# Constraining Vectorlike Quark Model from B Radiative Decays 

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#### Abstract

In the $S U(2)$ singlet down type vectorlike quark model，there exist a tree level coupling $z_{\mathrm{sb}}$ of $\mathrm{b} \rightarrow \mathrm{sZ}^{*}$ and an additional D quark．In the framework we evaluate the D quark effects on $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \gamma$ by running the Wilson coefficients of the effective Hamiltonian with the renormalization scale from $m_{\mathrm{D}}$ to weak scale．Using the recent measurements for $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}$，we extract rather stringent constraints on the size and CP violating phase of $z_{\mathrm{sb}}$ ，and find that the zero point of the forward－backward asymmetry may have large deviation from that of the standard model and is very sensitive to $z_{\mathrm{sb}}$ ，and therefore，it can be useful in probing the new physics．


Key words B radiative decays，tree level coupling of bsZ，vectorlike quark model

## 1 Introduction

The rare radiative decays $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \gamma$ and $\mathrm{B} \rightarrow$ $\mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}$are sensitive probes of new physics ${ }^{[1]}$ ．Unlike in the standard model（SM）where flavor changing neutral currents（FCNC）arise only at loop level，in the vector quark model（VQM）${ }^{[2,3]}$ ，the CKM matrix is necessarily non－unitarity，leading to interaction $\mathrm{Z} \overline{\mathrm{s}} \mathrm{b}$ at tree level，and hence potentially large new physics contributions can be expected．

There are some studies regarding the constraints on model with extra singlet quark ${ }^{[2,3]}$ ．In this work， （i）we first integrate out the heavy D quark．New op－ erators are introduced for $\mathrm{b} \rightarrow \mathrm{s} \gamma$ ；（ii）since vectorlike down－type quark contributions to $\mathrm{b} \rightarrow \mathrm{s} \gamma$ just occur at loop level as the case of the SM，the constraints from $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \gamma$ on $z_{\mathrm{sb}}$ ，the tree level FCNC coupling for $\mathrm{b} \rightarrow \mathrm{sZ}$ ，are less restrictive compared to those from those processes governed by $\mathrm{b} \rightarrow \mathrm{sl}^{+} \mathrm{l}^{-}$transition．Re－ cently，the rare decays $B \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}(\mathrm{l}=\mathrm{e}, \mu)$ also have been measured by BaBar and Belle ${ }^{[4,5]}$ ．In light of the improvements mentioned above，it is necessary to present a comprehensive analysis in this model．

## $2 \mathrm{~b} \rightarrow \mathrm{~s} \gamma$ and $\mathrm{b} \rightarrow \mathrm{sl}^{+} \mathrm{l}^{-}$transitions in vectorlike quark model

In VQM the difference between the new quark and ordinary quarks of the three SM generations is that both the left－and right－handed components of the former quark are $S U(2)$ singlets，leading to non－ unitarity as

$$
\begin{align*}
z_{\alpha \beta} \equiv & \sum_{i=1}^{3} V^{\alpha i} V^{\beta i *}=\sum_{i=1}^{3} V_{\mathrm{CKM}}^{i \alpha *} V_{\mathrm{CKM}}^{i \beta}= \\
& \delta_{\alpha \beta}-V^{\alpha 4} V^{\beta 4 *}, \tag{1}
\end{align*}
$$

where the matrix $V_{\text {CKM }}$ is enlarged to $3 \times 4$ and $V$ is a $4 \times 4$ unitary matrix which relates the weak－eigenstates $\tilde{q}_{\mathrm{L}}$ to mass－eigenstates $\mathrm{q}_{\mathrm{L}}$ ．The deviations from the standard unitary triangles are going to vanish as the down type singlet mass increases compared with elec－ troweak breaking scale $v$ ．The relevant interaction Lagrangian can be found in Ref．［2］．

In VQM，the down－type vector quark may be much heavier than weak scale．In a theory with differ－ ent mass scales，the heavier scale should be integrated out firstly，then Wilson coefficients are run with the

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renormalization scale from heavier scale to low scale by using renormalization group equation．Only when $m_{\mathrm{D}}$ is about the weak scale，can W，Z boson，Higgs boson and top quark be integrated out together．In this work，we consider two possibilities as follows：

