第 30 卷 第 3 期 高 能 物 理 与 核 物 理 Vol. 30, No. 3
2006 年 3 月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS Mar., 2006

Wake Field Calculation Studies in Constant Gradient Accelerating Structures

PEI Shi-Lun[1,2;1)] WANG Shu-Hong[1]

1 (Institute of High Energy Physics, CAS, Beijing 100049, China)

2 (Graduate School of the Chinese Academy of Sciences, Beijing 100049, China)

Abstract When charged particle beams traverse in an accelerating structure, the excited wake fields may dilute the beam energy spread and emittance. In order to estimate the wake field effects on the beam dynamics properly, it is very important to calculate the wake fields precisely. In this paper, the calculation methods of the short range, middle long range and super long range wake fields in constant gradient structures are systematically studied, and the expression of the super long range longitudinal wake field is first given. In addition, the wake fields for the BEPCⅡ Linac's disk-loaded structures are calculated by using analytical methods and various computer codes, the consistency and some deviations of the results obtained by different calculation methods are also discussed.

Key words wake field, constant accelerating structure, loss factor, kick factor

1 Introduction

When charged particle beams traverse in a disk-loaded structure, the wake fields induced by the head particles in a bunch (or the upstream bunches in a bunch train) will act on the tail particles in a bunch (or the downstream bunches in a bunch train), which may deteriorate the beam qualities. Longitudinal wake field can affect the energy and energy spread and transverse wake field can cause emittance dilution.

In order to estimate the wake field effects on the beam performance properly, it is very important to evaluate the wake fields in the accelerating structures precisely. In this paper, the calculation methods of the short range, middle long range and super long range wake fields in constant gradient structures are systematically studied, and the expression of super long range longitudinal wake field is first given. In addition, the wake fields of the BEPCⅡ Linac's accelerating structures are calculated by using analytical methods and various computer codes, such as ABCI[1] and MAFIA[2], the consistency and some deviations of the results obtained by different calculation methods are also discussed.

2 Basic equations

When a point charge travels through a disk loaded structure, the wake potentials can be written as the sum over all of the synchronous cavity modes[3—6], i.e.,

$$W_{||}(\boldsymbol{r},\boldsymbol{r}_{\mathrm{q}};s) = \sum_{m\geqslant 0}\left(\frac{r}{a}\right)^m\left(\frac{r_{\mathrm{q}}}{a}\right)^m\cos(m\varphi)\times \sum_n 2k_{\mathrm{mn}}\mathrm{e}^{-\frac{\omega_{\mathrm{mn}}s}{2Q_{\mathrm{mn}}\beta c}}\cos\left(\frac{\omega_{\mathrm{mn}}s}{\beta c}\right), \quad (1)$$

$$W_{\perp}(\boldsymbol{r},\boldsymbol{r}_{\mathrm{q}};s) = \sum_{m\geqslant 1}\left(\frac{r}{a}\right)^{m-1}\left(\frac{r_{\mathrm{q}}}{a}\right)^m(\boldsymbol{r}\cos(m\varphi)-\boldsymbol{\varphi}\sin(m\varphi))\times\sum_n 2\frac{cmk_{\mathrm{mn}}}{a\omega_{\mathrm{mn}}}\mathrm{e}^{-\frac{\omega_{\mathrm{mn}}s}{2Q_{\mathrm{mn}}\beta c}}\sin\left(\frac{\omega_{\mathrm{mn}}s}{\beta c}\right), \quad (2)$$

$$k_{\mathrm{mn}}(a) = \frac{(E_{\mathrm{zmn}}(r=a))^2}{4U_{\mathrm{mn}}(1-v_{\mathrm{gn}}/\beta c)} = \frac{\omega_{\mathrm{mn}}R_{\mathrm{mn}}(r=a)}{4Q_{\mathrm{mn}}(1-v_{\mathrm{gn}}/\beta c)}, \quad (3)$$

Received 16 June 2005, Revised 12 October 2005

1) E-mail: peisl@mail.ihep.ac.cn

where a is the aperture of the beam hole, β the normalized particle's velocity, φ the azimuthal angle, r and $r_{\rm q}$ the transverse positions of the test charge and the exciting charge respectively, $\omega_{\rm mn}$ the frequency of the resonant mode n, $k_{\rm mn}$ and $R_{\rm mn}$ the loss factor per unit length and the shunt impedance per unit length at $r=a$ respectively, $E_{\rm zmn}$ the amplitude of the longitudinal electric field, $Q_{\rm mn}$ the quality factor, $U_{\rm mn}$ the energy stored in the cavity, and $v_{\rm gn}$ the group velocity. The factor $(1-v_{\rm gn}/\beta c)$ in Eq. (3) is often ignored in the traveling wave structures due to the very low group velocity. For example, the group velocity of the accelerating mode for BEPCⅡ Linac's structures is changed from $0.0204c$ to $0.0065c$ along a structure, which is much smaller than the light speed c and can be ignored. The modes for $m=0,1,\cdots$ are called monopole, dipole and so on.

