

# Next-to-Leading Order QCD Corrections to Energy Loss Effect in Drell-Yan Process

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**Abstract** In this paper, by means of the nuclear parton distribution obtained from the DGLAP equation, taking into account the energy loss of the beam proton through the nucleus, we analyze the QCD corrections to the measured Drell-Yan production cross sections for an 800GeV proton beam incident on Be, Fe and W nuclear targets in the Glauber model. For the nuclear parton distribution being extracted from the leading order, the results show that the QCD corrections can't improve the theoretical results fitting to the experimental data, especially in the cross section ratio of p-W to p-Be verse  $x_1$ . Then we calculate the Drell-Yan cross sections using the shadowing effect, which is extracted from the next-to-leading order by Frankfurt et al., and find that direct photon production should be sensitive to energy loss in Compton scattering and annihilation process.

**Key words** Drell-Yan, energy loss, shadowing effect, Glauber model

## 1 Introduction

The Drell-Yan process<sup>[1]</sup> still remains an active area of theoretical and experimental research some thirty years later. It has played a key role in developing the mathematical technology of perturbative quantum chromodynamics (QCD), being one of the first processes to be calculated to next-to-leading-order (NLO) and remains one of a few processes to be calculated to next-to-next-to-leading-order (NNLO). Experimentally it has provided a wealth of information about nucleon structure; its confirmation of quark-parton model and its verification of the quark charge assignments are two notable early applications.

In 1999, the FNAL E866 had measured the energy loss effect of the Drell-Yan process<sup>[2]</sup>. Recently, Johnson et al.<sup>[3]</sup> gave an analysis of the nuclear Drell-Yan process. They examined the effect of initial state

energy loss on the Drell-Yan cross-section ratios versus the incident proton momentum fraction, by employing a new formulation of the Drell-Yan process in the rest frame of the nucleus. In this paper, having analysed the energy loss effect in high-energy Drell-Yan dimuon process at leading order<sup>[4]</sup>, we studied the QCD corrections to the energy loss effect in the nuclear Drell-Yan process in the center of mass system. Using the Glauber model with the introducing energy loss of the projectile proton through the nucleus, we analyse the QCD corrections to the energy loss effect at next-to-leading-order. For the nuclear parton distribution being extracted from leading order, the results show that the QCD corrections can't improve the theory results fitting to the experimental data, especially in the cross section ratio of p-W to p-Be verse  $x_1$ . Then we calculated the Drell-Yan cross sections using the shadowing effect, which is extracted from next-to-leading order by Frankfurt et al.<sup>[5]</sup>, for

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$0.01 < x_2 < 0.12$ , which is in the nuclear shadowing and antishadowing region. Unfortunately, only the shadowing function of C, Ca, Au and Pb are given by them, so we calculated the cross section ratio of p-Au to p-C verse  $x_1$ . The result show that direct photon production should be sensitive to energy loss in Compton scattering and annihilation process.

## 2 Method

The Glauber model<sup>[6-8]</sup> of multiple collision process provides a quantitative consideration of the geometrical configuration of the nuclei when they collide. The Glauber model basically describes the nucleus-nucleon interaction in terms of elementary nucleon-nucleon interaction. It is based on the assumption that the nucleus travels in a straight line path. At high energies this approximation is very good. In this paper, using the Glauber model, the QCD corrections calculated results of the energy loss effect in high-energy Drell-Yan dimuon process are given. The results show that direct photon production should be sensitive to energy loss in Compton scattering and annihilation process.

For high energy proton-nucleus Drell-Yan Collisions, according to the Glauber model, the projectile proton makes many collisions with nucleons in the nuclei( $A$ ), the probability of having  $n$  collisions at an impact parameter  $\mathbf{b}$  can be expressed as<sup>[7]</sup>

$$P(\mathbf{b}, n) = \frac{A!}{n!(A-n)!} [T(\mathbf{b})\sigma_{\text{in}}]^n [1 - T(\mathbf{b})\sigma_{\text{in}}]^{A-n}, \quad (1)$$

where  $\sigma_{\text{in}}$  ( $\sim 30\text{mb}$ ) is non-diffractive cross section for inelastic nucleon-nucleon collision, the thickness function is given in Ref. [8].

In Eq. (1), the first factor on the right-hand side represents the number of combinations for finding  $n$  collisions out of  $A$  possible nucleon-nucleon encounters, the second factor gives the probability of exactly  $n$  collisions and the third factor gives the probability of having  $A - n$  miss. For collisions of baryons which are not polarized, the collisions do not depend on the orientation of  $\mathbf{b}$  and  $T(\mathbf{b})$  depends only on the magnitude  $|\mathbf{b}| = b$ . We shall consider only this case of  $T(\mathbf{b}) = T(b)$ .

