

$\pi\pi$ Phase Shifts from $\psi' \rightarrow J/\psi \pi^+\pi^-$ Decays*

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Abstract The $\psi' \rightarrow J/\psi \pi^+\pi^-$ decay process provides a new way to extract the $\pi\pi$ S wave phase shifts up to 0.59GeV. In this paper we derive the formulae for extracting the $\pi\pi$ S wave phase shifts from the invariant mass spectrum of $\pi\pi$ in the $\psi' \rightarrow J/\psi \pi^+\pi^-$ decay.

Key words $\pi\pi$, S wave phase shifts, $\psi' \rightarrow J/\psi \pi^+\pi^-$

Meson-meson scattering is an important process for understanding the fundamental hadron-hadron interactions. $\pi\pi$ interaction, as one of the simplest processes, has substantially been studied experimentally and theoretically over many years. Up to now, the theoretical study of the $\pi\pi$ interaction has been made using perturbative^[? ? ?] and non-perturbative chiral effective theories^[? ? ?] and boson exchange models^[?]. The experimental investigations on this line were commonly carried out in terms of the reactions dominated by one pion exchange (for a review, see Ref. [6], for example), and the $\pi\pi$ phase shifts were extracted from the analysis of the reactions $\pi N \rightarrow \pi\pi N$ (e.g. $\pi^- p \rightarrow n\pi^+\pi^-$ ^[? ? ? ?] and $\pi^- p \rightarrow n\pi^0\pi^0$ ^[?]), $\pi^+ p \rightarrow \Delta^{++}\pi^+\pi^-$ and $\pi^+ p \rightarrow p\pi^+\pi^0\pi^0$ ^[?]. The off-shell extrapolation of the $\pi\pi$ interaction and the reaction mechanism cause uncertainties for the determination of the $\pi\pi$ phase shifts.

Many years ago, it was realized that when particles are produced in a reaction, some of these particles often interact strongly with each other before going outside the interaction range^[?], and this, the

so-called final state interaction (FSI), makes the analysis complicated. But, FSI does provide important information about the reaction mechanism and the interaction among the outgoing particles.

In this paper we discuss the S -wave isoscalar-channel $\pi\pi$ scattering in the $\psi' \rightarrow J/\psi \pi^+\pi^-$ decay process and provide a formalism for extracting the $\pi\pi$ S wave phase shifts from the invariant mass spectrum of the $\pi\pi$ in the $\psi' \rightarrow J/\psi \pi^+\pi^-$ decay.

For the decay process $\psi' \rightarrow J/\psi \pi^+\pi^-$, only $I=0$ channel contributes to the final $\pi\pi$ amplitude because the isospins of ψ' and J/ψ are zero. This isospin selection makes $\pi\pi$ phase shift analysis simpler and clearer. In the analysis, the phase space limits the kinematic region of the $\pi\pi$ invariant mass in a region below 0.59GeV, and Bose-statistics limits the angular-momentum between two pions to be 0 or 2, i.e., S wave or D wave. Because the D wave component in the $\pi\pi$ final state is smaller compared to the S wave component^[?], which is consistent with simple analysis, for simplicity, we ignore the D wave component and consider the S wave $\pi\pi$ FSI only in

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the analysis. As mentioned in Refs. [15,16], the Lagrangian for the $\psi' \rightarrow J/\psi \pi^+ \pi^-$ decay in Fig. 1(a) can be written in a form of contact term

$$g\psi'_\mu\psi^\mu\partial_\nu\phi\partial^\nu\phi' \quad (1)$$

Due to FSI, one has to consider an additional diagram Fig. 1(b) for the $\psi' \rightarrow J/\psi \pi^+ \pi^-$ decay. Let β_i and m_i denote the velocity and the mass of particle i in the final state, respectively. It is easy to derive a relation between β_i and m_i

$$\beta_i^2 \leq 1 - \frac{m_i^2}{(M - \sum_{j \neq i} m_j)^2}, \quad (2)$$

where M is the mass of the initial particle, ψ' in this case. This inequality shows that the possible maximal velocity of J/ψ is much smaller than that of the corresponding pions. Therefore, we can neglect the FSI between J/ψ and pions.