Since the charged particles usually move very close to the axis, r/a and $r_{\rm q}/a$ in Eqs. (1) and (2) will be very small, and the longitudinal and transverse wake potentials are dominated by the monopole and dipole modes, respectively. If the azimuthal angle φ is chosen to be 0°, Eqs. (1) and (2) can be rewritten as[3—6]

$$W_{||}(s)=\sum_n 2k_{0\rm n}{\rm e}^{-\frac{\omega_{0\rm n}s}{2Q_{0\rm n}\beta c}}\cos(\frac{\omega_{0\rm n}s}{\beta c}),\quad s\geqslant 0, \tag{4}$$

$$\boldsymbol{W}_{\perp}(s)=\boldsymbol{r}\frac{r_{\rm q}}{a}\sum_n 2\frac{ck_{1\rm n}}{a\omega_{1\rm n}}{\rm e}^{-\frac{\omega_{1\rm n}s}{2Q_{1\rm n}\beta c}}\sin(\frac{\omega_{1\rm n}s}{\beta c}),\quad s\geqslant 0. \tag{5}$$

Three points should be kept in mind in this paper. First, we only consider monopole and dipole modes in the wake field calculations. Second, we only consider the calculation method of delta wake potentials instead of the bunch length dependent wake fields. And third, the calculation results given in this paper are all for β=1. Some methods mentioned in this paper can also be used to calculate the wake fields for β<1. In addition, it is worth to note that the group velocity damping effect is not included in all of the equations in this section.

3 Short range wake field

In the short range wake field calculation, the constant gradient structure can be regarded as a constant impedance structure with average dimensions, so one can consider only one cavity period. For the short range wake field with $s\ll a$, one has to find cavity modes with frequencies higher than the cutoff frequency $\omega\gg c/a$[7], and in this case the short range wake field can only be obtained by the impedance analysis method. For the short range wake field with $s\sim a$, the contributions of cavity modes with frequencies higher than the cutoff frequency can be neglected, one can only consider cavity modes with frequencies lower than the cutoff frequency, which can be solved by the numerical method. Besides the impedance analysis method and the numerical method, the pillbox approximation method[5] developed by J. GAO can also be used to calculate the short range wake field without any limitation to s and β.

3.1 Impedance analysis method

Usually, the high frequency longitudinal and transverse impedances of the disk-loaded accelerating structure can be generalized as[8, 9]

$$\begin{cases} Z_0^{||}(k)=\dfrac{{\rm i}Z_0}{\pi ka^2}\left[1+(1+{\rm i})\dfrac{\alpha(g/L)L}{a}\sqrt{\dfrac{\pi}{kg}}\right]^{-1} \\ Z_1^{\perp}(k)=\dfrac{2}{ka^2}Z_0^{||}(k) \end{cases} \tag{6}$$

with $k=\omega/c$ being the wave number, g the gap length, L the period length, and $Z_0\approx 377\Omega$. The parameter α can be represented as $\alpha\approx 1-\alpha_1(\gamma)^{1/2}-(1-2\alpha_1)\gamma$ with $\alpha_1=0.4648$ and $\gamma=g/L$. Using the Fourier transformation in Eq. (6), we can write the short range wake fields as[8]

$$\begin{cases} W_{||}(s)\approx\ \dfrac{Z_0c}{\pi a^2}H(s)\exp\left(-\sqrt{\dfrac{8\alpha^2gs}{a^2}}\right) \\ \boldsymbol{W}_{\perp}(s)\approx \boldsymbol{r}\dfrac{r_{\rm q}Z_0c}{2g\alpha^2\pi a^2}H(s)\times \\ \qquad\qquad \left[1-\left(1+\sqrt{\dfrac{8\alpha^2gs}{a^2}}\right){\rm e}^{-\sqrt{\frac{8\alpha^2gs}{a^2}}}\right] \end{cases} \tag{7}$$

with $H(s)=1$ for $s>0$, and 0 for $s<0$. On the other hand, K. Bane developed another fitted model, which has the same form as Eq. (7) with a better accuracy when $0\leqslant s/L\leqslant 0.16$, $0.34\leqslant a/L\leqslant 0.69$ and

$0.54 \leqslant g/L \leqslant 0.89$[8, 10], i.e.,

$$\begin{cases} W_{||}(s) \approx \dfrac{Z_0 c}{\pi a^2} H(s) \exp\left(-\sqrt{\dfrac{sL^{2.4}}{0.41 g^{1.6} a^{1.8}}}\right) \\ \boldsymbol{W}_{\perp}(s) \approx \boldsymbol{r} \dfrac{0.676 Z_0 c r_{\rm q}}{\pi a^{2.21} L^{1.17} g^{-0.38}} H(s) \times \\ \qquad \left[1 - \left(1 + \sqrt{\dfrac{sL^{1.17} g^{-0.38}}{0.169 a^{1.79}}}\right) {\rm e}^{-\sqrt{\frac{sL^{1.17} g^{-0.38}}{0.169 a^{1.79}}}}\right] \end{cases} . \tag{8}$$

Both the generalized Eq. (7) and Eq. (8) can be used to calculate the short range wake fields of the SLAC-like disk-loaded structure.

