear coherent motions even in the first few thousand turns at $\nu_s=0.038$ and 0.039, where $\nu_{x,\pi}$ is farther away from the resonance line.

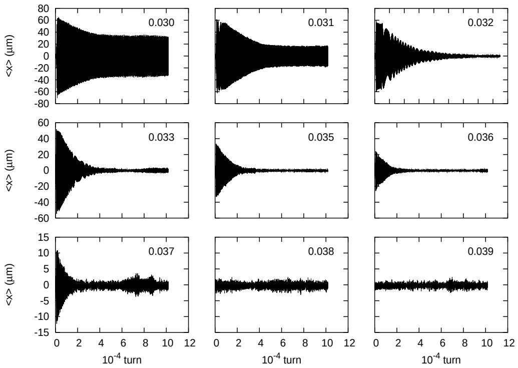

Fig. 2. Evolution of the horizontal dipole amplitude of the bunch. The corresponding synchrotron tune is written in each figure. The betatron tunes are $(\nu_x,\nu_y)=(0.51,0.57)$.

3.2 SBR versus bunch current

It is straightforward to think that the SBRs due to the BBI would be impacted by the bunch current. This is partly proved during the beam-beam limit study at $(\nu_x,\nu_y)=(0.51,0.57)$. The evolution of $\langle x\rangle$ for various bunch currents was shown in Fig. 11 in Ref. [5]. The coherent oscillations exist when the current $I_b\in[2.94\text{mA},4.90\text{mA}]$, and dispear at a higher current $I_b\in[5.88\text{mA},7.84\text{mA}]$. The coherent motions are damped after several ten-thousand turns near $I_b=9.8$mA, and not damped when $I_b>12$mA. Fig. 3(a) shows the corresponding horizontal π-mode tune versus the bunch current. The relation between SBRs and $\nu_{x,\pi}$ is similar as that mentioned before.

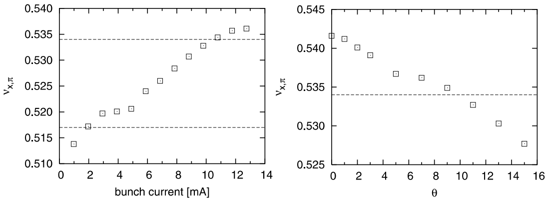

Fig. 3. (a) The horizontal π-mode tune versus the bunch current at $(\nu_x,\nu_y) = (0.51,0.57)$. The two straight lines correspond to $\nu_{x,\pi} = 0.517$ and 0.534, respectively; (b) The horizontal π-mode tune versus the half horizontal crossing angle at $(\nu_x,\nu_y) = (0.51,0.57)$. The straight line corresponds to $\nu_{x,\pi} = 0.534$.

3.3 SBR versus crossing angle

The SBR versus the horizontal crossing angle at $(\nu_x,\nu_y) = (0.51,0.57)$ is also studied by simulation. The evolution of $\langle x \rangle$ for various crossing angles was shown in Fig. 14 in Ref. [5]. There exist no clear coherent motions when the half horizontal crossing angle $\theta < 5$mrad. The resonances are excited when $\theta \in [5\text{mrad}, 9\text{mrad})$, and damped when $\theta \in [9\text{mrad}, 15\text{mrad})$. The coherent motion disapears completely when $\theta = 15$mrad. Fig. 3(b) shows the corresponding horizontal π-mode tune versus the half horizontal crossing angle. Here the rules of the SBRs are also similar as that mentioned before.

4 Discussion

It is generally believed that the SBR frequencies are given by $\nu_\beta = (p + r\nu_s)/q$. In our simulations, the horizontal SBRs due to the BBI are excited when $\nu_{x,\pi} \gtrsim (p + r\nu_s)/q$, and the SBRs would be damped when $\nu_{x,\pi} \lesssim (p + r\nu_s)/q$. The SBRs would limit the choice of the working points of BEPCⅡ.

It should be mentioned that we can not prove that the SBRs due to the BBI are caused by the non-zero crossing angle in our simulation. It is proved in Ref. [2]. Due to the nonlinearity of the beam-beam kick, the crossing-angle collisions in the horizontal plane can cause the coupling between the synchrotron and vertical betatron oscillations of particles, which is discussed in Ref. [8].

The author would like to thank Prof. CHEN Limin for his continuous support. The author would also like to thank OHMI Kazuhito in KEK for his help during the development of the simulation code.

References

1 Piwinski A. CERN 87-03, 187 (1987)

2 Piwinski A. IEEE Trans. Nucl. Sci., 1977, **NS-24**: 1408

3 ZHANG Y. HEP & NP, 2004, **28**: 1116 (in Chinese) (张源. 高能物理与核物理, 2004, **28**: 1116)

4 ZHANG Y, Ohmi K. in PAC'05 Proceedings, 2005,

5 ZHANG Y, Ohmi K, CHEN L. Phys. Rev. ST Accel. Beams 8, 074402 (2005)

6 Design Report of BEPCⅡ(Accelerator Part), 2nd ed, unpublished (2003).

7 Suzuki T. Part. Accel., 1990, **27**: 157

8 Pestrikov D V. Nucl. Instr. Methods, 1993, **A336**: 427

水平交叉对撞时束束作用导致的水平Synchro-Betatron共振*

张源[1)]

(中国科学院高能物理研究所　北京　100049)

摘要　在交叉对撞时, 束束作用可能会激发synchro-betatron共振. 通过数值模拟对此进行了研究. 结果表明, 当$\nu_{x,\pi} \gtrsim (p+r\nu_s)/p$时, 可能会出现水平方向的synchro-betatron共振.

关键词　synchro-betatron共振　束束作用　交叉角

---

*国家自然科学基金(10275079), 中日大学群合作项目和中国科学院超级计算中心资助

1) E-mail: zhangy@mail.ihep.ac.cn