# Covariant Tensor Formalism for Partial Wave Analyses of $\psi$ Decays into $\gamma \mathrm{B} \overline{\mathrm{B}}^{*}$ 

Sayipjamal Dulat ${ }^{2,3 ; 1)}$ LIU Bo－Chao ${ }^{2,4 ; 2)}$ ZOU Bing－Song ${ }^{1,2}$ WU Ji－Min ${ }^{1,2}$<br>1 （CCAST（World Laboratory），Beijing 100080，China）<br>2 （Institute of High Energy Physics，CAS，Beijing 100049，China）<br>3 （Department of Physics，Xinjiang University，Urumqi 830046，China）<br>4 （Graduate School of Chinese Academy of Sciences，Beijing 100049，China）


#### Abstract

Recently BES II collaboration observed an enhancement near the p $\bar{p}$ invariant mass spectrum． Using the covariant tensor formalism，here we provide theoretical formulae for the partial wave analysis （PWA）of the $\psi$ radiative decay channels $\psi \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}, \gamma \Lambda \bar{\Lambda}, \gamma \Sigma \bar{\Sigma}, \gamma \Xi \bar{\Xi}$ ．By performing the Monte Carlo simulation，we give the angular distributions for the photon and proton in the process of $\mathrm{J} / \psi \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ ，which may serve as a useful reference for the future PWA on these channels．


Key words $\mathrm{PWA}, \mathrm{J} / \psi, \gamma \mathrm{B} \overline{\mathrm{B}}$

## 1 Introduction

$\mathrm{J} / \psi$ and $\psi^{\prime}$ radiative decay to $\mathrm{B} \overline{\mathrm{B}}$（baryon and antibaryon pair）is a good channel to study the pos－ sible bound or resonant states of the $B \bar{B}$ system ${ }^{[1-4]}$ ． Abundant $\mathrm{J} / \psi$ and $\psi^{\prime}$ events have been collected at the Beijing Electron Positron Collider（BEPC）． More data will be collected at the upgraded BEPC and CLEO－C．Based on the analysis of the 58 mil－ lion $\mathrm{J} / \psi$ events accumulated by the BES II detector at the BEPC，recently BES II reported ${ }^{[5]}$ that they have observed a strong，narrow enhancement near the threshold in the invariant mass spectrum of $\mathrm{p} \overline{\mathrm{p}}$（pro－ ton－antiproton）pairs from $J / \psi \rightarrow \gamma p \bar{p}$ radiative de－ cays．The structure has the properties consistent with either an $S\left(0^{-+}\right)$－or $P\left(0^{++}\right)$－wave Breit－Wigner reso－ nance function．In the $S$－wave case，the peak mass is blow $2 M_{\mathrm{p}}$ around $M=1859 \mathrm{MeV}$ with a total width $\Gamma<30 \mathrm{MeV}$ ．In order to get more useful informa－ tion about properties of the resonances such as their
$J^{P C}$ quantum numbers，mass，width，production and decay rates，etc．，partial wave analyses（PWA）are necessary．PWA is an effective method for analysing the experimental data of hadron spectrum．There are two types of PWA：one is based on the covariant ten－ sor（also named Rarita－Schwinger）formalism ${ }^{[6]}$ and the other is based on the helicity formalism ${ }^{[7]}$ ．In the work ${ }^{[8]}$ ，it showed the connection between the co－ variant tensor formalism and the covariant helicity one．In the article ${ }^{[9]}$ ，the author provided PWA for－ mulae in a covariant tensor formalism for $\psi$ decays to mesons，which have been used for a number of chan－ nels already published by $\mathrm{BES}^{[10-15]}$ and are going to be used for more channels．A similar approach has been used in analyzing other reactions ${ }^{[16-18]}$ ．An－ other work ${ }^{[19]}$ provided explicit formulae for the an－ gular distribution of photon of the $\psi$ radiative decays in the covariant tensor formalism，and also discussed helicity formalism of the angular distribution of the $\psi$ radiative decays to two pseudoscalar mesons，and

[^0]its relation to the covariant tensor formalism．Now we extend the covariant tensor formalism to the $\psi \rightarrow$ $\gamma \mathrm{B} \overline{\mathrm{B}}$ with B represents baryons p，$\Lambda, \Sigma, \Xi$ etc．，and derive the decay amplitudes for various intermediate resonant states in the framework of the relativistic covariant tensor formalism．

In this paper we study the phenomenological spin parity determination of resonances．The plan of this article is as follows：in Section 2，we present the nec－ essary tools for the calculation of the tensor ampli－ tudes，within a covariant tensor formalism．This will allow us to derive covariant amplitudes for all pos－ sible processes．In Section 3，we present covariant tensor formalism for $\psi$ radiative decays to baryon an－ tibaryon pairs．Since covariance is a general require－ ment of any decay amplitude，all possible amplitudes are written in terms of covariant tensor form．All amplitudes include a complex coupling constant and Blatt－Weisskopf centrifugal barriers where necessary． In Section 3，we provide the angular distribution of the photon and proton．The conclusions are given in Section 5.