3.2 Pill-box approximation method

Each cell of the disk-loaded structure can be regarded as a single pill-box cavity, so the characteristics of the disk-loaded structure can be given by making some modifications to the analytical expressions of one single pill-box cavity. Usually, the general expression of the loss factor $k_{\rm mnl}$ for the $TM_{\rm mnl}$ mode can be given by Eq. (16) in Ref. [5]. After obtaining the loss factors and the angular frequencies of monopole and dipole modes respectively, one can get the short range wake fields by substituting the results into Eqs. (4) and (5).

3.3 Numerical method

Computer codes ABCI, TBCI[11], TRANSVERSE[12] and KN7C[13] can be used to calculate the wake fields when s is not much smaller than a. The advantage of the numerical method is that it can be used to obtain the wake fields of some complicated structures very quickly, which cannot be solved by analytical methods.

3.4 Short range wake field for the BEPCⅡ Linac

For short range wake fields of the BEPCⅡ Linac, the disk-loaded structure can be regarded as a constant impedance structure with the average dimensions of $a = 0.012$m, $g = 0.029$m, $L = 0.035$m, $b = 0.041$m. When $\beta = 1$, the short range longitudinal and transverse wake potentials can be shown in Fig. 1 and Fig. 2, respectively.

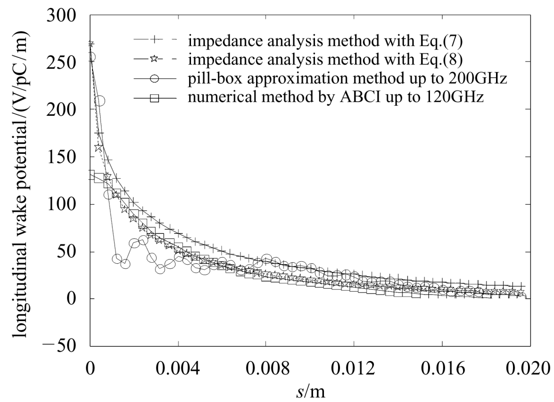

Fig. 1. Short range longitudinal wake potential for BEPCⅡ Linac.

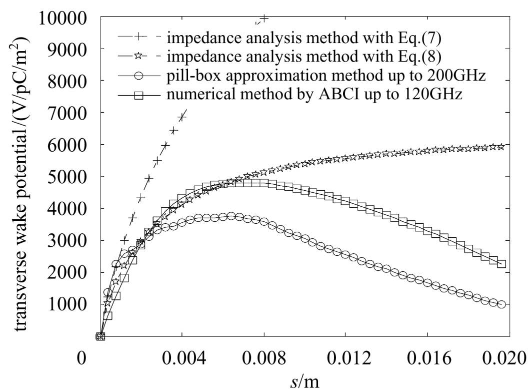

Fig. 2. Short range transverse wake potential for BEPCⅡ Linac.

By comparing the two figures, one can get the following conclusions:

1) When $s \ll a$, the agreement of the impedance method and the pill-box method is very good for both longitudinal and transverse wake potentials. While the difference of the numerical method by ABCI from the other methods is very large, this is because the cavity modes with frequencies higher than the cut-off frequency are not included in the numerical ABCI method, which exceeds the capability of ABCI.

2) For the longitudinal wake potential, all of the methods agree well except for some small deviations which can be accepted when $s \gg 0.5a$, but there is a relative big disagreement of the pill-box method from other methods when s is not much smaller than a but smaller than $0.5a$, this is due to the overestimation of the detuning effect for cavity modes with higher frequencies.

In addition, we can see there is a small oscillation of longitudinal short range wake potential calculated by pill-box method, and the amplitude of the oscillation decreases as s increases. When $s \sim a$, the amplitude of the oscillation decreases to almost zero. Fig. 3 shows the wake potentials calculated by the pill-box method with different upper limit of the calculated cavity modes' frequencies. The wavelength of the oscillation also varies when the upper limit of the calculated cavity modes' frequencies changes. Furthermore, the frequency corresponding to the wavelength of the oscillation is almost equal to the upper limit frequency of the calculated cavity modes. The appearance of the oscillation is due to the overestimation of the loss factors for the cavity modes with higher frequencies, while the decreasing of the amplitude as increasing the s is due to the detuning effect.

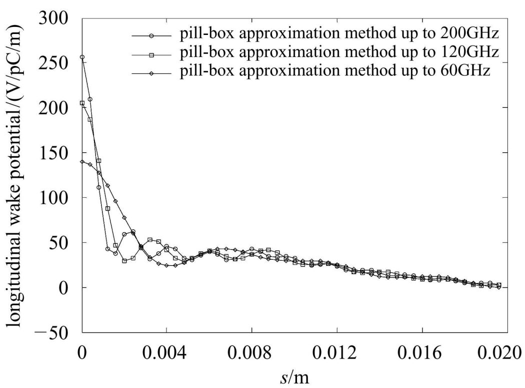

Fig. 3. Short range longitudinal wake potential calculated by pill-box approximation method with different frequency upper limit.