Scenario A：$m_{\mathrm{W}}^{2} \ll m_{\mathrm{D}}^{2}$
By keeping only the leading order terms of $\delta_{\mathrm{D}}=$ $m_{\mathrm{W}}^{2} / m_{\mathrm{D}}^{2}$ ，we obtain the effective Hamiltonian for $\mathrm{b} \rightarrow \mathrm{s} \gamma^{*}\left(\mathrm{~g}^{*}\right)$ as：

$$
\begin{equation*}
\mathscr{H}_{\mathrm{eff}}^{\text {new }}\left[\mathrm{b} \rightarrow \gamma^{*}\left(\mathrm{~g}^{*}\right)\right]=\frac{4 G_{\mathrm{F}}}{\sqrt{2}} z_{4 \mathrm{~s}}^{*} z_{4 \mathrm{~b}} \sum_{i} C_{i}(\mu) \mathscr{O}_{i}(\mu) \tag{2}
\end{equation*}
$$

A complete basis for the local operators is listed below ${ }^{1)}$ ：

$$
\begin{align*}
\mathscr{O}_{\mathrm{LR}}^{1} & =-\frac{1}{16 \pi^{2}} m_{\mathrm{b}} \bar{s}_{\mathrm{L}} \mathscr{D}^{2} b_{\mathrm{R}}, \\
\mathscr{O}_{\mathrm{LR}}^{2} & =\frac{1}{16 \pi^{2}} g_{\mathrm{s}} m_{\mathrm{b}} \bar{s}_{\mathrm{L}} \sigma^{\mu \nu} T^{a} b_{\mathrm{R}} G_{\mu \nu}, \\
\mathscr{O}_{\mathrm{LR}}^{3} & =\frac{1}{16 \pi^{2}} e e_{\mathrm{d}} m_{\mathrm{b}} \bar{s}_{\mathrm{L}} \sigma^{\mu \nu} b_{\mathrm{R}} F_{\mu \nu}, \\
Q_{\mathrm{LR}}^{\phi^{0}} & =\frac{1}{2} g_{\mathrm{s}}^{2} m_{\mathrm{b}} \phi^{0} \phi^{0} \bar{s}_{\mathrm{L}} b_{\mathrm{R}}  \tag{3}\\
P_{\mathrm{L}}^{1, A} & =-\mathrm{i} \frac{1}{16 \pi^{2}} \bar{s}_{\mathrm{L}} T_{\mu \nu \sigma}^{A} \mathscr{D}^{\mu} \mathscr{D}^{\nu} \mathscr{D}^{\sigma} b_{\mathrm{R}}, \\
P_{\mathrm{L}}^{2} & =\frac{1}{16 \pi^{2}} \bar{s}_{\mathrm{L}} \gamma^{\mu} b_{\mathrm{R}} \partial^{\nu} F_{\mu \nu}, \\
R_{\mathrm{L}}^{1, \phi^{0}} & =\mathrm{i} \frac{1}{2} g_{\mathrm{s}}^{2} \phi^{0} \phi^{0} \bar{s}_{\mathrm{L}} \not \mathscr{D} b_{\mathrm{L}}, \\
R_{\mathrm{L}}^{2, \phi^{0}} & =\mathrm{i} g_{\mathrm{s}}^{2}\left(\partial^{\sigma} \phi^{0}\right) \phi^{0} \bar{s}_{\mathrm{L}} \gamma_{\sigma} b_{\mathrm{L}},
\end{align*}
$$

where $T^{a}$ stands for the $S U(3)_{\text {color }}$ generator，$F_{\mu \nu}$ and $G_{\mu \nu}$ are field strengths of photon and gluon respec－ tively． $\mathscr{D}_{\mu}=\partial_{\mu}-\mathrm{i} g_{\mathrm{s}} G_{\mu}^{a} T^{a}-\mathrm{i} e e_{\mathrm{d}} A_{\mu}$ is the covariant derivative．$\phi^{0}=H^{0}, \mathrm{G}^{0}$ ，and $\mathrm{G}^{0}$ stands for Goldstone boson．The tensors $T_{\mu \nu \sigma}^{A}(A=1,2,3,4)$ appearing in $P_{\mathrm{L}}^{1, A}$ have the following Lorentz structures ${ }^{[6]}$ ：

$$
\begin{array}{lc}
T_{\mu \nu \sigma}^{1}=g_{\mu \nu} \gamma_{\sigma}, & T_{\mu \nu \sigma}^{2}=g_{\mu \sigma} \gamma_{\nu}  \tag{4}\\
T_{\mu \nu \sigma}^{3}=g_{\nu \sigma} \gamma_{\mu}, & T_{\mu \nu \sigma}^{4}=-\mathrm{i} \epsilon_{\mu \nu \sigma} \gamma^{\tau} \gamma_{5}
\end{array}
$$