## 2 Prescriptions for the construction of covariant tensor amplitudes

In this section we present the necessary tools for the construction of covariant tensor amplitudes．The partial wave amplitudes $U_{i}^{\mu \nu \alpha}$ in the covariant Rarita－ Schwinger tensor formalism can be constructed by us－ ing pure orbital angular momentum covariant tensors $\tilde{t}_{\mu_{1} \cdots \mu_{L_{\mathrm{bc}}}}^{\left(L_{\mathrm{bc}}\right)}$ and covariant spin wave functions $\phi_{\mu_{1} \cdots \mu_{\mathrm{s}}}$ to－ gether with metric tensor $g^{\mu \nu}$ ，totally antisymmetric Levi－Civita tensor $\varepsilon_{\mu \nu \lambda \sigma}$ and momenta of parent par－ ticles．For a process $\mathrm{a} \rightarrow \mathrm{bc}$ ，if there exists a relative orbital angualar momentum $\boldsymbol{L}_{\mathrm{bc}}$ between the parti－ cle a and b ，then the pure orbital angular momentum $\boldsymbol{L}_{\mathrm{bc}}$ state can be represented by covariant tensor wave functions $\tilde{t}_{\mu_{1} \cdots \mu_{L_{\mathrm{bc}}}}^{\left(L_{\mathrm{bc}}\right)}{ }^{[7]}$ which is built out of relative mo－ mentum．Thus here we give only covariant tensors that correspond to the pure $S_{-}, P_{-}, D_{-}$，and $F$－wave orbital angular momenta：

$$
\begin{equation*}
\tilde{t}^{(0)}=1, \tag{1}
\end{equation*}
$$

$$
\begin{gather*}
\tilde{t}_{\mu}^{(1)}=\tilde{g}_{\mu \nu}\left(p_{\mathrm{a}}\right) r^{\nu} B_{1}\left(Q_{\mathrm{abc}}\right) \equiv \tilde{r}_{\mu} B_{1}\left(Q_{\mathrm{abc}}\right),  \tag{2}\\
\tilde{t}_{\mu \nu}^{(2)}=\left[\tilde{r}_{\mu} \tilde{r}_{\nu}-\frac{1}{3}(\tilde{r} \cdot \tilde{r}) \tilde{g}_{\mu \nu}\left(p_{\mathrm{a}}\right)\right] B_{2}\left(Q_{\mathrm{abc}}\right),  \tag{3}\\
\tilde{t}_{\mu \nu \lambda}^{(3)}=\left[\tilde{r}_{\mu} \tilde{r}_{\nu} \tilde{r}_{\lambda}-\frac{1}{5}(\tilde{r} \cdot \tilde{r})\left(\tilde{g}_{\mu \nu}\left(p_{\mathrm{a}}\right) \tilde{r}_{\lambda}+\right.\right. \\
\left.\left.\tilde{g}_{\nu \lambda}\left(p_{\mathrm{a}}\right) \tilde{r}_{\mu}+\tilde{g}_{\lambda \mu}\left(p_{\mathrm{a}}\right) \tilde{r}_{\nu}\right)\right] B_{3}\left(Q_{\mathrm{abc}}\right), \tag{4}
\end{gather*}
$$

where $r=p_{\mathrm{b}}-p_{\mathrm{c}}$ is the relative four－momentum of the two decay products in the parent particle rest frame； $(\tilde{r} \cdot \tilde{r})=-\boldsymbol{r}^{2}$ ．and

$$
\begin{equation*}
\tilde{g}_{\mu \nu}\left(p_{\mathrm{a}}\right)=g_{\mu \nu}-\frac{p_{\mathrm{a} \mu} p_{\mathrm{a} \nu}}{p_{\mathrm{a}}^{2}} \tag{5}
\end{equation*}
$$

$B_{L_{\mathrm{bc}}}\left(Q_{\mathrm{abc}}\right)$ is a Blatt－Weisskopf barrier factor ${ }^{[7,20]}$ ， here $Q_{\mathrm{abc}}$ is the magnitude of $\boldsymbol{p}_{\mathrm{b}}$ or $\boldsymbol{p}_{\mathrm{c}}$ in the rest system of a，

$$
\begin{equation*}
Q_{\mathrm{abc}}^{2}=\frac{\left(s_{\mathrm{a}}+s_{\mathrm{b}}-s_{\mathrm{c}}\right)^{2}}{4 s_{\mathrm{a}}}-s_{\mathrm{b}} \tag{6}
\end{equation*}
$$

with $s_{\mathrm{a}}=E_{\mathrm{a}}^{2}-\boldsymbol{p}_{\mathrm{a}}^{2}$ ．
The spin－1 and spin－2 particles wave functions $\phi_{\mu}\left(p_{\mathrm{a}}, m\right)$ and $\phi_{\mu \nu}\left(p_{\mathrm{a}}, m\right)$ satisfy the following condi－ tions

$$
\begin{align*}
& p_{\mathrm{a}}^{\mu} \phi_{\mu}\left(p_{\mathrm{a}}, m\right)=0, \quad \phi_{\mu}\left(p_{\mathrm{a}}, m\right) \phi^{* \mu}\left(p_{\mathrm{a}}, m\right)=-\delta_{m m^{\prime}} \\
& \sum_{m} \phi_{\mu}\left(p_{\mathrm{a}}, m\right) \phi_{\nu}^{*}\left(p_{\mathrm{a}}, m\right)=-g_{\mu \nu}+\frac{p_{\mathrm{a} \mu} p_{\mathrm{a} \nu}}{p_{\mathrm{a}}^{2}} \equiv-\tilde{g}_{\mu \nu}\left(p_{\mathrm{a}}\right) \tag{7}
\end{align*}
$$

and

$$
\begin{align*}
& p_{\mathrm{a}}^{\mu} \phi_{\mu \nu}\left(p_{\mathrm{a}}, m\right)=0, \quad \phi_{\mu \nu}=\phi_{\nu \mu}, \\
& g^{\mu \nu} \phi_{\mu \nu}=0, \quad \phi_{\mu \nu}(m) \phi^{* \mu \nu}\left(m^{\prime}\right)=\delta_{m m^{\prime}} \\
& P_{\mu \nu \mu^{\prime} \nu^{\prime}}^{(2)}\left(p_{\mathrm{a}}\right)=\sum_{m} \phi_{\mu \nu}\left(p_{\mathrm{a}}, m\right) \phi_{\mu^{\prime} \nu^{\prime}}^{*}\left(p_{\mathrm{a}}, m\right)= \\
& \frac{1}{2}\left(\tilde{g}_{\mu \mu^{\prime}} \tilde{g}_{\nu \nu^{\prime}}+\tilde{g}_{\mu \nu^{\prime}} \tilde{g}_{\nu \mu^{\prime}}\right)-\frac{1}{3} \tilde{g}_{\mu \nu} \tilde{g}_{\mu^{\prime} \nu^{\prime}} \tag{8}
\end{align*}
$$

