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Quark Number Density Susceptibilities in Lattice QCD*

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Abstract We measured charge fluctuation and fermion number density susceptibility using lattice QCD with two light flavors of staggered quarks on $16\times 8^2\times 4$ lattice. Below the critical temperature, we simulated them with the sub-volume for $V=(2a)^3\sim(8a)^3$, where $\left(a=\dfrac{1}{4T}\right)$. We find that new aspect is observation of volume dependence in the fluctuations. The charge fluctuation and fermion number density susceptibility vary with the volume for V. This suggests the existence of hadronic length scale in the charge fluctuation and fermion number density susceptibility. Above critical temperature, charge fluctuation and fermion number density susceptibility are almost volume independent for $V\geqslant(2a)^3$.

Key words lattice quantum field theory, quark number density susceptibility, charge fluctuation

1 Introduction

Fluctuations of quark number density and other conserving quantities have been payed much attention as signals of formation of quark gluon plasma (QGP) in the heavy ion collision experiments[1—5]. Several authors have discussed that the fluctuations are experimentally accessible and sensible quantities on phase of QCD matter[6, 7]. Those arguments concern with the change of dynamical modes, specifically those of mass and granularity of charge and baryon number.

It is well known that the numerical simulation of finite density lattice QCD is a difficult problem, because of the complex nature of fermionic determinant with finite chemical potential μ. Since the naive quenched approximation does not suitable to the $N_{\rm f}\to 0$ limit of dynamical configurations at finite temperature[8], one needs to incorporate dynamical quarks to extract relevant physics. On the other hand, the quark number susceptibilities at $\mu=0$ can be calculated[9] in lattice QCD, and give us strong enhancement of it. In this work, we examine charge fluctuation as well as fermion number susceptibilities by focusing on the length scale of them.

2 Quark number density susceptibilities

We employ the staggered quark formulation with two flavors. The iso-singlet and iso-non-singlet susceptibilities are

$$\chi_{\rm S}\equiv\frac{1}{16T^2}\frac{\partial}{\partial\mu_{\rm S}}(n_{\rm u}+n_{\rm d})\bigg|_{\mu=0}=\frac{1}{32TV}\left\{\left\langle{\rm Tr}\left[\ddot{D}G\right]\right\rangle-\left\langle{\rm Tr}\left[\dot{D}G\dot{D}G\right]\right\rangle\right\}+\frac{1}{64TV}\left\{\left\langle{\rm Tr}\left[\dot{D}G\right]{\rm Tr}\left[\dot{D}G\right]\right\rangle\right\},$$

$$\chi_{\rm V}\equiv\frac{1}{16T^2}\frac{\partial}{\partial\mu_{\rm V}}(n_{\rm u}-n_{\rm d})\bigg|_{\mu=0}=\frac{1}{32TV}\left\{\left\langle{\rm Tr}\left[\ddot{D}G\right]\right\rangle-\left\langle{\rm Tr}\left[\dot{D}G\dot{D}G\right]\right\rangle\right\},$$

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$$\chi_{\mathrm{q}} \equiv \frac{1}{16T^2}\frac{\partial}{\partial \mu_{\mathrm{q}}}(Q_{\mathrm{u}}n_{\mathrm{u}}+Q_{\mathrm{d}}n_{\mathrm{d}})\bigg|_{\mu=0} = \frac{1}{16TV}\frac{Q_{\mathrm{u}}^2+Q_{\mathrm{d}}^2}{4}\left\{\left\langle \mathrm{Tr}\left[\ddot{D}G\right]\right\rangle - \left\langle \mathrm{Tr}\left[\dot{D}G\dot{D}G\right]\right\rangle\right\} + \frac{1}{16TV}\frac{(Q_{\mathrm{u}}-Q_{\mathrm{d}})^2}{16}\left\{\left\langle \mathrm{Tr}\left[\dot{D}G\right]\mathrm{Tr}\left[\dot{D}G\right]\right\rangle\right\} = \frac{Q_{\mathrm{u}}^2+Q_{\mathrm{d}}^2}{2}\chi_{\mathrm{V}} + \frac{(Q_{\mathrm{u}}-Q_{\mathrm{d}})^2}{4}(\chi_{\mathrm{S}}-\chi_{\mathrm{V}}) \quad , \tag{1}$$