The coefficients of operators $Q_{\mathrm{LR}}$ and $R_{\mathrm{L}}$ at scale $m_{\mathrm{D}}$ can be obtained by matching the diagrams of full theory with effective theory at tree level while for those of $\mathscr{O}_{\mathrm{LR}}, P_{\mathrm{L}}$ ，by matching one loop diagrams shown in Fig．1．They read

$$
C_{Q_{\mathrm{LR}}^{\mathrm{H}^{0}}}\left(m_{\mathrm{D}}\right)=-C_{Q_{\mathrm{LR}}^{\chi^{0}}}\left(m_{\mathrm{D}}\right)=-\frac{1}{g_{\mathrm{s}}^{2}},
$$

$$
\begin{align*}
C_{R_{\mathrm{L}}^{1, \phi^{0}}}\left(m_{\mathrm{D}}\right) & =2 C_{R_{\mathrm{L}}^{2, \phi^{0}}}\left(m_{\mathrm{D}}\right) \\
C_{O_{\mathrm{LR}}^{i}}\left(m_{\mathrm{D}}\right) & =-\frac{1}{2 g_{\mathrm{s}}^{2}} \\
C_{P_{\mathrm{L}}^{1,2}}\left(m_{\mathrm{D}}\right) & =\frac{8}{9}, C_{P_{\mathrm{L}}^{1,1}}\left(m_{\mathrm{D}}\right)=C_{P_{\mathrm{L}}^{1,4}}\left(m_{\mathrm{D}}\right)=-\frac{1}{2}, C_{P_{\mathrm{L}}}^{2}\left(m_{\mathrm{D}}\right)=-\frac{11}{18}, \tag{5}
\end{align*}
$$

where $i$ runs from 1 to 3 ．The values for operators $O$ ， $P$ are understood as the sum of $\mathrm{H}^{0}$ and $\mathrm{G}^{0}$ contribu－ tions．


Fig．1．Matching conditions at scale $m_{\mathrm{D}}$ in full theory（left）and in the intermediate effective field theory（right）．Note that $\phi^{0}$ in the sec－ ond line of diagrams is not integrated out yet．

To obtain the coefficients of the operators at weak scale，we extract the anomalous dimensions by calcu－ lating one－loop diagrams in unitary gauge with oper－ ator insertions．Then we solve renormalization group equation（RGE）and have the coefficients at $m_{\mathrm{W}}$ scale as follows：

$$
\begin{align*}
C_{O_{\mathrm{LR}}^{1}}= & \frac{247}{548} \zeta^{-\frac{4}{21}}+\frac{336}{8905} \zeta^{\frac{113}{126}}-\frac{511}{780} \zeta^{\frac{8}{21}}+\frac{1}{6} \zeta^{\frac{2}{3}}, \\
C_{O_{\mathrm{LR}}^{2}}= & \frac{247}{1096} \zeta^{-\frac{4}{21}}+\frac{168}{8905} \zeta^{\frac{113}{126}}-\frac{223}{780} \zeta^{\frac{8}{21}}+\frac{1}{24} \zeta^{\frac{2}{3}}, \\
C_{O_{\mathrm{LR}}^{3}}= & \frac{247}{1096} \zeta^{-\frac{4}{21}}+\frac{168}{8905} \zeta^{\frac{113}{126}}-\frac{223}{780} \zeta^{\frac{8}{21}}+ \\
& \frac{5}{12} \zeta^{\frac{2}{3}}-\frac{3}{8} \zeta^{\frac{16}{21}}  \tag{6}\\
C_{P_{\mathrm{L}}^{1,1}}= & C_{P_{\mathrm{L}}^{1,3}}=-\frac{247}{548} \zeta^{-\frac{4}{21}}-\frac{791}{4932} \zeta^{\frac{113}{126}}, \\
C_{P_{\mathrm{L}}^{1,2}}= & \frac{247}{548} \zeta^{-\frac{4}{21}}+\frac{1}{12} \zeta^{\frac{8}{21}}+\frac{875}{2466} \zeta^{\frac{113}{63}}, \\
C_{P_{\mathrm{L}}^{1,4}}= & -\frac{247}{598} \zeta^{-\frac{8}{21}}-\frac{1}{12} \zeta^{\frac{8}{21}}+\frac{14}{411} \zeta^{\zeta^{113}}, \\
C_{P_{\mathrm{L}}^{2}}= & 0,
\end{align*}
$$