In an nucleon-nucleus collision without impact parameter selection, the number of nucleon-nucleon collisions  $n$  (for  $n = 1$  to  $A$ ) has a probability distribution  $P(n)$ .

$$P(n) = \frac{\int d\mathbf{b} P(n, \mathbf{b})}{\sum_{n=1}^A \int d\mathbf{b} P(n, \mathbf{b})}. \quad (2)$$

In the multiple-collision Glauber model, the basic process is proton-nucleon collision with dimuon pair production. The parton-model cross section for the process at leading-order is given by

$$\frac{d^2\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9M^2} \sum_f e_f^2 [q_f^p(x_1)\bar{q}_f^A(x_2) + \bar{q}_f^p(x_1)q_f^A(x_2)], \quad (3)$$

where  $\alpha$  is the fine-structure constant,  $e_f$  is the fractional charge of the quark of flavor  $f$ ,  $q_f^{p(A)}(x)$  and  $\bar{q}_f^{p(A)}(x)$  are the quark and anti-quark distributions in the proton (nucleon in the target nuclei  $A$ ), respectively. In addition, one has the kinematic relations  $x_1 x_2 = \frac{M^2}{s}$ ,  $x_F \approx x_1 - x_2$ , with the Feynman scaling variable  $x_F \approx \frac{2p_1}{\sqrt{s}}$ , where  $\sqrt{s}$  is the center-of-mass system (cms) energy and  $p_1$  is the longitudinal momentum of the virtual photon.

Now let us take into account the energy loss effect initially. In proton-nucleus collision, the projectile proton may change its identity during its passage through the nucleons, but its baryon number remains unchanged. Because the “soft” collisions exist, the projectile proton imparts energy to the struck nucleon in nuclei, and therefore must lose energy before making collision with the dimuon pair production. Thus the energy loss affects the measured cross section. After the projectile proton has  $n$  collisions with nucleons in the nuclei, the cms energy of the DY collisions can be expressed as  $\sqrt{s'} = \sqrt{s} - (n-1)\Delta\sqrt{s}$ , where  $\Delta\sqrt{s}$  is the cms energy loss per collision in the initial state. Therefore, the cross section for DY process at leading order can be rewritten as

$$\frac{d^2\sigma^{(n)}}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9M^2} \sum_f e_f^2 [q_f^p(x'_1)\bar{q}_f^A(x'_2) + \bar{q}_f^p(x'_1)q_f^A(x'_2)], \quad (4)$$

where the rescaled quantities are defined as  $x'_F = \frac{2p_1}{\sqrt{s'}} = r_s x_F$  and  $x'_{1,2} = r_s x_{1,2}$ , with the cms energy

ratio  $r_s = \frac{\sqrt{s}}{\sqrt{s'}}$ . It is noteworthy that  $r_s$  is always greater than one if there exists an energy loss in the collisions of protons with the nucleus  $A$ .

Now, let us review the  $\alpha_s$  order corrections to the DY mechanism of lepton pair production<sup>[9, 10]</sup>. These corrections consist of annihilation(Ann), which include the vertex contribution and the gluon production, and Compton(C) terms. The next-to-leading-order QCD corrected DY differential cross section to the first order in the strong coupling constant  $\alpha_s$  can be expressed by

$$\frac{d^2\sigma}{dx_1 dx_2} = \frac{d^2\sigma^{\text{DY}}}{dx_1 dx_2} + \frac{d^2\sigma^{\text{Ann}}}{dx_1 dx_2} + \frac{d^2\sigma^{\text{C}}}{dx_1 dx_2}, \quad (5)$$

where

$$\frac{d^2\sigma^i}{dx_1 dx_2} = \int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \left[ \frac{d^2\hat{\sigma}^i(t_1, t_2)}{dx_1 dx_2} Q^i(t_1, t_2) + \frac{d^2\hat{\sigma}^i(t_2, t_1)}{dx_1 dx_2} \tilde{Q}^i(t_1, t_2) \right]. \quad (6)$$

The superscript  $i = \text{DY, Ann or C}$  denotes the various subprocesses;  $t_1$  and  $t_2$  are the momentum fractions of incident hadron and target nucleon, respectively, which are taken by quark (anti-quark) or gluon (we only consider the longitudinal momentum because the transversal momentum is smaller than the longitudinal in the DY process);  $d^2\sigma^i(t_1, t_2)/dx_1 dx_2$  is the differential cross section for the process with index  $i$  given in Ref. [11].  $Q^i(t_1, t_2)$  is the joint distribution of quarks (anti-quarks) and gluons in the colliding hadrons for the  $i$  process and for  $i = \text{DY, Ann and C}$  read

$$Q_A^{\text{DY}}(t_1, t_2) = Q_A^{\text{Ann}}(t_1, t_2) = \sum_f e_f^2 q_f^p(t_1, Q^2) \bar{q}_f^A(t_2, Q^2), \quad (7)$$