where $\frac{\partial}{\partial \mu_{\mathrm{S}}} = \frac{\partial}{\partial \mu_{\mathrm{u}}} + \frac{\partial}{\partial \mu_{\mathrm{d}}}$, $\frac{\partial}{\partial \mu_{\mathrm{V}}} = \frac{\partial}{\partial \mu_{\mathrm{u}}} - \frac{\partial}{\partial \mu_{\mathrm{d}}}$, $\frac{\partial}{\partial \mu_{\mathrm{Q}}} = Q_{\mathrm{u}}\frac{\partial}{\partial \mu_{\mathrm{u}}} + Q_{\mathrm{d}}\frac{\partial}{\partial \mu_{\mathrm{d}}}$, n_{u} and n_{d} are quark number densities. The thermodynamical quantities $n_\alpha = \frac{T}{V}\frac{\partial \ln Z}{\partial \mu_\alpha}$ ($\alpha = \mathrm{u,d}$), Z is partition function defined as $Z = \int \mathscr{D}U \prod_\alpha \det D_\alpha \exp[-S_{\mathrm{G}}] = \int \mathscr{D}U \exp\left[-S_{\mathrm{G}} + \sum_\alpha \frac{1}{4}\mathrm{Tr}\ln D_\alpha\right]$ in lattice QCD, in which D_α is the fermion operators, and G is quark propagator with $G = \frac{1}{D}$, and $Q_{\mathrm{u}} = \frac{2}{3}$ and $Q_{\mathrm{d}} = -\frac{1}{3}$.

3 Numerical results

The numerical simulations are performed on lattices of size $16 \times 8 \times 8 \times 4$ with standard Wilson gauge action and two dynamical flavors of staggered quarks. The configurations are generated with the R-algorithm with molecular dynamical step size $\delta = 0.01$. Quark mass parameter ma is taken to be 0.0125 and 0.025. We study charge fluctuations and fermion number density susceptibilities below and above the critical temperature.

In this work, we measure fluctuations and susceptibilities in finite domains: $L^3 \times N_{\mathrm{t}}$, $L = 2, 3, \cdots, 8$. The sub-volume is $V = (aL)^3 = \left(\frac{L}{4T}\right)^3$. Fig. 1 shows the numerical results of χ_{Q}, χ_{S} and χ_{V} for $ma = 0.0125$, $ma = 0.025$, and $\beta = 5.26$ (low temperature phase), $\beta = 5.34$ (high temperature phase).

In the low temperature phase, in investigating χ_{Q} and χ_{V}, when $L > 4$, we find the numerical results become close to the value of total volume. A minimum is found around $L = 4$. In physical length unit, it corresponds to 1.2—2fm (Here we consider $T_{\mathrm{C}} = 170$MeV, lattice spacing $a \approx 0.3$fm). Above $L \geqslant 6$, the fluctuations saturate to the constant value which is irrespective of the box size. Therefore, we find the length scale of hadronic size which couples to charge fluctuation and non-singlet fermion number density susceptibilities. For χ_{S}, the smallest simulation results are seen for $L = 6$. Though there are large errors and difference for $ma = 0.0125$ and $ma = 0.025$, we still can find length scale of hadronic size which couples to singlet fermion number density susceptibilities.