where $\zeta=\alpha_{\mathrm{s}}\left(m_{\mathrm{D}}\right) / \alpha_{\mathrm{s}}\left(m_{\mathrm{W}}\right)$ ．
To match the operator set in（3）onto these oper－ ators obtained by integrating out the $\mathrm{W}, \mathrm{Z}$ bosons， Goldstone boson，Higgs boson and top quark as in SM，we use the equations of motion to reduce all the

1）Strictly speaking，$\phi^{0}$ in Fig． 1 can be $Z^{0}$ boson，which indicates that there exists operator $Z_{\mu} Z^{\mu} \bar{s}_{\mathrm{R}} b_{\mathrm{L}}$ ．However，since its coefficient is suppressed by large scale $m_{\mathrm{D}}$ ，its contribution can be neglected safely in Scenario A
remaining two－quark operators to the gluon and pho－ ton magnetic moment operators $\mathscr{O}_{\mathrm{LR}}^{2}$ and $\mathscr{O}_{\mathrm{LR}}^{3}$ which are redefined as $\mathscr{Q}_{8 G}$ and $\mathscr{Q}_{7 \gamma}$ ．Now the effective Hamiltonian describing $\mathrm{b} \rightarrow \mathrm{sl}^{+} \mathrm{l}^{-}$transition reads ${ }^{[7]}$

$$
\begin{align*}
\mathscr{H}_{\text {eff }}= & -\frac{4 G_{\mathrm{F}}}{\sqrt{2}} K_{\mathrm{tb}} K_{\mathrm{ts}}^{*}\left[\sum_{i=1}^{10} \widetilde{C}_{i}(\mu) \mathscr{Q}_{i}(\mu)+\right. \\
& \widetilde{C}_{7 \gamma}(\mu) \mathscr{Q}_{7 \gamma}(\mu)+\widetilde{C}_{8 \mathrm{G}}(\mu) \mathscr{Q}_{8 \mathrm{G}}(\mu)+ \\
& \left.\mathscr{C}_{9}(\mu) \mathscr{O}_{9}(\mu)+\mathscr{C}_{10}(\mu) \mathscr{O}_{10}(\mu)\right] \tag{7}
\end{align*}
$$

where $\mathscr{Q}_{i}(i=1-10)$ are four－quark operators，$K_{i j} \equiv$ $\left(V_{\text {CKM }}\right)_{i j}$ for $i, j=1,2,3$ ．

Now we rewrite the Wilson coefficients as $C=$ $C^{\mathrm{SM}}+C^{\text {new }}$ where $C^{\mathrm{SM}}$ stands for that of $\mathrm{SM}^{[7]}$ while $C^{\text {new }}$ denotes the deviation of the values between VQM and SM．After straightforward calculations，at $m_{\mathrm{W}}$ scale，we have non－vanishing coefficients for new physics contributions：