3) For the transverse wake potential, when s is not much smaller than a and not larger than $0.5a$ (about $0.16L$), the impedance method with Eq. (8) and the numerical ABCI method agree well, the small difference of the pill-box method from other methods is due to the overestimation of the detuning effect for cavity modes with higher frequencies.

When $s > 0.5a$, the agreement between any two methods is not reasonably good. Because $s > 0.5a$ is not within the parameter range of validity of impedance method with Eq. (8), this method is not right any more. The difference between the pill-box method and the numerical ABCI method when $s > 0.5a$ is almost constant, which is about $1000\mathrm{V/pC/m^2}$. Considering the kick factor of the first dipole mode calculated by each method, we find the kick factor of the first dipole mode calculated by ABCI is about $1350\mathrm{V/pC/m^2}$, and is about $925\mathrm{V/pC/m^2}$ for the pill-box method. This will result in a difference of $850\mathrm{V/pC/m^2}$ between the short range wake fields calculated by the pill-box method and the numerical ABCI method when $s > 0.5a$. So we think the difference between these two methods is caused by the cavity modes with lower frequencies. In addition, we also calculated the kick factor of the first dipole mode by MAFIA, which is about $1175\mathrm{V/pC/m^2}$, and is almost equal to the average value of $1350\mathrm{V/pC/m^2}$ and $925\mathrm{V/pC/m^2}$. Therefore, when $s > 0.5a$, we think the average of the wake fields calculated by the pill-box method and the numerical ABCI method is right.

4) Analyzing the conclusions 1) to 3), we know the pill-box method overestimates the loss factors and the detuning effect of the cavity modes with higher frequencies, and underestimates the kick factors of the cavity modes with lower frequencies; while the numerical ABCI method overestimates the kick factors of the cavity modes with lower frequencies and do not have the capability to calculate cavity modes with frequencies higher than the cutoff frequency. The impedance analysis method with Eq. (7) and Eq. (8) can be used to calculate the short range longitudinal wake potential without limitation of s and transverse wake potential with $s \ll 0.5a$, and Eq. (8) can also be used to calculate the short range transverse wake potential when s is not much smaller than a and not larger than $0.5a$.

It is worth to note that the above conclusions can be applied in SLAC-like disk-loaded structures, and need to be verified when they are applied in other kinds of disk-loaded structures.

4 Middle long range wake field

If the wake field to be calculated is in the region of $L \leqslant s \ll \beta c T_{\mathrm{f}}$, it can be called middle long range wake field. T_{f} is the filling time of the whole structure.

4.1 Middle long range wake field with $s \sim L$

When s is not much larger than L, the constant gradient structure can still be regarded as a constant impedance structure. In addition, both the damping factor $e^{-\frac{\omega_{1n}s}{2Q_{1n}\beta c}}$ and the contributions of cavity modes with frequencies higher than the cutoff frequency can still be neglected in Eqs. (4) and (5). Both the pill-box method and the numerical method can be used to calculate the middle long range wake field with $s\sim L$.

The longitudinal and transverse middle long range wake fields with $s\sim L$ for the BEPCⅡ Linac's disk-loaded structures are shown in Fig. 4 and Fig. 5, respectively. In our ABCI calculation, one cavity period and flat shape approximation of the disk nose are used, the characteristic bunch length σ_z is 1mm, and only the wake field in $0<s<1.5$m is calculated. In the MAFIA calculation, E module is used to calculate the dispersion curve, and then the loss factors and the frequencies of the synchronous modes can be found.

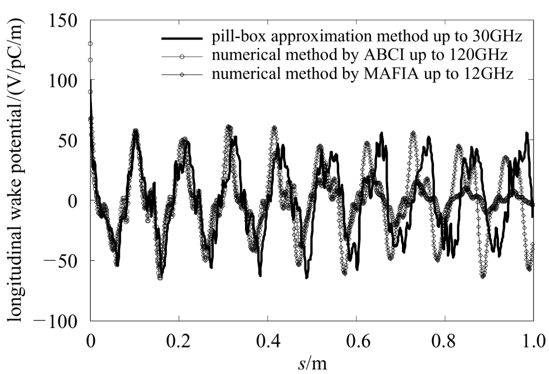

Fig. 4.　Middle long range longitudinal wake potential with $s\sim L$ for BEPCⅡ Linac.

