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Origin of Cosmological Constant from Bulk Manifold

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Abstract The problem about cosmological constant is a difficult and important problem, people even don't know what it is really originated from. In this letter, we show up a kind of origin of the cosmological constant from the viewpoint of some extra dimensional spaces, obtain different values of the cosmological constant under different circumstances, acquire the evolution function with time t. And we achieve a cosmological constant that may be fitted with modern astronomic observation.

Key words cosmological constant, cosmology, bulk manifold, Einstein equation

1 Introduction

The physical origin of the cosmological constant is considered to be one of the most difficult and important problems in modern physics[1—3], especially in cosmology. According to the recent observation, it should be a very small but nonzero value. In quantum field theory (QFT), vacuum energy density is believed to contribute to the cosmological constant[4, 5], but there is a very huge discrepancy between their orders of magnitude. Though many people propose some explanations, it now is still being considered a problem.

In general relativity, one deduces Einstein equation with cosmological constant through adding a free λ into action, but one can not explain what it is. An old but still exciting idea is to assume that we live in a fundamentally higher-dimensional spacetime[6—10], but most papers just give volumes of some extra dimensional spaces. In this letter, we propose a new idea about cosmological constant and some other new ideas about some extra dimensional spaces.

2 Bulk spacetime manifold

For a n-dimensional bulk spacetime manifold, we assume that its' infinitesimal square line element has the form as follows

$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 + g_{ij}\mathrm{d}x^i\mathrm{d}x^j + g_{ab}(t)\mathrm{d}x^a\mathrm{d}x^b \quad , \qquad (1)$$

where $i,j=1,2,3$; $a,b=1,2,\cdots,n-4$.

By carefully and long calculating in terms of the metrics of the bulk spacetime manifold, we obtain that the Ricci scalar of the n-dimensional bulk spacetime manifold is

$$\tilde{R}_{\mathrm{total}} = g^{AB}\tilde{R}_{AB} = R + R' + \Delta \quad , \qquad (2)$$

where

$$R = g^{\mu\nu}(\Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{\mu\nu} - \Gamma^{\alpha}_{\beta\nu}\Gamma^{\beta}_{\mu\alpha}) \quad , \qquad (3)$$

$$R' = g^{ab}(\Gamma^{c}_{ab,c} - \Gamma^{c}_{ac,b} + \Gamma^{c}_{dc}\Gamma^{d}_{ab} - \Gamma^{c}_{db}\Gamma^{d}_{ac}) \quad , \qquad (4)$$

$$\Delta = g^{\mu\nu}(-\Gamma^{c}_{\mu c,\nu} + \Gamma^{c}_{\beta c}\Gamma^{\beta}_{\mu\nu} - \Gamma^{c}_{\beta\nu}\Gamma^{\beta}_{\mu c} - \Gamma^{c}_{a\nu}\Gamma^{a}_{\mu c}) + g^{ab}(\Gamma^{\alpha}_{ab,\alpha} + \Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{ab} - \Gamma^{\alpha}_{db}\Gamma^{d}_{a\alpha} + \Gamma^{c}_{\beta c}\Gamma^{\beta}_{ab} - \Gamma^{c}_{\beta b}\Gamma^{\beta}_{ac}) \quad . \qquad (5)$$

We consider a 3-dimensional extra space. This space has a big symmetric form which is similar to the space part of Robertson-Walker spacetime manifold with $k=1$ as follows

$$g_{ab}\mathrm{d}x^a\mathrm{d}x^b = B^2(t)\left[\frac{\mathrm{d}r^2}{1-r^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)\right] . \qquad (6)$$

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Because taking $k=1$ may result in positive curvature, further may lead to a positive cosmological constant, see Eq. (4.5), as the positive cosmological constant may result in cosmic accelerative expansion[11, 12].

After carefully and long calculating in terms of the metrics of the concrete bulk spacetime manifold, we have

$$R'=\frac{6}{B^2}\quad , \tag{7}$$

$$\varDelta=-6\left(\frac{\dot{B}}{B}\right)^2 g^{00}-6\left(\frac{\ddot{B}}{B}\right) g^{00}+3\frac{\dot{B}}{B}g^{\alpha\beta}\varGamma^0_{\alpha\beta}-3\frac{\dot{B}}{B}g^{\alpha 0}\varGamma^{\nu}_{\alpha\nu}. \tag{8}$$

Using the above researches, we can further deduce Einstein equation.

3 Einstein equation

Then the geometrical action of the (4+3)-dimensional bulk spacetime is

$$I_{\rm g}=\int {\rm d}^4x{\rm d}^3y\sqrt{-\tilde{g}}\tilde{R}=\int {\rm d}^4x{\rm d}^3y\sqrt{-g}\sqrt{g'}\tilde{R}\quad . \tag{9}$$

According to the expression of Eq. (9), we can carry on the integral in the additional dimensions, thus we obtain

$$\int {\rm d}^3y\sqrt{g'}=B^3V\quad , \tag{10}$$

where V is the volume. We define

$$R'+\varDelta=L'\quad , \tag{11}$$

Then we can have

$$I_{\rm g}=\int {\rm d}^4x{\rm d}^3y\sqrt{-\tilde{g}}\tilde{R}= V\int {\rm d}^4x\, {\rm d}^3y\sqrt{g'}\sqrt{-g}(R+L')\quad . \tag{12}$$

According to variational principle, we have $\delta I_{\rm g}=0$, we, then, can deduce the gravitational field equation of the 4-dimensional spacetime as follows

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\frac{1}{2}L'g_{\mu\nu}=\kappa T_{\mu\nu}\quad , \tag{13}$$

where

$$T_{\mu\nu}=\frac{3}{\kappa}\left[2\left(\frac{\dot{B}^2}{B^2}+\frac{\ddot{B}}{B}\right)\delta^0_\mu\delta^0_\nu+\frac{\dot{B}}{B}(\delta^0_\mu\delta^0_\nu g^{\alpha\beta}g_{\alpha\beta,0}+ g^{00}g_{\mu\nu,0})+\frac{\partial_0(B^2\dot{B}\sqrt{-g}g^{00}g_{\mu\nu})}{\sqrt{-g}B^3}\right]\quad , \tag{14}$$

$T_{\mu\nu}$ is the effective energy-momentum tensor depending on the conformal factor of the extra dimensional space and the derivatives of the conformal factor.