Note that for a given decay process $a \rightarrow b c$ ，the total angular momentum should be conserved，which means

$$
\begin{equation*}
\boldsymbol{J}_{\mathrm{a}}=\boldsymbol{S}_{\mathrm{bc}}+\boldsymbol{L}_{\mathrm{bc}} \tag{9}
\end{equation*}
$$

where

$$
\begin{equation*}
\boldsymbol{S}_{\mathrm{bc}}=\boldsymbol{S}_{\mathrm{b}}+\boldsymbol{S}_{\mathrm{c}} \tag{10}
\end{equation*}
$$

In addition parity should also be conserved，which means

$$
\begin{equation*}
\eta_{\mathrm{a}}=\eta_{\mathrm{b}} \eta_{\mathrm{c}}(-1)^{L_{\mathrm{bc}}} \tag{11}
\end{equation*}
$$

where $\eta_{\mathrm{a}}, \eta_{\mathrm{b}}$ ，and $\eta_{\mathrm{c}}$ are the intrinsic parities of parti－ cles a，b，and c，respectively．From this relation，one knows whether $L_{\mathrm{bc}}$ should be even or odd．Then from Eq．（9）one can find out how many different $L_{\mathrm{bc}}-S_{\mathrm{bc}}$ combinations determine the number of independent couplings．Also note that in the construction of the covariant tensor amplitude，for $S_{\mathrm{bc}}-L_{\mathrm{bc}}-J_{\mathrm{a}}$ coupling， if $S_{\mathrm{bc}}+L_{\mathrm{bc}}+J_{\mathrm{a}}$ is an odd number，then $\varepsilon_{\mu \nu \lambda \sigma} p_{\mathrm{a}}^{\sigma}$ with $p_{\mathrm{a}}$ the momentum of the parent particle is needed； otherwise it is not needed．

## 3 Covariant tensor formalism for $\psi$ decay into $\gamma \mathrm{B} \overline{\mathrm{B}}$

The general form of the decay $\psi \rightarrow \gamma \mathrm{X} \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ amplitude can be written as follows by using the po－ larization four－vectors of the initial and final states，

$$
\begin{align*}
A^{(s)}= & \psi_{\mu}\left(m_{\mathrm{J}}\right) e_{\nu}^{*}\left(m_{\gamma}\right) \psi_{\alpha_{s}}\left(p_{\mathrm{b}}, S_{\mathrm{b}} ; p_{\mathrm{c}}, S_{\mathrm{c}}\right) A^{\mu \nu \alpha_{s}}= \\
& \psi_{\mu}\left(m_{\mathrm{J}}\right) e_{\nu}^{*}\left(m_{\gamma}\right) \psi_{\alpha_{s}}\left(p_{\mathrm{b}}, S_{\mathrm{b}} ; p_{\mathrm{c}}, S_{\mathrm{c}}\right) \sum_{i} \Lambda_{i} U_{i}^{\mu \nu \alpha_{s}} \tag{12}
\end{align*}
$$

where $\psi_{\mu}\left(m_{J}\right)$ is the polarization four－vector of the $\psi$ with spin projection of $m_{\mathrm{J}} ; e_{\nu}\left(m_{\gamma}\right)$ is the polariza－ tion four－vector of the photon with spin projections of $m_{\gamma} ; U_{i}^{\mu \nu \alpha_{s}}$ is the $i$－th partial wave amplitude with coupling strength determined by a complex parame－ ter $\Lambda_{i}$ ．The spin－1 polarization vector $\psi_{\mu}\left(m_{J}\right)$ for $\psi$ with four－momentum $p_{\mu}$ satisfies

$$
\begin{equation*}
\sum_{m_{\mathrm{J}}=1}^{3} \psi_{\mu}\left(m_{\mathrm{J}}\right) \psi_{\nu}^{*}\left(m_{\mathrm{J}}\right)=-g_{\mu \nu}+\frac{p_{\mu} p_{\nu}}{p^{2}} \equiv-\tilde{g}_{\mu \nu}(p) \tag{13}
\end{equation*}
$$

with $p^{\mu} \psi_{\mu}=0$ ．Here the Minkowsky metric tensor has the form

$$
g_{\mu \nu}=\operatorname{diag}(1,-1,-1,-1)
$$

For $\psi$ production from $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation，the elec－ trons are highly relativistic，with the result that $J_{z}= \pm 1$ for the $\psi$ spin projection taking the beam direction as the $z$－axis．This limits $m_{J}$ to 1 and 2 ， i．e．components along $x$ and $y$ ．Then one has the following relation

$$
\begin{equation*}
\sum_{m_{\mathrm{J}}=1}^{2} \psi_{\mu}\left(m_{\mathrm{J}}\right) \psi_{\mu^{\prime}}^{*}\left(m_{\mathrm{J}}\right)=\delta_{\mu \mu^{\prime}}\left(\delta_{\mu 1}+\delta_{\mu 2}\right) \tag{14}
\end{equation*}
$$