$$
\begin{align*}
\widetilde{C}_{2}^{\text {new }}= & -\kappa, \quad \widetilde{C}_{3}\left(m_{\mathrm{W}}\right)=-\frac{1}{6} \kappa, \\
\widetilde{C}_{7}^{\text {new }}= & -\frac{2}{3} \sin ^{2} \theta_{\mathrm{W}} \kappa, \quad \widetilde{C}_{9}^{\text {new }}=-\frac{2}{3} \cos ^{2} \theta_{\mathrm{W}} \kappa, \\
\widetilde{C}_{7 \gamma}^{\mathrm{new}}= & {\left[\frac{23}{36}-\frac{1}{4} e_{\mathrm{d}}\left(C_{O_{\mathrm{LR}}^{1}}-4 C_{O_{\mathrm{LR}}^{3}}+C_{P_{\mathrm{L}}^{1,1}}+\right.\right.} \\
& \left.\left.C_{P_{\mathrm{L}}^{1,2}}-C_{P_{\mathrm{L}}^{1,4}}\right)+e_{\mathrm{d}}\left(\frac{1}{3}+\frac{1}{9} \sin ^{2} \theta_{\mathrm{W}}\right)\right] \kappa, \\
\widetilde{C}_{8 \mathrm{G}}^{\mathrm{new}}= & {\left[\frac{1}{3}-\frac{3}{4} e_{\mathrm{d}}\left(C_{O_{\mathrm{LR}}^{1}}-4 C_{O_{\mathrm{LR}}^{3}}+C_{P_{\mathrm{L}}^{1,1}}+\right.\right.} \\
& \left.\left.C_{P_{\mathrm{L}}^{1,2}}-C_{P_{\mathrm{L}}^{1,4}}\right)-3 e_{\mathrm{d}}\left(\frac{1}{3}+\frac{1}{9} \sin ^{2} \theta_{\mathrm{W}}\right)\right] \kappa, \\
\mathscr{C}_{9}^{\mathrm{new}}= & \frac{\pi}{\alpha_{\mathrm{em}}} \kappa\left(-1+4 \sin ^{2} \theta_{\mathrm{W}}\right), \quad \mathscr{C}_{10}^{\mathrm{new}}=\frac{\pi}{\alpha_{\mathrm{em}}} \kappa, \tag{8}
\end{align*}
$$

where $\kappa \equiv \frac{z_{\mathrm{sb}}}{K_{\mathrm{tb}} K_{\mathrm{ts}}^{*}}$ ．In deriving the above equation， we have used the unitarity relation $z_{4 \mathrm{~b}} z_{4 \mathrm{~s}}^{*}=-z_{\mathrm{sb}}$ which is a direct result of Eq．（1）．


Fig．2．Tree level Feynman diagram contribut－ ing to $\mathrm{b} \rightarrow \mathrm{s} \gamma$ and $\mathrm{b} \rightarrow \mathrm{sl}^{+} \mathrm{l}^{-}$．

At this moment，we would like to point out that these new contributions have different sources．The terms proportional to $\frac{z_{\mathrm{sb}}}{K_{\mathrm{tb}} K_{\mathrm{ts}}^{*}}$ in $\widetilde{C}_{3,7,9}$ and $\mathscr{C}_{9,10}$ come from the tree－level diagram as displayed in Fig． 2 whereas in $\widetilde{C}_{2}$ ，from tree diagram due to the non－
unitarity of CKM matrix in $\mathrm{VQM}^{[2]}$ ．In expressions of $\widetilde{C}_{7 \gamma, 8 \mathrm{G}}$ ，the first constant term comes from the charged current one－loop diagrams due to the non－ unitarity of CKM matrix in VQM，other terms pro－ portional to $\frac{z_{\mathrm{sb}}}{K_{\mathrm{tb}} K_{\mathrm{ts}}^{*}}$ come from the neutral current one－loop diagrams．

Scenario B：$m_{\mathrm{W}}^{2} \sim m_{\mathrm{D}}^{2}$
In this scenario，the t and D quark， W and Z can be integrated out together．The corresponding initial values of Wilson coefficients $\widetilde{C}_{7 \gamma, 8 G}\left(m_{\mathrm{W}}\right)$ are changed to ${ }^{[2]}$