4 Discussion and conclusion

1. When B is a constant, so $\dot{B}=0$, then Eq. (13) can be rewritten as

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\frac{1}{2}g_{\mu\nu}\frac{6}{B^2}=0\quad . \tag{15}$$

Compare Eq. (15) with general Einstein equation with cosmological constant in de Sitter spacetime

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-g_{\mu\nu}\lambda=0\quad , \tag{16}$$

we obtain $\lambda=\frac{3}{B^2}$, then we conclude, if there is the extra dimensional space, or there are some extra dimensional spaces similar to the extra dimensional space, and these extra dimensional spaces do not vary with time, then the cosmological constant would be the curvature of the extra dimensional spaces, and $\frac{3}{B^2}$ may be a small value or a big value. For a big value, this is contrast to the traditional viewpoint of string theory. Refs. [6, 7] gave some explanations why we can't see these large extra dimensions.

2. When B is a function of time t, we choose $B(t)=b_0t^\eta$, then we have

$$B^2\propto t^{2\eta},\quad \dot{B}\propto \eta t^{\eta-1},\quad \ddot{B}\propto\eta(\eta-1)t^{\eta-2},$$

$$\frac{\dot{B}}{B}=\frac{\eta}{t},\quad \frac{\ddot{B}}{B}=\frac{\eta(\eta-1)}{t^2}\quad , \tag{17}$$

where η is a parameter. Eq. (13) changes to

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\frac{1}{2}g_{\mu\nu}\left[\frac{6}{(b_0t^\eta)^2}- 6\frac{\eta(\eta-1)+\eta^2}{t^2}g^{00}-3\frac{\eta}{t}g^{00}g^{\alpha\beta}g_{\alpha\beta,0}\right]=\kappa T_{\mu\nu}, \tag{18}$$

where $t\neq 0$, because the cosmic birth during the very closest to $t=0$ may be described by quantum cosmology.

1) When $\eta>0$ and t is big enough, we acquire $\frac{6}{(b_0t^\eta)^2}$, $\frac{\eta}{t}$, $\frac{\eta^2+\eta(\eta-1)}{t^2}$ are all small.

Compare Eq. (18) with Einstein equation, we have

$$\lambda=\frac{1}{2}\left[\frac{6}{(b_0t^\eta)^2}-6\frac{\eta(\eta-1)+\eta^2}{t^2}g^{00}-3\frac{\eta}{t}g^{00}g^{\alpha\beta}g_{\alpha\beta,0}\right]\quad , \tag{19}$$

$$T_{\mu\nu}=\frac{3}{\kappa}\left[2\frac{\eta^2+\eta(\eta-1)}{t^2}\delta^0_\mu\delta^0_\nu+\frac{\eta}{t}(\delta^0_\mu\delta^0_\nu g^{\alpha\beta}g_{\alpha\beta,0}+g^{00}g_{\mu\nu,0})+\frac{\partial_0(\eta t^{3\eta-1}\sqrt{-g}g^{00}g_{\mu\nu})}{\sqrt{-g}t^{3\eta}}\right] . \quad (20)$$

The cosmological constant is not only relative to the curvature of the extra dimensional space, but also relative to our usual universe. The constant is function about t, and its value becomes smaller and smaller as time t gets bigger and bigger. In the other words, when t is very small, the constant is so large that we must pay attention to it's effect on the process of the early universe's varying. Furthermore, energy-momentum tensor is also related to the parameter of some extra space.

2) When $\eta<0$ and t is big enough, we obtain that $\frac{\eta}{t}$, $\frac{\eta^2+\eta(\eta-1)}{t^2}$ are approach to zero, and $\frac{3}{(b_0t^\eta)^2}$ is so large that it contrast to the observation result.

According to the expression of the cosmological constant, we deduce, the cosmological constant is relative to the scale factor of some extra dimensional spaces and metrics of the 4-dimensional usual space-time. And the energy-momentum tensor of the universe is also relative to the contribution of some extra dimensional spaces.

3) When η approaches to zero, we have $\frac{\eta}{t}$, $\frac{\eta^2+\eta(\eta-1)}{t^2}$ near to zero, but the cosmological constant $\frac{3}{b_0^2t^{2\eta}}$ approaches to a small quantity Eq. (19), b_0 may be taken some value so that $\frac{3}{b_0^2}$ is fitted with modern astronomic observation. Thus，we can explain why cosmological constant is not zero but a small quantity.

Because our researches are general, the more researches and their generalization about this letter will be written in other papers due to length limit of letter's article.

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宇宙常数的Bulk流形的起源

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摘要 关于宇宙常数问题是个至今没有解决的问题, 它的来源至今还没有一个共识. 从额外维的流形出发, 给出了宇宙常数的bulk流形起源的理论, 得到了不同情况下宇宙常数的取值和宇宙常数随时间演化的函数, 并且得到了可拟合现代天文观测的宇宙常数.

关键词 宇宙常数 宇宙学 bulk流形 爱因斯坦方程

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