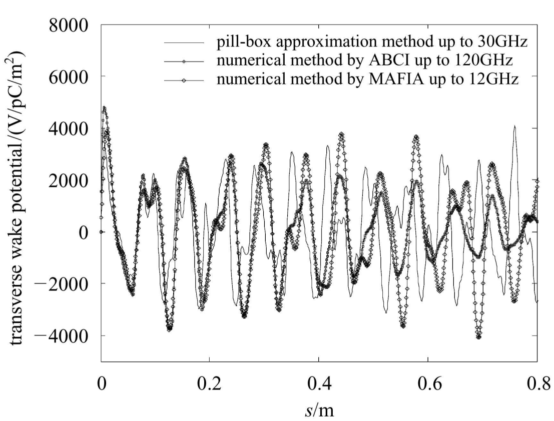

Fig. 5.　Middle long range transverse wake potential with $s\sim L$ for BEPCⅡ Linac.

By analyzing Fig. 4 and Fig. 5, the following conclusions can be obtained:

1) For the longitudinal wake potential, all of the results show very good agreement when $L\leqslant s\leqslant 10L$; while for the transverse wake potential, only when $L\leqslant s\leqslant 5L$ the agreement is good. In addition, when $s\sim L$ the agreement between the results of ABCI and MAFIA calculations further confirms that the contributions of cavity modes with frequencies higher than the first several cavity modes can be neglected for the calculation of long range wake fields.

2) When $s>10L$, neither the pill-box method nor the numerical method using ABCI can be applied. For the pill-box method, the peaks of the wake potential appear earlier or later than those got from ABCI and MAFIA, which is due to the large frequency approximation errors between the real disk-loaded structure and the closed pill-box cavity. For example, the frequencies of the first monopole and dipole modes calculated by the pill-box method are 2776MHz and 4424MHz, while they are 2856MHz and 4229MHz by MAFIA. If we want to improve the accuracy of the pill-box method, the effect of the beam hole on the resonant frequency must be included in the frequency calculation of the cavity modes, this work can be done by some codes like MAFIA, HFSS[14] and so on. While for the numerical ABCI method, the wake potential damps very fast to a lower level, since there is some frequency spread for each synchronous mode in ABCI results. For example, Fig. 6 shows the dk/df dependence on the longitudinal mode's frequency of the cell with the average dimensions. The effect of the frequency

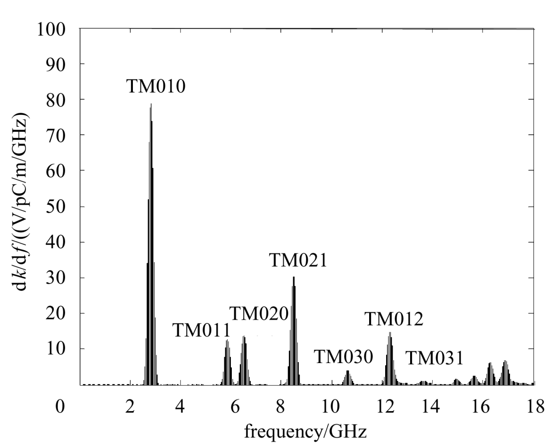

Fig. 6.　dk/df dependence on the longitudinal mode's frequency.

spread in ABCI is caused by the algorithm of Fourier analysis, and just like the detuning effect in constant gradient structure. If we want to mitigate the effect of the frequency spread and obtain the discrete frequency and the corresponding loss factor or kick factor for each mode, we need to use a structure with more cavity periods and calculate the wake fields in a longer distance in our calculation, but this is limited by the computer capability. The more the cavity periods used, the longer the distance of the wake field to be calculated, and the more the CPU time needed, the higher the requirement to the computer capability.

3) In Section 3.4, we found the kick factors of cavity modes with lower frequencies calculated by ABCI are higher than those calculated by the pill-box method and MAFIA, but in Fig. 5 the amplitude of the wake field calculated by ABCI is not very higher than those calculated by the other two methods. This is because the detuning effect of the frequency spread for each synchronous mode in ABCI has exceeded the effect of the overestimation of the kick factor. On the other hand, the amplitude of the wake field calculated by MAFIA is truly a little higher than those calculated by the pill-box method, which is caused by the difference of the calculated kick factors of cavity modes with lower frequencies.

4.2 Middle long range wake field with $s \gg L$

For middle long range wake field with $L \ll s \ll \beta c T_{\mathrm{f}}$, constant gradient structure can no longer be regarded as a constant impedance structure, due to the fact that both the detuning effect and the damping effect of the damping factor begin to be effective, and the wake field might be damped to a lower level quickly.

Because the cell dimensions differ from cell to cell in the constant gradient accelerating structure, we have to calculate the loss factor (or kick factor), quality factor and resonant frequency for each cavity mode of each cell in the calculation of middle long range wake field with $s \gg L$, which is a very big and tedious job. However, we know the long range wake fields in the conventional disk-loaded accelerating structures are dominated by the first several longitudinal modes and the first several dipole modes, which have been verified by Fig. 4 and Fig. 5 when $s > 10L$. So in this case we can only calculate the synchronous cavity modes in the first several pass bands for each cell, and substitute the results into Eqs. (4) and (5), then average them according to the number of the calculated cells to get the middle long range wake field with $s \gg L$.