$$\tilde{Q}_A^{\text{DY}}(t_1, t_2) = \tilde{Q}_A^{\text{Ann}}(t_1, t_2) = \sum_f e_f^2 \bar{q}_f^p(t_1, Q^2) q_f^A(t_2, Q^2), \quad (8)$$

$$Q_A^{\text{C}}(t_1, t_2) = \sum_f e_f^2 g_1^p(t_1, Q^2) [q_f^A(t_2, Q^2) + \bar{q}_f^A(t_2, Q^2)], \quad (9)$$

$$\tilde{Q}_A^{\text{C}}(t_1, t_2) = \sum_f e_f^2 [q_f^p(t_1, Q^2) + \bar{q}_f^p(t_1, Q^2)] g_2^A(t_2, Q^2), \quad (10)$$

where  $Q^2 = M^2 = x_1 x_2 s = \tau s$  and  $g_1$  and  $g_2$  are the distribution functions of gluons in free nucleon and the target nucleon, respectively.

Based on QCD corrections, Eq. (4) should be

changed into

$$\frac{d^2\sigma^{(n)}}{dx_1 dx_2} = \frac{d^2\sigma^{\text{DY}(n)}}{dx_1 dx_2} + \frac{d^2\sigma^{\text{Ann}(n)}}{dx_1 dx_2} + \frac{d^2\sigma^{\text{C}(n)}}{dx_1 dx_2}, \quad (11)$$

where

$$\frac{d^2\sigma^{i(n)}}{dx_1 dx_2} = \int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \left[ \frac{d^2\hat{\sigma}^i(r_s t_1, r_s t_2)}{dx_1 dx_2} \times Q^i(r_s t_1, r_s t_2) + \frac{d^2\hat{\sigma}^i(r_s t_2, r_s t_1)}{dx_1 dx_2} \tilde{Q}^i(r_s t_1, r_s t_2) \right]. \quad (12)$$

The average cross section for the dimuon production in nuclear DY process can be expressed as

$$\left\langle \frac{d^2\sigma}{dx_1 dx_2} \right\rangle = \sum_{n=1}^A P(n) \frac{d^2\sigma^{(n)}}{dx_1 dx_2}. \quad (13)$$

### 3 Discussions and conclusions

We calculated the investigative results of the energy loss effect in nuclear Drell-Yan process based on the Glauber Model. The nuclear-parton distributions are obtained from the DGLAP equation by K.J.Eskola et al.<sup>[12]</sup>, and the parton distributions of nucleon are based on the CTEQ6<sup>[13]</sup>. For comparison with the experimental data from the E866 collaboration<sup>[2]</sup>, we introduce the nuclear Drell-Yan ratios as

$$R^{A_1/A_2}(x_1) = \frac{\int dx_2 \left\langle \frac{d^2\sigma^{p-A_1}}{dx_1 dx_2} \right\rangle}{\int dx_2 \left\langle \frac{d^2\sigma^{p-A_2}}{dx_1 dx_2} \right\rangle}. \quad (14)$$

The integral range is determined according to the E866 experimental kinematic region<sup>[2]</sup>. The values of  $\Delta\sqrt{s} = 0.02\text{GeV}^{[4]}$ . The calculated results are shown in Fig. 1 and Fig. 2, which show the ratios of the cross section per nucleon for W or Fe to Be versus  $x_1$ , respectively. The solid line is the calculated results of the leading order without energy loss<sup>[4]</sup>. The dashed and dotted lines correspond to the leading-order and next-to-leading-order calculated results taking into account energy loss. Theoretically, because of the energy loss existence, the momentum fractions  $x_1$  and  $x_2$  turn into  $r_s x_1$  and  $r_s x_2$ , where  $r_s \geq 1$ . The sea quark and gluon will decrease with the increase of  $x_1$  and  $x_2$ , so the cross section decreases. Obviously, the theoretical results taking into account the QCD corrections can't improve the fitting to the Fermi Na-

tional Accelerator Laboratory (FNAL) E866 experimental data, especially in the cross section ratio of p-W to p-Be verse  $x_1$ . The reason is the nuclear parton distribution, which is obtained from the DGLAP equation by K.J.Eskola, extracted from the leading order.

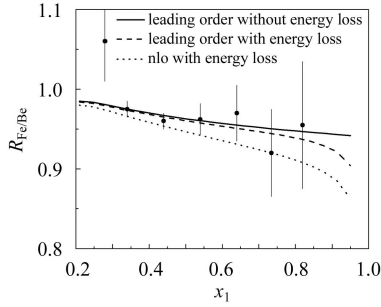


Fig. 1. The calculated results are the ratios of the cross section per nucleon for Fe to Be versus  $x_1$ . The solid line is the calculated results of the leading-order without considering energy loss. The dashed and dotted lines are correspond to the leading-order and next-to-leading-order calculated results by taking into account energy loss. The experimental data come from Ref. [2].