$$
\begin{align*}
\widetilde{C}_{7 \gamma}^{\text {new }}= & {\left[\frac{23}{36}+\left(f_{\mathrm{D}}^{\mathrm{Z}}\left(y_{\mathrm{D}}\right)+f_{\mathrm{D}}^{\mathrm{H}}\left(w_{\mathrm{D}}\right)+f_{\mathrm{D}}^{\mathrm{G}}\left(y_{\mathrm{D}}\right)\right)+\right.} \\
& \left.e_{\mathrm{d}}\left(\frac{1}{3}+\frac{1}{9} \sin ^{2} \theta_{\mathrm{W}}\right)\right] \kappa, \\
\widetilde{C}_{8 \mathrm{G}}^{\mathrm{new}}= & {\left[\frac{1}{3}-3\left(f_{\mathrm{D}}^{\mathrm{Z}}\left(y_{\mathrm{D}}\right)+f_{\mathrm{D}}^{\mathrm{H}}\left(w_{\mathrm{D}}\right)+f_{\mathrm{D}}^{\mathrm{G}}\left(y_{\mathrm{D}}\right)\right)-\right.}  \tag{9}\\
& \left.3 e_{\mathrm{d}}\left(\frac{1}{3}+\frac{1}{9} \sin ^{2} \theta_{\mathrm{W}}\right)\right] \kappa,
\end{align*}
$$

where $y_{\mathrm{D}}=m_{\mathrm{D}}^{2} / m_{Z}^{2}, w_{\mathrm{D}}=m_{\mathrm{D}}^{2} / m_{\mathrm{H}}^{2}$ ．Other Wilson coefficients are the same as Scenario A we discussed． The function $f_{\mathrm{y}}^{\mathrm{x}}$ stands for the contribution from bo－ son x mediated penguin one－loop diagram with quark y in loops．They have forms as

$$
\begin{align*}
& f_{\mathrm{D}}^{\mathrm{Z}}(x)=-\frac{5 x^{2}+5 x-4}{72(x-1)^{3}}+\frac{x(2 x-1)}{12(x-1)^{4}} \ln x, \\
& f_{\mathrm{D}}^{\mathrm{H}}(x)=-e_{\mathrm{d}} x\left[\frac{7 x^{2}-29 x+16}{48(x-1)^{3}}+\frac{3 x-2}{8(x-1)^{4}} \ln x\right], \\
& f_{\mathrm{D}}^{\mathrm{G}}(x)=e_{\mathrm{d}} x\left[\frac{5 x^{2}-19 x+20}{48(x-1)^{3}}+\frac{x-2}{8(x-1)^{4}} \ln x\right], \tag{10}
\end{align*}
$$

As a consistency check，in Scenario B in limit of $m_{\mathrm{D}} \gg m_{\mathrm{W}}$ ，from Eqs．（9），（10）one can infer that the term $f_{\mathrm{D}}^{\mathrm{Z}}\left(y_{\mathrm{D}}\right)+f_{\mathrm{D}}^{\mathrm{H}}\left(w_{\mathrm{D}}\right)+f_{\mathrm{D}}^{\mathrm{G}}\left(y_{\mathrm{D}}\right) \rightarrow \frac{1}{24} e_{\mathrm{d}}$ ，which is the value of the term $-\frac{1}{4} e_{\mathrm{d}}\left[C_{O_{\mathrm{LR}}^{1}}-4 C_{O_{\mathrm{LR}}^{3}}+C_{P_{\mathrm{L}}^{1,1}}+\right.$ $\left.C_{P_{\mathrm{L}}^{1,2}}-C_{P_{\mathrm{L}}^{1,4}}\right]$ in（8）if the QCD running of the co－ efficients with the renormalization scale from $m_{\mathrm{D}}$ to $m_{\mathrm{W}}$ is negligible．Therefore，under the approxima－ tion，values of $\widetilde{C}_{7 \gamma}\left(m_{\mathrm{W}}\right)$ and $\widetilde{C}_{8 \mathrm{G}}\left(m_{\mathrm{W}}\right)$ in Scenario A would be equivalent to those in Scenario B．Our calcu－ lation also shows that the running effect on the parts from neutral current in Scenario A is large；however， at $m_{\mathrm{W}}$ scale，since the new dominant contribution comes from the charged current diagrams，the total
values of $\widetilde{C}_{7 \gamma, 8 \mathrm{G}}\left(m_{\mathrm{W}}\right)$ are changed slightly．Since the coefficients are insensitive to the mass of $m_{\mathrm{D}}$ ，in the follows we will focus on Scenario A and study how to constrain the interaction coupling of Z FCNC $z_{\mathrm{sb}}$ in VQM using B radiative decays．

## 3 Constraints on $z_{\mathrm{sb}}$ in VQM from $\mathbf{B} \rightarrow \mathbf{X}_{\mathrm{s}} \mathbf{1}^{+} \mathbf{l}^{-}$