The E module of MAFIA is used to calculate the synchronous modes, the corresponding loss factors (or kick factors), and the quality factors of the BEPCⅡ Linac's accelerating structures. The longitudinal synchronous modes in the first seven pass bands and transverse synchronous modes in the first six pass bands are calculated. Fig. 7 shows the loss factor and quality factor dependence on the frequency of the longitudinal modes, Fig. 8 shows the kick factor and quality factor dependence on the frequency of the transverse modes.

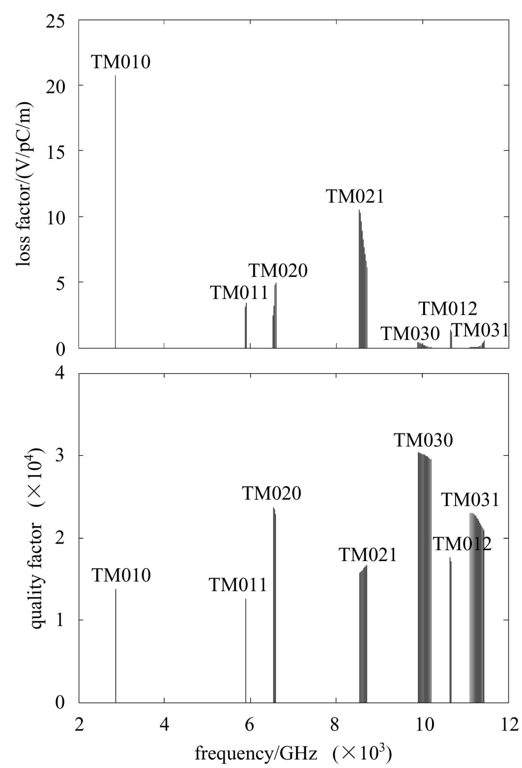

Fig. 7. Loss factor and quality factor dependence on frequency.

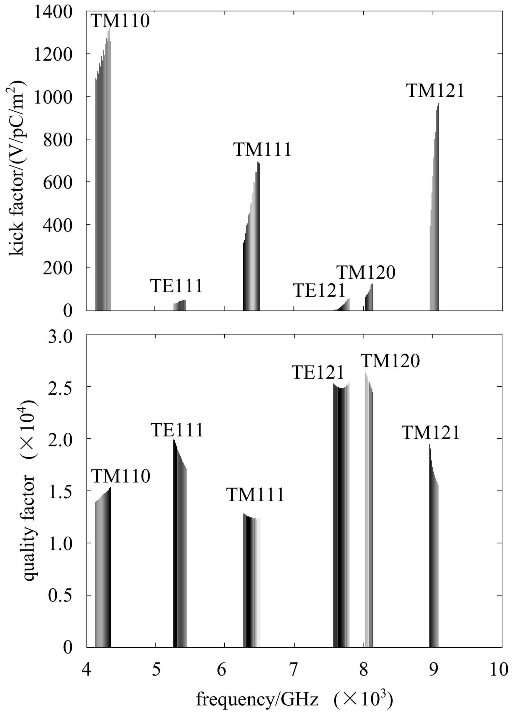

Fig. 8. Kick factor and quality factor dependence on frequency.

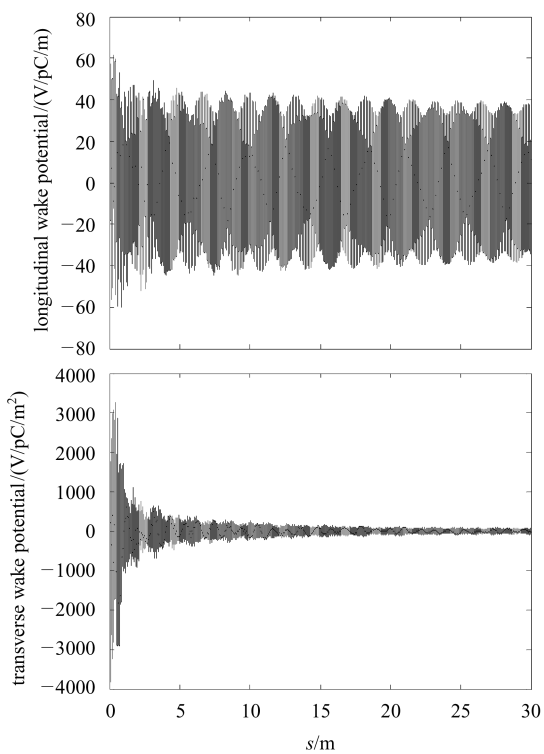

Fig. 9. Middle long range wake potentials with $s \gg L$ for BEPCⅡ Linac.

Substituting the calculated loss factors (or kick factors), frequencies and quality factors into Eqs. (4) and (5), and averaging them according to the number of calculated cells, we can get the middle long range longitudinal and transverse wake potentials with $s \gg L$ shown in Fig. 9.