For the photon polarization four－vector，there is the usual Lorentz orthogonality conditions．Namely，the polarization four－vector $e_{\nu}\left(m_{\gamma}\right)$ of the photon with momenta $q$ satisfies

$$
\begin{equation*}
q^{\nu} e_{\nu}\left(m_{\gamma}\right)=0 \tag{15}
\end{equation*}
$$

which states that spin－1 wave function is orthogonal to its own momentum．The above relation is the same as for a massive vector meson．However，for the pho－ ton，there is an additional gauge invariance condition． Here we assume the Coulomb gauge in the $\psi$ rest sys－ tem，i．e．，$p^{\nu} e_{\nu}=0$ ．Then we have ${ }^{[21]}$

$$
\begin{align*}
\sum_{m_{\gamma}} e_{\mu}^{*}\left(m_{\gamma}\right) e_{\nu}\left(m_{\gamma}\right)= & -g_{\mu \nu}+\frac{q_{\mu} K_{\nu}+K_{\mu} q_{\nu}}{q \cdot K}- \\
& \frac{K \cdot K}{(q \cdot K)^{2}} q_{\mu} q_{\nu} \equiv-g_{\mu \nu}^{(\perp \perp)} \tag{16}
\end{align*}
$$

with $K=p-q$ and $K^{\nu} e_{\nu}=0$ ．For $\mathrm{X} \rightarrow \mathrm{p} \overline{\mathrm{p}}$ ，the total spin of $\mathrm{p} \overline{\mathrm{p}}$ system can be either 0 or 1 ．These two states can be represented by $\psi$ and $\psi_{\alpha}{ }^{[22]}$ ．where

$$
\begin{equation*}
\psi=\bar{u}\left(p_{\mathrm{b}}, S_{\mathrm{b}}\right) \gamma_{5} v\left(p_{\mathrm{c}}, S_{\mathrm{c}}\right), \quad \text { if } \quad s=0 \tag{17}
\end{equation*}
$$

$$
\begin{equation*}
\psi_{\alpha}=\bar{u}\left(p_{\mathrm{b}}, S_{\mathrm{b}}\right)\left(\gamma_{\alpha}-\frac{r_{\alpha}}{m_{X}+2 m}\right) v\left(p_{\mathrm{c}}, S_{\mathrm{c}}\right), \quad \text { if } \quad s=1 \tag{18}
\end{equation*}
$$

One can see that both $\psi$ and $\psi_{\alpha}$ have no dependence on the direction of the momentum $\hat{\boldsymbol{p}}$ ，hence corre－ spond to pure spin states with the total spin of 0 and 1 ，respectively．Where $p_{\mathrm{b}}, p_{\mathrm{c}}$ ，and $S_{\mathrm{b}}, S_{\mathrm{c}}$ are momenta and spin of the proton antiproton pairs，respectively． $m_{\mathrm{X}}$ and $m$ are the masses of X and $\mathrm{p}, \overline{\mathrm{p}}$ ，respectively； $u\left(p_{\mathrm{b}}, S_{\mathrm{b}}\right)$ and $v\left(p_{\mathrm{c}}, S_{\mathrm{c}}\right)$ are the standard Dirac spinor． If we sum over the polarization，we have the two pro－ jection operators：

$$
\begin{align*}
& \sum_{S_{\mathrm{b}}} u_{\alpha}\left(p_{\mathrm{b}}, S_{\mathrm{b}}\right) \bar{u}_{\beta}\left(p_{\mathrm{b}}, S_{\mathrm{b}}\right)=\left(\frac{\not p_{\mathrm{b}}+m}{2 m}\right)_{\alpha \beta} \\
& \sum_{S_{\mathrm{c}}} v_{\alpha}\left(p_{\mathrm{c}}, S_{\mathrm{c}}\right) \bar{v}_{\beta}\left(p_{\mathrm{c}}, S_{\mathrm{c}}\right)=\left(\frac{\not p_{\mathrm{c}}-m}{2 m}\right)_{\alpha \beta} \tag{19}
\end{align*}
$$

To compute the differential cross section，we need an expression for $|A|^{2}$ ．Note that the square mod－ ulus of the decay amplitude，which gives the decay probability of a certain configuration should be inde－ pendent of any particular frame，that is，a Lorentz scalar．Thus by using Eqs．（14）and（16），the dif－ ferential cross section for the radiative decay to an

3－body final state is：

$$
\begin{align*}
\frac{\mathrm{d} \sigma^{(s)}}{\mathrm{d} \Phi_{3}}= & \left.\frac{1}{2} \sum_{S_{\mathrm{b}}, S_{\mathrm{c}}} \sum_{m_{\mathrm{J}}=1}^{2} \sum_{m_{\gamma}=1}^{2} \right\rvert\, \psi_{\mu}\left(m_{\mathrm{J}}\right) e_{\nu}^{*}\left(m_{\gamma}\right) \times \\
& \left.\psi_{\alpha_{s}}\left(p_{\mathrm{b}}, S_{\mathrm{b}} ; p_{\mathrm{c}}, S_{\mathrm{c}}\right) A^{\mu \nu \alpha_{s}}\right|^{2}= \\
& -\frac{1}{2} \sum_{S_{\mathrm{b}}, S_{\mathrm{c}}} \sum_{\mu=1}^{2} A^{\mu \nu \alpha_{s}} g_{\nu \nu^{\prime}}^{(\perp \perp)} A^{* \mu \nu^{\prime} \alpha_{s}^{\prime}} \psi_{\alpha_{s}}^{*} \psi_{\alpha_{s}^{\prime}}= \\
& -\frac{1}{2} \sum_{i, j} \Lambda_{i} \Lambda_{j}^{*} \sum_{\mu=1}^{2} U_{i}^{\mu \nu \alpha_{s}} g_{\nu \nu^{\prime}}^{(\perp \perp)} U_{j}^{* \mu \nu^{\prime} \alpha_{s}^{\prime}} \times \\
& \sum_{S_{\mathrm{b}}, S_{\mathrm{c}}} \psi_{\alpha_{s}}^{*} \psi_{\alpha_{s}^{\prime}} \equiv \sum_{i, j} P_{i j} \cdot F_{i j}^{(s)} \tag{20}
\end{align*}
$$