Now we constrain the parameter $z_{\mathrm{sb}}$ from $\mathrm{B} \rightarrow$ $\mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}$．The invariant dilepton distribution is

$$
\begin{align*}
& \frac{\mathrm{d} \Gamma\left(\mathrm{~B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}\right)}{\mathrm{d} s}=\left(\frac{\alpha_{\mathrm{em}}}{4 \pi}\right)^{2} \frac{G_{\mathrm{F}}^{2} m_{\mathrm{b}}^{5}\left|K_{\mathrm{ts}}^{*} K_{\mathrm{tb}}\right|^{2}}{48 \pi^{3}}(1-s) R_{0} \\
& R_{0}= \\
& 4\left(1+\frac{2}{s}\right)\left|\widetilde{C}_{7 \gamma}^{\mathrm{eff}}\right|^{2}+(1+2 s)\left(\left|\mathscr{C}_{9}^{\mathrm{eff}}\right|^{2}+\left|\mathscr{C}_{10}^{\mathrm{eff}}\right|^{2}\right)+  \tag{11}\\
& \\
& \quad 12 \operatorname{Re}\left(\widetilde{C}_{7 \gamma}^{\mathrm{eff}} \mathscr{C}_{9}^{\mathrm{eff} *}\right)
\end{align*}
$$

It depends on the tree level FCNC coupling $z_{\text {sb }}$ and $\left|K_{\mathrm{tb}} K_{\mathrm{ts}}\right|$ which is determined by

$$
\begin{equation*}
\left|K_{\mathrm{tb}} K_{\mathrm{ts}}^{*}\right| \simeq\left|K_{\mathrm{cb}} K_{\mathrm{cs}}^{*}\right|+\left|z_{\mathrm{sb}}\right| \cos \theta, \quad \theta=\arg (\kappa) \tag{12}
\end{equation*}
$$

where $\left|K_{\mathrm{cb}} K_{\mathrm{cs}}\right|$ can be used as input of the direct experimental values ${ }^{[8]}$ ．

There is some progress in predicting $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}$． The complete computations of NLL and NNLL pre－ cision of the decay for small dilepton mass can be found in［9］and［10］，respectively．Recently，the first calculation of the NNLL contributions for arbitrary dilepton invariant mass is also available ${ }^{[11]}$ ．For con－ sistency，we use NLL prediction ${ }^{[9]}$ in our calculation， and exclude the resonances $\mathrm{J} / \psi, \psi^{\prime}$ contributions by using the same cuts as experiments ${ }^{[5]}$ so we can com－ pare our predication with experiments．In addition， we also consider theoretical errors which come mainly from the uncertainties of $m_{\mathrm{t}}, m_{\mathrm{b}}$ and $m_{\mathrm{c}} / m_{\mathrm{b}}{ }^{[8]}$ ，then combine the experimental and theoretical relative er－ rors together．

Using the current average value for $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-[12]}$

$$
\begin{equation*}
\mathscr{B} 2^{\mathrm{ex}}\left(\mathrm{~B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}\right)=\left(6.2 \pm 1.1_{-1.3}^{+1.6}\right) \times 10^{-6}, \tag{13}
\end{equation*}
$$

in Fig． 3 we display the corresponding $2 \sigma$ experimen－ tal bounds on the size of $z_{\mathrm{sb}}$ and phase $\theta$ ．From this figure，we obtain

$$
\begin{equation*}
\left|z_{\mathrm{sb}}\right| \leqslant 1.40 \times 10^{-3}(95 \% \text { C.L. }) \tag{14}
\end{equation*}
$$



Fig．3．The $\left(\left|z_{\mathrm{sb}}\right|, \theta\right)$ contour in Scenario A con－ strained by $\mathscr{B} \imath^{\text {ex }}\left(\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} 1^{+} \mathrm{l}^{-}\right)$．The dashed， dotted lines correspond to $1 \sigma$ and $2 \sigma$ experi－ mental bounds of $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}$in（13），respec－ tively．The solid line denotes the experimental central value $6.2 \times 10^{-6}$ ．The region between dot lines is allowed at $1 \sigma$ level．