One can see that the longitudinal wake potential damps very slowly, while the transverse wake potential damps very fast. This is due to the fact that the dominated longitudinal mode is the accelerating mode, and all of the cells have the same frequency for this mode. So the damping of the longitudinal fundamental mode only relies on the damping factor, which is very small. But for the transverse wake potential, the frequency of each dipole mode in every cell is different from that of each dipole mode in the other cells, i.e., every dipole mode has detuning effect, so the detuning effect of the transverse wake potential is very large.

5 Super long range wake field

Super long range wake field is defined to be the wake field in the region of $s \sim \beta c T_{\mathrm{f}}$. For the super long range wake field, besides the detuning effect of the high order modes and the damping effect of the damping factor, another damping effect related to the group velocity of the cavity modes needs to be considered, so the equations representing the wake functions need to be modified. It is worth to point out that the damping effect related to the group velocity mentioned in this section is not the group velocity effect on the calculation of loss factor, which is given in Eq. (3). In Eq. (3), the group effect can only affect the calculation of loss factor, and cannot damp the wake fields as increasing the s.

Actually, the wake fields are not stationary after they are induced in disk-loaded structures. The behavior of the wake fields is just like that of a travelling wave, travelling forward or backward to the output coupler or input coupler with the corresponding group velocity. If they travel backward and get to the input coupler, they will be reflected and then travel forward to the output coupler. Anyway, they will get to the output coupler finally, and be absorbed by the load installed at the end of the structure, and then the amount of wake fields' energy left in the structure will be decreased. The absorption of wake fields into the load can be considered as the damping effect related to the group velocity.

5.1 Super long range longitudinal wake field

When $s \sim \beta c T_{\rm f}$, the contribution of the high order modes with frequencies higher than the fundamental mode to the longitudinal wake field is much smaller than that of the fundamental mode due to the detuning effect and the group velocity effect. We can consider the group velocity effect only on the fundamental mode and not on the other modes in the calculation of super long range longitudinal wake field. So when s is almost equal to $\beta c T_{\rm f}$, there should be no any longitudinal wake fields in the disk-loaded structure if the trapped modes are not considered. In fact, the contribution of the trapped modes to the longitudinal wake field is very small compared with the longitudinal fundamental mode.

For a constant gradient structure, if a bunch traverses through the structure at time $t=0$, then at time $t=s/c$ the wake field of the fundamental mode will travel a distance of z_0[15],

$$z_0 = L_{\rm s}\frac{1-{\rm e}^{\frac{2s\tau}{\beta c T_{\rm f}}}}{1-{\rm e}^{-2\tau}}, \qquad (9)$$

where τ is the attenuation coefficient, $L_{\rm s}$ the length of the whole structure. Therefore, at time $t=s/c$, we can think the wake field of the fundamental mode only exists in $z_0<z<L_{\rm s}$ of the accelerating structure. If only the group velocity effect on the fundamental mode is considered, we can add a factor related to Eq. (9) in the fundamental mode part of Eq. (4), and then the super long range longitudinal wake field can be written as

$$W_{||}(s) = 2k_{01}{\rm e}^{-\frac{\omega_{01}s}{2Q_{01}\beta c}}\cos\left(\frac{\omega_{01}s}{\beta c}\right)\left(1-\frac{1-{\rm e}^{\frac{2s\tau}{\beta c T_{\rm f}}}}{1-{\rm e}^{-2\tau}}\right)+ \sum_{n>1}2k_{0n}{\rm e}^{-\frac{\omega_{0n}s}{2Q_{0n}\beta c}}\cos\left(\frac{\omega_{0n}s}{\beta c}\right). \qquad (10)$$

With Eq. (10) and MAFIA calculation results for the longitudinal synchronous modes in the first several pass bands of each cell, the super long range longitudinal wake field for BEPCⅡ Linac's accelerating structure can be obtained and shown in Fig. 10.

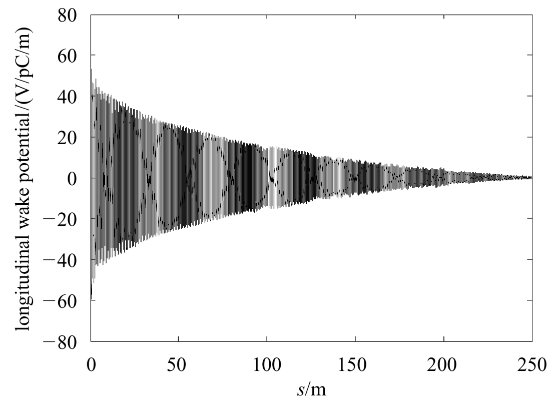

Fig. 10. Super long range longitudinal wake field for BEPCⅡ Linac with $\tau=0.57$, $\beta=1$ and $T_{\rm f}=0.83\mu s$.