where

$$
\begin{align*}
P_{i j} & =P_{j i}^{*}=\Lambda_{i} \Lambda_{j}^{*} \\
F_{i j}^{(s)} & =F_{j i}^{*(s)}=-\frac{1}{2} \sum_{\mu=1}^{2} U_{i}^{\mu \nu \alpha_{s}} g_{\nu \nu^{\prime}}^{(\perp \perp)} U_{j}^{* \mu \nu^{\prime} \alpha_{s}^{\prime}} \sum_{S_{\mathrm{b}}, S_{\mathrm{c}}} \psi_{\alpha_{s}}^{*} \psi_{\alpha_{s}^{\prime}} \tag{21}
\end{align*}
$$

$\mathrm{d} \Phi_{3}$ is the standard Lorentz invariant 3－body phase space given by

$$
\begin{gather*}
\mathrm{d} \Phi_{3}\left(p ; q, p_{\mathrm{b}}, p_{\mathrm{c}}\right)= \\
\delta^{4}\left(p-q-p_{\mathrm{b}}-p_{\mathrm{c}}\right) \times  \tag{22}\\
\frac{\mathrm{d}^{3} \boldsymbol{q}}{(2 \pi)^{3} 2 E_{\gamma}} \frac{m^{2} \mathrm{~d}^{3} \boldsymbol{p}_{\mathrm{b}} \mathrm{~d}^{3} \boldsymbol{p}_{\mathrm{c}}}{(2 \pi)^{3} E_{\mathrm{b}}(2 \pi)^{3} E_{\mathrm{c}}} . \\
F_{i j}^{(0)}= \\
F_{j i}^{*(0)}=-\frac{1}{2} \sum_{\mu=1}^{2} U_{i}^{\mu \nu} g_{\nu \nu^{\prime}}^{(\perp \perp)} U_{j}^{* \mu \nu^{\prime}} \sum_{S_{\mathrm{b}}, S_{\mathrm{c}}} \psi^{*} \psi=  \tag{23}\\
\\
\frac{1}{2} \sum_{\mu=1}^{2} U_{i}^{\mu \nu} g_{\nu \nu^{\prime}}^{(\perp \perp)} U_{j}^{* \mu \nu^{\prime}} \operatorname{Tr}\left(\frac{\not p_{\mathrm{b}}+m}{2 m} \gamma_{5} \frac{\not p_{\mathrm{c}}-m}{2 m} \gamma_{5}\right)= \\
\\
-\frac{m_{\mathrm{X}}^{2}}{4 m^{2}} \sum_{\mu=1}^{2} U_{i}^{\mu \nu} g_{\nu \nu^{\prime}}^{(\perp \perp)} U_{j}^{* \mu \nu^{\prime}} .
\end{gather*}
$$

The spin sums can be performed using the complete－ ness relations from Eq．（19）：

$$
\begin{aligned}
F_{i j}^{(1)}= & F_{j i}^{*(1)}=-\frac{1}{2} \sum_{\mu=1}^{2} U_{i}^{\mu \nu \alpha} g_{\nu \nu^{\prime}}^{(\perp \perp)} U_{j}^{* \mu \nu^{\prime} \alpha^{\prime}} \sum_{S_{\mathrm{b}}, S_{\mathrm{c}}} \psi_{\alpha}^{*} \psi_{\alpha^{\prime}}= \\
& -\frac{1}{2} \sum_{\mu=1}^{2} U_{i}^{\mu \nu \alpha} g_{\nu \nu^{\prime}}^{(\perp \perp)} U_{j}^{* \mu \nu^{\prime} \alpha^{\prime}} \times \\
& {\left[\operatorname{Tr}\left(\frac{\not p_{\mathrm{b}}+m}{2 m} \gamma_{\alpha} \frac{\not \mathrm{c}_{\mathrm{c}}-m}{2 m} \gamma_{\alpha^{\prime}}\right)-\right.} \\
& \frac{r_{\alpha}}{m_{\mathrm{X}}+2 m} \operatorname{Tr}\left(\frac{\not p_{\mathrm{b}}+m}{2 m} \frac{\not \mathrm{c}_{\mathrm{c}}-m}{2 m} \gamma_{\alpha^{\prime}}\right)- \\
& \frac{r_{\alpha^{\prime}}}{m_{\mathrm{X}}+2 m} \operatorname{Tr}\left(\frac{\not \mathrm{~b}_{\mathrm{b}}+m}{2 m} \gamma_{\alpha} \frac{\not \mathrm{p}_{\mathrm{c}}-m}{2 m}\right)+
\end{aligned}
$$