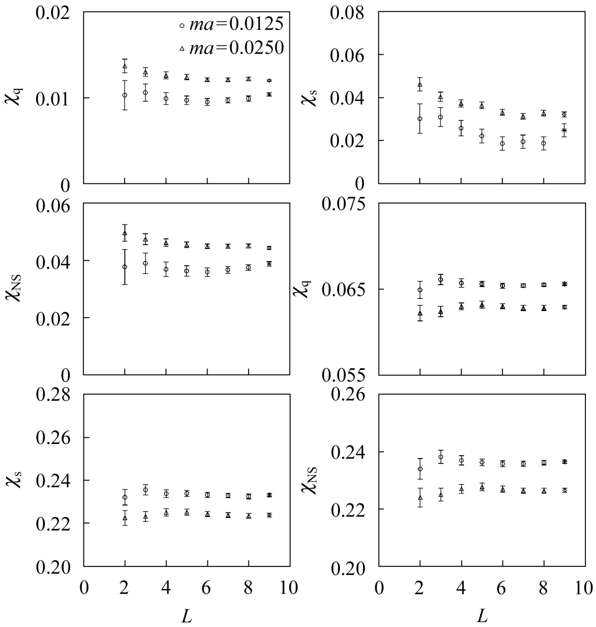

Fig. 1. χ_{q}, χ_{S} and χ_{V} as a function of sub-volume. $\beta = 5.26$ (top, low temperature phase), $\beta = 5.34$ (bottom, high temperature phase). Horizontal axis is sub-bolume as $(La)^3$.

In the high temperature phase, the data are rather flat in variation of box size and we do not see characteristic length for L=2—8. For $V \geqslant (2a)^3$ the χ_{Q}, χ_{S} and χ_{V} are independent of sub-volume.

4 Conclusions

In this work, we have developed a framework to study the response of hadrons to the chemical potential. It is based on Taylor expanding hadronic quantities around $\mu = 0$. We show the first results of the first and second derivatives of pseudoscalar and vector meson masses with respect to μ. As shown in the previous sections, the second order responses are sizable and reveal several characteristic features. For the pseudoscalar meson, the behaviour of the responses seems to have close contact to chiral symmetry restoration. For the isoscalar chemical potential μ_S, the dependence of the pseudoscalar mass on μ_S in the chiral limit is consistent with zero, reflecting the fact that at low temperature and small μ_S the pion is still a goldstone. For the isovector chemical potential, we show features that point towards the phase structure studied by Son and Stephanov[10]. The $u\bar{d}$ pseudoscalar mass tends to decrease as a function of μ_V at a much stronger rate in the low temperature phase.

It is notable that a single hadron pole gives a good description for the response as well as for the correlator at $\beta = 5.34$ ($T/T_c \approx 1.1$) in the pseudoscalar channel.

On the other hand, the results for the vector meson response are still too noisy. It is encouraging that the questions touched in the introduction appear to be addressable within our approach, but improved statistics are necessary for quantitative conclusions.

Since the present study is a first trial, our simulations have been performed on a rather small lattice. However, differences between the dynamics of $N_t = 4$ and $N_t = 6$ lattices have been reported[11]. Thus, further investigations on larger lattices are indispensable.

The chemical potential response of the nucleon is also an interesting issue. That of the quark condensate is even more important. An exploratory study on these topics is in progress.

Simulations were performed on the Nankai Start at Nankai University.

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夸克数密度敏感性的格点研究*

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摘要　应用格点QCD理论对电荷涨落和夸克数密度敏感性进行了研究. 研究选取了两个轻味夸克, 应用staggered作用量, 并选取了$16\times8^2\times4$格点. 对电荷涨落和夸克数密度敏感性在$V=(2a)^3\sim(8a)^3$(这里$a=\frac{1}{4T}$)区间进行了数值计算. 结果显示, 在相变温度以下的区域内, 上述参数对体积V具有依赖性, 这表明了可以通过对上述参数的研究来确定强子的尺度. 在高于相变温度的区域内, 对体积V没有表现出依赖性.

关键词　格点量子场理论　夸克数密度敏感性　电荷涨落

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