5.2 Super long range transverse wake field

For the super long range transverse wake field, although one can use the same method as that for the longitudinal wake field, it will also be a tedious job. Fortunately, due to the large detuning effects of all the transverse modes, the transverse wake field has already damped to a very low level near zero with the super large value of s, which can be seen from Fig. 9. In addition, the long range transverse wake field is dominated by the dipole modes in the first several pass bands of the disk-loaded structures, and the beam phase velocities of the synchronous modes in the first pass band are very close to π mode, whose group velocity is 0 in the pass band of TM110 mode. So the group velocities of the synchronous dipole modes in the first pass band can be considered to be 0, and the method used to calculate the middle long range transverse wake field with $s \gg L$ can still be used to calculate the super long range transverse wake field, i.e., the damping effect related to the group velocity can be neglected in the calculation of super long range transverse wake field. Of course, if we neglect the damping effect related to the group velocity in the calculation, there must be some difference between the calculated and the real super long range transverse wake field, but the difference must be in the acceptable range.

Neglecting the group velocity effect on all of the dipole modes, one can obtain the super long range transverse wake field for BEPCⅡ, shown in Fig. 11.

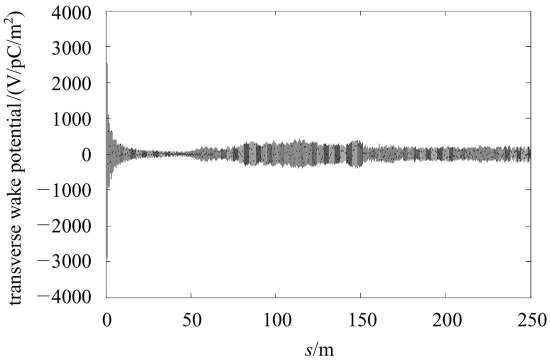

Fig. 11.　Super long range transversal wake field for BEPCⅡ Linac.

6　Summary

Using analytical methods and various computer codes, the ways to evaluate the longitudinal and transverse wake fields of constant gradient structures are studied and analyzed, and the expression of the super long range longitudinal wake field is first given. In addition, the wake field calculation results for the BEPCⅡ Linac's disk-loaded structures are also given, which will be used in the beam dynamics studies of two bunch acceleration on the BEPCⅡ Linac and discussed in the future papers.

The authors would like to thank Prof. Jie Gao from IHEP for his very helpful advices.

References

1 Chin Y H. User's Guide for ABCI (Version 8.7): CERN SL/94-02 (AP). Geneva, Swizerland, 1994
2 The MAFIA Collaboration. MAFIA User Manual (Version 4.106). Germany: CST Inc, 2000
3 Takao M et al. Evaluation of Wake Fields of Disk Loaded Structure for JLC: KEK Report 91-4. 1991
4 XIAO Li-Ling. R&D of Accelerating Structure for NLC. Postdoctor Thesis, Tsinghua University, 1995.11 (in Chinese)
(肖利苓. 下一代LC加速结构的研究进展. 博士后出站报告, 清华大学, 1995.11)
5 GAO Jie. NIM, 2000, **A447**: 301—308
6 JIN C. Group Velocity Effect on Wakefield Calculation: WF-222. ANL, 2004
7 Chao A. Physics of Collective Beam Instabilities in High Energy Accelerators. New York: Wiley Interscience. 1993
8 Bane K. Short-Range Dipole Wakefields in Accelerating Structures for the NLC: SLAC-PUB-9663. SLAC, 2003
9 Gluckstern R L. Phys. Rev., 1989, **D39**: 2780—2783
10 Bane K et al. Measurement of the Longitudinal Wakefield in the SLAC Linac for Extremely Short Bunches: SLAC-PUB-9905. 2003
11 Weiland T. NIM, 1983, **212**: 13—21
12 Bane K et al. Proceedings of the 11th Int. Conf. on High Energy Accelerators. CERN, 1980. 581
13 Keil E. NIM, 1972, **100**: 419
14 The Ansoft High Frequency Structure Simulator. Copyright Ansoft Corporation
15 GU Peng-Da. Studies on New RF Pulse Compressor. PhD Thesis, IHEP, 1999 (in Chinese)
(顾鹏达. 新型RF脉冲压缩器研究. 博士论文, 中国科学院高能物理研究所, 1999)

等梯度加速结构中的尾场计算研究

裴士伦[1,2;1)]　王书鸿[1]
1 (中国科学院高能物理研究所　北京　100049)
2 (中国科学院研究生院　北京　100049)

摘要　带电粒子束在穿过加速结构时激起的尾场，会导致束流能散和发射度的增大. 准确计算尾场对正确估计尾场效应对束流性能的影响具有重要意义. 本文对等梯度加速结构中短程尾场、中长程尾场和超长程尾场的计算方法进行了较系统的研究，并首次给出了超长程尾场的计算公式. 此外，本文还给出了BEPCⅡ直线加速器等梯度加速结构中短程尾场、中长程尾场以及超长程尾场的计算结果，并对多种方法计算结果的一致性和在某些范围内的差别做了分析说明.

关键词　尾场　等梯度加速结构　损失因子　冲击因子

2005 – 06 – 16 收稿，　2005 – 10 – 12 收修改稿

1) E-mail: peisl@mail.ihep.ac.cn