$$
\begin{align*}
& \left.\frac{r_{\alpha} r_{\alpha^{\prime}}}{\left(m_{\mathrm{X}}+2 m\right)^{2}} \operatorname{Tr}\left(\frac{\not p_{\mathrm{b}}+m}{2 m} \frac{\not p_{\mathrm{c}}-m}{2 m}\right)\right]= \\
& -\frac{1}{4 m^{2}} \sum_{\mu=1}^{2} U_{i}^{\mu \nu \alpha} g_{\nu \nu^{\prime}}^{(\perp \perp)} U_{j}^{* \mu \nu^{\prime} \alpha^{\prime}} \times \\
& {\left[p_{b \alpha} p_{b \alpha^{\prime}}+p_{c \alpha} p_{c \alpha^{\prime}}+p_{b \alpha} p_{c \alpha^{\prime}}+p_{b \alpha^{\prime}} p_{c \alpha}-m_{\mathrm{X}}^{2} g_{\alpha \alpha^{\prime}}\right] .} \tag{24}
\end{align*}
$$

where we have used traces of $\gamma$ matrics，

$$
\begin{array}{ll}
\operatorname{tr}(1)=4, \quad \operatorname{tr}\left(\gamma^{5}\right)=0, & \operatorname{tr}\left(\gamma^{\mu} \gamma^{\nu}\right)=4 g^{\mu \nu} \\
\operatorname{tr}\left(\text { any odd } \sharp \gamma^{\prime} s\right)=0, & \operatorname{tr}\left(\gamma^{\mu} \gamma^{\nu} \gamma^{5}\right)=0 . \tag{25}
\end{array}
$$

## Amplitudes for the radiative decay $\psi \rightarrow \gamma \mathrm{p} \overline{\mathbf{p}}$

We consider the decay of a $\psi$ state into two steps： $\psi \rightarrow \gamma \mathrm{X}$ with $\mathrm{X} \rightarrow \mathrm{p} \overline{\mathrm{p}}$ ．The possible $J^{P C}$ for X are $0^{++}, 0^{-+}, 1^{++}, 2^{++}, 2^{-+}$，etc．For $\psi \rightarrow \gamma \mathrm{X}$ ，we choose two independent momenta $p$ for $\psi$ and $q$ for the pho－ ton to be contracted with spin wave functions．We denote the four－momentum of X by K ．The tensor describing the first and second steps will be denoted by $\tilde{T}_{\mu_{1} \cdots \mu_{L}}^{(L)}$ and $\tilde{t}_{\mu_{1} \cdots \mu_{l}}^{(l)}$ ，respectively．

For $\psi \rightarrow \gamma 0^{++} \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ ，there is one independent covariant tensor amplitude：

$$
\begin{equation*}
U^{\mu \nu \alpha}=g^{\mu \nu} \tilde{t}^{(1) \alpha} \tag{26}
\end{equation*}
$$

For $\psi \rightarrow \gamma 0^{-+} \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ ，there is one independent covariant tensor amplitude：

$$
\begin{equation*}
U^{\mu \nu}=\varepsilon^{\mu \nu \lambda \sigma} p_{\lambda} q_{\sigma} B_{1}\left(Q_{\psi \gamma \mathrm{X}}\right) \tag{27}
\end{equation*}
$$

For $\psi \rightarrow \gamma 1^{++} \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ ，there are two independent covariant tensor amplitudes：

$$
\begin{align*}
U_{1}^{\mu \nu \alpha} & =\varepsilon^{\mu \nu \lambda \sigma} p_{\lambda} \varepsilon_{\sigma}^{\alpha \beta \rho} K_{\rho} \tilde{t}_{\beta}^{(1)}  \tag{28}\\
U_{2}^{\mu \nu \alpha} & =\varepsilon^{\nu \lambda \sigma \gamma} p_{\lambda} q^{\mu} q_{\gamma} \varepsilon_{\sigma}^{\alpha \beta \rho} K_{\rho} \tilde{t}_{\beta}^{(1)} B_{2}\left(Q_{\psi \gamma \mathrm{X}}\right) \tag{29}
\end{align*}
$$

For $\psi \rightarrow \gamma 1^{-+}$，the exotic $1^{-+}$meson cannot decay into $\mathrm{p} \overline{\mathrm{p}}$ ．

For $\psi \rightarrow \gamma 2^{++} \rightarrow p \bar{p}$ ，there are three independent covariant tensor amplitudes：

$$
\begin{align*}
U_{1}^{\mu \nu \alpha} & =P^{(2) \mu \nu \alpha \beta}(K) \tilde{t}_{\beta}^{(1)}  \tag{30}\\
U_{2}^{\mu \nu \alpha} & =g^{\mu \nu} P^{(2) \alpha \beta \lambda \sigma} p_{\beta} p_{\lambda} \tilde{t}_{\sigma}^{(1)} B_{2}\left(Q_{\psi \gamma \mathrm{X}}\right)  \tag{31}\\
U_{3}^{\mu \nu \alpha} & =P^{(2) \nu \alpha \beta \lambda} q^{\mu} p_{\beta} \tilde{t}_{\lambda}^{(1)} B_{2}\left(Q_{\psi \gamma \mathrm{X}}\right) \tag{32}
\end{align*}
$$

For $2^{++}$decaying to $\mathrm{p} \overline{\mathrm{p}}$ ，the orbital angular momen－ tum between the proton and antiproton $l$ could be 1 and 3 ；but we ignore $l=3$ contribution because of the strong centrifugal barrier．

For $\psi \rightarrow \gamma 2^{-+} \rightarrow p \bar{p}$ ，the possible partial wave amplitudes are the following：

$$
\begin{align*}
U_{1}^{\mu \nu} & =\varepsilon^{\mu \nu \lambda \sigma} p_{\lambda} q^{\gamma} \tilde{t}_{\gamma \sigma}^{(2)} B_{1}\left(Q_{\psi \gamma \mathrm{X}}\right)  \tag{33}\\
U_{2}^{\mu \nu} & =\varepsilon^{\mu \nu \lambda \sigma} p_{\lambda} q_{\sigma} p_{\gamma} p_{\delta} \tilde{t}^{(2) \gamma \delta} B_{3}\left(Q_{\psi \gamma \mathrm{X}}\right)  \tag{34}\\
U_{3}^{\mu \nu} & =\varepsilon^{\nu \gamma \lambda \sigma} p_{\lambda} q_{\sigma} q^{\mu} p^{\delta} \tilde{t}_{\gamma \delta}^{(2)} B_{3}\left(Q_{\psi \gamma \mathrm{X}}\right) \tag{35}
\end{align*}
$$