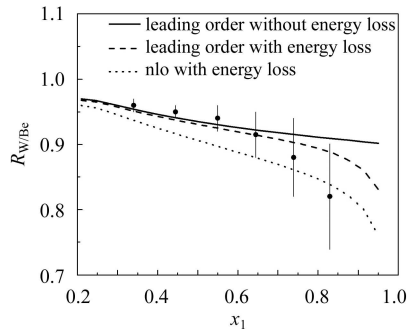


Fig. 2. Same legend as Fig.1 for the ratios of the cross section per nucleon for W to Be.

For  $0.01 < x_2 < 0.12$ , which is in the nuclear shadowing and antishadowing region, we calculated the Drell-Yan cross sections using the nuclear shadowing effect, which is extracted from the next-to-leading order by Frankfurt et al.<sup>[5]</sup>. Unfortunately, only the shadowing function of C, Ca, Au and Pb are given by them, so we calculated the cross section ratio of p-Au to p-C verse  $x_1$ . The results are shown in Fig. 3. For the nucleus number of C is more than that of Be and the nuclear shadowing effect of the nuclei with few nucleus number is more dependent on the nucleus

number than that of the nuclei with lots of nucleus number, i.e. Au and W, we can expect that this ratio should be bigger than the ratio of p-W to p-Be and less than 1. The solid line and dashed line are calculated in the Glauber model and correspond to  $\Delta\sqrt{s} = 0.02\text{GeV}$  or  $0.04\text{GeV}$  respectively. We can see from Fig. 3 that the solid line is bigger than 1 in small  $x_1$  region and is not consistent with our expectation, but the dashed line accords with our expectation. In Ref. [14], S.Gavin et al expected that high- $x_F$  direct photon production was sensitive to energy loss through the parton density dependence in Compton scattering and annihilation process and assumed  $\Delta x_g = 9/4\Delta x_q$ , where  $\Delta x_g$  and  $\Delta x_q$  is the energy loss of gluon and quark through the nuclei, and this also is applicable to the energy loss effect derived from quantum mechanical by S.J.Brodsky et al.<sup>[15]</sup> and they assumed  $\Delta x_g \approx 2\Delta x_q$ . We think these effect may be also adapted in the Glauber model. In order to compare with the Glauber model, the result calculated by the method given in Ref. [14] is also shown in Fig. 3 by the dotted line. Obviously, this result accords with our expectation.

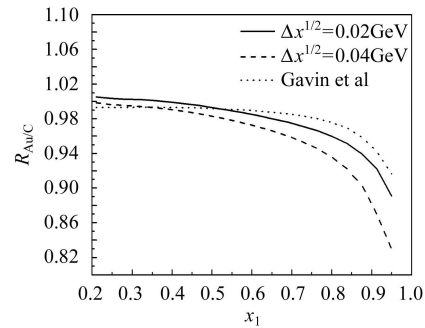


Fig. 3. The solid line and dashed line are calculated in the Glauber model and correspond to  $\Delta\sqrt{s} = 0.02\text{GeV}$  or  $0.04\text{GeV}$  respectively. The dotted line is the result calculated by the method given in Ref. [14].

In summary, the Drell-Yan cross sections are calculated in the Glauber Model at next-to-leading-order. Especially, the energy loss effect in the nuclear Drell-Yan process is investigated within the Glauber model by taking into account the energy loss of the beam proton. Through calculation, we can conclude that the effect expected by S.Gavin et al. in Compton scattering and annihilation process may be also adapted in Glauber model. The nuclear effect and

initial state energy loss can occur in both Drell-Yan production and  $J/\psi$  formation. Hence, this research

should also further the understanding of  $J/\psi$  production.

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# 微扰 QCD 对 Drell-Yan 过程能量损失效应的修正

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**摘要** 利用 Glauber 模型以及 DGLAP 方程下的核内核子的部分子分布函数, 在次领头阶 QCD 下计算了 Drell-Yan 过程中的能量损失效应, 计算表明 QCD 修正并不能改善理论结果与试验结果的符合, 尤其是 p-W 与 p-Be 以  $x_1$  为变量的微分截面比. 原因是所用的核内核子部分子的分布函数是以领头阶近似为基础并通过演化方程得到的. 于是利用在次领头阶微扰 QCD 下得到的核遮蔽效应核内核子的部分子分布函数重新计算了次领头阶 QCD 修正对 Drell-Yan 过程能量损失的贡献. 计算结果表明康普顿散射过程与湮灭过程中应该有更多的能量损失.

**关键词** Drell-Yan 过程 能量损失效应 核遮蔽效应 Glauber 模型