Now we turn to study the correlation of the branching ratios of $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \gamma^{[13]}$ and $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}$pre－ dicted in VQM．Our numerical result shows that（1） $\mathscr{B} \imath\left(\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \gamma\right)$ is not so sensitive to the phase，which is not the case for $\mathscr{B} \imath\left(\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}\right)$as stated earlier； （2）within the experimental bounds of $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}$， the corresponding branching ratio of $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \gamma$ pre－ dicted in VQM is consistent with the current average of the $\mathrm{CLEO}^{[14]}$ and Belle ${ }^{[15]}$ measurements

$$
\begin{equation*}
\mathscr{B} \imath^{\mathrm{ex}}\left(\mathrm{~B} \rightarrow \mathrm{X}_{\mathrm{s}} \gamma\right)=(3.3 \pm 0.4) \times 10^{-4} \tag{15}
\end{equation*}
$$

It is very interesting to analyze how the zero of the forward－backward（FB）asymmetry $\left(s_{0}\right)$ is modi－ fied in VQM which is determined by equation

$$
\begin{equation*}
\operatorname{Re}\left[\left(s_{0} \mathscr{C}_{9}^{\mathrm{eff}}+2 \mathscr{C}_{7 \gamma}^{\mathrm{eff}}\right) \mathscr{C}_{10}^{\mathrm{eff} *}\right]=0 \tag{16}
\end{equation*}
$$

Unlike the case of SM where $\mathscr{C}_{10}^{\text {eff }}$ is real，the coeffi－ cient $\mathscr{C}_{10}^{\text {eff }}$ is complex generally in VQM．Furthermore， the contributions to $\mathscr{C}_{9,10}^{\text {eff }}$ from tree level FCNC dia－ gram

$$
\begin{equation*}
\left|\mathscr{C}_{10}^{\text {new }}\right| \gg\left|\mathscr{C}_{9}^{\text {new }}\right| \tag{17}
\end{equation*}
$$

and subject to the constraints on $z_{\text {sb }}$ from $\mathrm{B} \rightarrow$ $\mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-},\left|\mathscr{C}_{10}^{\text {new }}\right|$ still can be larger $\left|\mathscr{C}_{10}^{\mathrm{SM}}\right|$ ，indicating that $\mathscr{C}_{10}^{\text {eff }}$ can have large imaginary part．Therefore， $s_{0}$ in VQM will have large deviation from that in SM．


Fig．4．The $\left(\left|z_{\mathrm{sb}}\right|, s_{0}\right)$ contour in VQM subject to the $1 \sigma$ bounds of $\mathscr{B} \imath^{e x}\left(\mathrm{~B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{I}^{+} \mathrm{l}^{-}\right)$．For specified $\left|z_{\mathrm{sb}}\right|$ ，the phase $\theta$ effect on $s_{0}$ is also shown．
Fig． 4 indicates that，subject to the experimental
measurement for branching ratio of $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}$，the zero point of FB asymmetry is very sensitive to the parameters $z_{\mathrm{sb}}$ and phase $\theta$ ，especially in the region $0.6 \times 10^{-3}<\left|z_{\mathrm{sb}}\right|<1.2 \times 10^{-3}$ ．

Considering that the B factories such as BaBar and Belle are running，measurements for inclusive and exclusive B decays with high precision are ex－ pected．Therefore，the VQM will be tested in the near future．

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# B 稀有辐射衰变对似矢量夸克模型约束的研究 

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摘要 在似矢量夸克模型中，具有一个为 $S U(2)$ 单态的下夸克 D 以及树图 $\mathrm{b} \rightarrow \mathrm{sZ} \mathrm{Z}^{*}$ 相互作用 $z_{\mathrm{sb}}$ 。在此框架下， （1）通过考察描述 $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \gamma$ 的Wilson系数随重正化标度从 $m_{\mathrm{D}}$ 到弱标度的跑动，研究了 D 夸克对此衰变的影响。 （2）使用最近对 $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{s}} \mathrm{l}^{+} \mathrm{l}^{-}$的测量，获得了对 $z_{\mathrm{sb}}$ 相当严格的限制。发现前后不对称的零点值可以与标致模型的预言有大的偏离，并对 $z_{\mathrm{sb}}$ 十分敏感，可以用来探测新物理。

关键词 B 稀有辐射衰变 树图相互作用 bsZ 似矢量夸克模型

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