It is worth mentioning here that the above partial wave amplitudes for the process $\mathrm{J} / \psi \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ are ap－ plicable to the processes $\mathrm{J} / \psi \rightarrow \gamma \Lambda \bar{\Lambda}, \gamma \Sigma \bar{\Sigma}$ ，and $\gamma \Xi \bar{\Xi}$ as well．

## 4 Monte Carlo simulation for $\mathrm{J} / \Psi \rightarrow$ $\gamma \bar{p}$

We perform Monte Carlo simulation for the de－ cay process $\mathrm{J} / \psi \rightarrow \gamma \mathrm{X} \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ ，with the $J^{P C}$ of X to be $0^{++}, 0^{-+}, 1^{++}, 2^{++}, 2^{-+}$．The predicted angular distributions for various $J^{P C}$ intermediate resonances could serve as a useful reference for people performing partial wave analysis of $\psi \rightarrow \gamma \mathrm{B} \overline{\mathrm{B}}$ channels．

BES II reported the observation of a narrow en－ hancement near $2 m_{\mathrm{p}}$ in the invariant mass spectrum of $\mathrm{p} \overline{\mathrm{p}}$ pairs from radiative $\mathrm{J} / \psi \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ decays．The peak has properties consistent with either a $J^{P C}=$ $0^{-+}$or $0^{++}$．We simulate these two processes with the Breit－Wigner mass and width of X from $\mathrm{BES}^{[5]}$ ． The angular distributions of the photon and proton are shown in Fig．1，while the invariant mass of $\mathrm{p} \overline{\mathrm{p}}$ and momentum distribution of the proton are shown in Fig．2．The two fits of $0^{-+}$and $0^{++}$are indeed hardly distinguishable from these distributions．How－ ever，if the narrow structure is really due to a narrow $0^{-+}$or $0^{++}$resonance，it can be distinguished by its other decay modes．While a $0^{-+}$resonance can de－ cay into $\eta \pi \pi$ ， $\mathrm{K} \overline{\mathrm{K}} \pi$ ，a $0^{++}$resonance cannot．On the other hand，while a $0^{++}$resonance can decay into two pseudoscalar mesons，such as $\pi \pi, \mathrm{K} \overline{\mathrm{K}}$ ，a $0^{-+}$res－ onance cannot．Both $0^{-+}$and $0^{++}$resonances can decay into $4 \pi$ and $\pi \pi \mathrm{K} \overline{\mathrm{K}}$ channels．Previous data on these channels ${ }^{[10-15]}$ have not seen such narrow struc－ ture around $2 m_{\mathrm{p}}$ ．This gives support to the expla－ nation of $\mathrm{p} \overline{\mathrm{p}} 0^{-+}$final state interaction ${ }^{[23,24]}$ for the observed narrow peak structure in $\mathrm{p} \overline{\mathrm{p}}$ channel only．


Fig．1．Angular distributions of the photon and proton in the process $\mathrm{J} / \psi \rightarrow \gamma 0^{ \pm+} \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ ．


Fig．2．Momentum distributions of proton and invariant mass of $\mathrm{p} \overline{\mathrm{p}}$ in the process $\mathrm{J} / \psi \rightarrow$ $\gamma 0^{ \pm+} \rightarrow \gamma p \bar{p}$ ．The dashed curve is the case of $0^{++}$；the solid curve is the case of $0^{-+}$．

The processes with other $J^{P C}$ intermediate X res－ onances are simulated by assuming the mass of X at 2.15 GeV with a width of 0.15 GeV ．The angular distri－ bution of photon and proton are shown in Figs．3－5． The $\theta_{\gamma}$ and $\theta_{\mathrm{p}}$ are given in $\mathrm{J} / \psi$ rest frame and X rest frame，respectively．The $\theta_{\gamma}$ angular distributions coincide with general analytic formulae given in the previous work ${ }^{[19]}$ as it should be．One can see that various partial wave amplitudes give different angular distributions．By fitting the theoretical differential cross section given by Eq．（20）with parameters $\Lambda_{i}$ to the data，one can get the magnitudes of each partial wave contribution．


Fig．3．Angular distributions of the photon and proton for independent amplitudes of the pro－ cess $\mathrm{J} / \psi \rightarrow \gamma 1^{++} \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ ．The solid curve and dashed curve correspond to Eqs．（28）and （29），respectively．


Fig．4．Angular distributions of the photon and proton given by independent amplitudes Eq． （30）（stars），Eq．（31）（solid lines），and Eq． （32）（dashed lines）of the process $\mathrm{J} / \psi \rightarrow$ $\gamma 2^{++} \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ ．


Fig．5．Angular distributions of the photon and proton given by independent amplitudes Eq． （33）（stars），Eq．（34）（solid lines），and Eq． （35）（dashed lines）of the process $\mathrm{J} / \psi \rightarrow$ $\gamma 2^{-+} \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ ．

## 5 Conclusion

In this paper we provided a theoretical formalism and a Monte Carlo study of the partial wave anal－ ysis for the radiative decay $\mathrm{J} / \psi \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ ，which are also applicable to the processes $\mathrm{J} / \psi \rightarrow \gamma \Lambda \bar{\Lambda}, \gamma \Sigma \bar{\Sigma}$ and $\gamma \Xi \bar{\Xi}$ ．We have constructed all possible covariant tensor amplitudes for intermediate resonant states of $J \leqslant 2$ ．For intermediate resonant states of $J \geqslant 3$ ，the production vertices need $L \geqslant 2$ and are expected to be suppressed ${ }^{[9]}$ ．The formulae here can be directly used to perform partial wave analysis of forthcom－ ing high statistics data from CLEO－c and BESIII on these channels to extract useful information on the baryon－antibaryon interactions．

S．Dulat would like to thank Prof．Kaoru Hagi－ wara，who has given productive comments．Both the presentation at KEK and content of this paper have greatly benefited from his insightful comments．

## References

Edwards C．PhD Thesis，Cal．Tech．Preprint CALT－68－ 1165
2 Coffman D，DeJongh F，Dubois G et al．Phys．Rev．，1990， D41： 1410
Augustin J E et al．Orsay Preprint LAL／85－27
BES Collaboration．Nucl．Phys．，2000，A675：337－340
5 BES Collaboration．Phys．Rev．Lett．，2003，91： 022001
6 Rarita W，Schwinger J．Phys．Rev．，1941，60： 61
7 Chung S U．Phys．Rev．，1993，D48：1225；Chung S U． Phys．Rev．，1998，D57： 431
8 Filippini V，Fontana A，Rotondi A．Phys．Rev．，1995，D51： 2247
9 ZOU B S，Bugg D V．Eur．Phys．J．，2003，A16： 537
10 BES Collaboration．Phys．Lett．，1998，B440： 217
11 BES Collaboration．Phys．Lett．，1999，B446： 356
12 BES Collaboration．Phys．Lett．，2000，B472： 200
13 BES Collaboration．Phys．Lett．，2000，B472： 208
14 BES Collaboration．Phys．Lett．，2000，B476： 25
15 BES Collaboration．Phys．Rev．，2003，D68： 052003
16 Anisovich V V，Bugg D V，Sarantsev A V．Nucl．Phys．，

1992，A537：501；Anisovich V V，Kobrinsky M N，Me－ likhov D I et al．Nucl．Phys．，1992，A544： 747
17 Anisovich A V et al．Phys．Lett．，1999，B452：173；Phys． Lett．，1999，B452：180；Phys．Lett．，1999，B452：187； Phys．Lett．，1999，B468：304；Phys．Lett．，1999，B468： 309；Phys．Lett．，2001，B500：222；Phys．Lett．，2001， B507： 23
18 Anisovich A V，Sadovnikova V A．Euro．Phys．J．，1998， A2：199；Anisovich A V et al．J．Phys．，2002，G28： 15
19 Sayipjamal Dulat，ZOU Bing－Song，WU Ji－Min．HEP \＆ NP，2004，28：350－358（in Chinese）
（沙依甫加马力－达吾来提，邹冰松，吴济民．高能物理与核物理， 2004，28：350—358）
20 Greiner W et al．Quantum Chromodynamics．Springer． 2002．242；Behtends E，Fronsdal C．Phys．Rev．，1957，106： 345
21 von Hippel F，Quigg C．Phys．Rev．，1972，D5： 624
22 ZOU B S，Hussain F．Phys．Rev．，2003，C67： 015204
23 ZOU B S，Chiang H C．Phys．Rev．，2004，D69： 034004
24 Kerbikov B，Stavinsky A，Fedotov V．Phys．Rev．，2004， C69： 055205

# 基于协变张量方法的 $\psi$ 衰变到 $\gamma \mathrm{B} \overline{\mathrm{B}}$ 的分波分析 ${ }^{*}$ 

沙依甫加马力•达吾来提 ${ }^{2,3 ; 1)}$ 刘伯超 ${ }^{2,4 ; 2)}$ 邹冰松 ${ }^{1,2}$ 吴济民 ${ }^{1,2}$<br>1 （中国高等科技中心 北京 100080）<br>2 （中国科学院高能物理研究所 北京 100049）<br>3 （新疆大学物理系 乌鲁木齐 830046）<br>4 （中国科学院研究生院 北京 100049）

摘要 最近BESII合作组在 $\mathrm{p} \overline{\mathrm{p}}$ 不变质量谱阈值附近观测到一个增强。利用协变张量方法，给出了 $\psi$ 辐射衰变道 $\psi \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}, \gamma \Lambda \bar{\Lambda}, \gamma \Sigma \bar{\Sigma}, \gamma \Xi \Xi$ 分波分析的理论公式。通过蒙特卡罗模拟，还给出了 $\mathrm{J} / \psi \rightarrow \gamma \mathrm{p} \overline{\mathrm{p}}$ 中光子和质子的角分布，它们可以作为将来对这些道分波分析的有用参考。

关键词 分波振幅 $J / \psi \quad \gamma B \bar{B}$

[^1]
[^0]:    Received 25 August 2005，Revised 28 November 2005
    ＊Supported by NSFC（10465004，10225525，10055003，90103012，10265003），Xinjiang University and Knowledge Innovation Project of CAS（KJCX2－SW－N02）

    1）E－mail：dulat98＠yahoo．com
    2）E－mail：liubc＠ihep．ac．cn

[^1]:    2005－08－25 收稿，2005－11－28 收修改稿
    ＊国家自然科学基金 $(10465004,10225525,10055003,90103012,10265003$ ），新疆大学和中国科学院知识创新工程重大项目（KJCX2－ SW－N02）资助

    1）E－mail：dulat98＠yahoo．com
    2）E－mail：liubc＠ihep．ac．cn