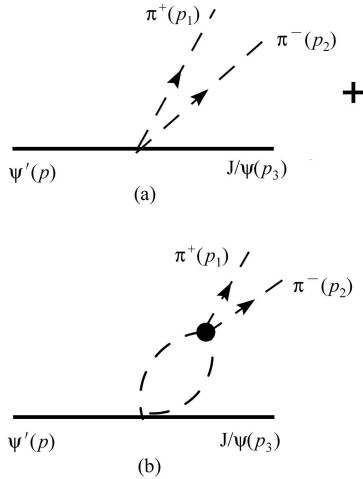


Fig. 1. Diagrams for the $\psi' \rightarrow J/\psi \pi^+ \pi^-$ decay. (a) represents the contact interaction; (b) denotes the process with $\pi\pi$ FSI included.

The decay amplitude for Fig. 1 can be written in terms of $\pi\pi$ amplitudes obtained by using the factorization approximation

$$T \equiv \langle J/\psi \pi^+ \pi^- | t | \psi' \rangle = P(1 + G t_{\pi^+ \pi^-, \pi^+ \pi^- + \pi^- \pi^+ + \pi^0 \pi^0}^{I=0}), \quad (3)$$

where P is the amplitude of the contact term, G represents the loop propagator, and

$$t_{\pi^+ \pi^-, \pi^+ \pi^- + \pi^- \pi^+ + \pi^0 \pi^0}^{I=0} = \langle \pi^+ \pi^- | t | \pi^+ \pi^- + \pi^- \pi^+ + \pi^0 \pi^0 \rangle$$

is the amplitude of the process $\pi^+ \pi^- + \pi^- \pi^+ + \pi^0 \pi^0 \rightarrow \pi^+ \pi^-$. If we denote the four-momenta of ψ' , π^+ , π^-

and J/ψ by p , p_1 , p_2 and p_3 , respectively, the term $P G t_{\pi^+ \pi^-, \pi^+ \pi^- + \pi^- \pi^+ + \pi^0 \pi^0}^{I=0}$ actually means an integration

$$\int \frac{d^4 q}{(2\pi)^4} P(p, p_3; q) G(p - p_3, q) t(q; p_1, p_2), \quad (4)$$

where q represents the pion momentum on one of the loop lines.

The amplitude for the S wave contact term reads^[? ?]

$$P(s) = -\frac{4g}{f_\pi^2} \varepsilon'^{(\lambda')} \cdot \varepsilon^{(\lambda)*} p_1 \cdot p_2 = -\frac{2g}{f_\pi^2} \varepsilon'^{(\lambda')} \cdot \varepsilon^{(\lambda)*} (s - 2m_\pi^2), \quad (5)$$

where g is the coupling constant and f_π is the pion decay constant with the experimental value of 92.4 MeV. From Eq. (??), one sees that P is not q -dependent. This implies the direct production term P can be factorized from the loop integration.

With the phase convention $|\pi^+\rangle = -|1, 1\rangle$, the wave function for the $I = 0$ $\pi\pi$ system can be written as

$$|\pi\pi\rangle^{I=0} = -\frac{1}{\sqrt{6}} (|\pi^+\rangle|\pi^-\rangle + |\pi^-\rangle|\pi^+\rangle + |\pi^0\rangle|\pi^0\rangle). \quad (6)$$

Then the amplitude $t_{\pi^+ \pi^-, \pi^+ \pi^- + \pi^- \pi^+ + \pi^0 \pi^0}^{I=0}$ can be rewritten as

$$\begin{aligned} \langle \pi^+ \pi^- | t | \pi^+ \pi^- + \pi^- \pi^+ + \pi^0 \pi^0 \rangle &= \\ \langle \pi^+ \pi^- | t | (-\sqrt{6}) \pi\pi \rangle^{I=0} &= \\ {}^{I=0} \langle (-2/\sqrt{6}) \pi\pi | t | (-\sqrt{6}) \pi\pi \rangle^{I=0} &= 2 t_{\pi\pi, \pi\pi}^{I=0}, \end{aligned} \quad (7)$$

where $t_{\pi\pi, \pi\pi}^{I=0}$ is the $I = 0$ $\pi\pi$ scattering amplitude which can be related to the S wave $I = 0$ $\pi\pi$ scattering phase shifts if we neglect the D wave contribution. Note that with the normalization ${}^{I=0} \langle \pi\pi | \pi\pi \rangle^{I=0} = 1$, $|\pi^+\rangle|\pi^-\rangle$ should be normalized to 2.

On the other hand, we take the normalization for partial wave amplitudes in such a way that the unitary relation for the partial amplitudes satisfies

$$\text{Im} T_l(s) = T_l^\dagger(s) \rho(s) T_l(s) \quad (8)$$

with the phase space factor

$$\rho(s) = \frac{p_{\text{cm}}}{8\pi\sqrt{s}} = \frac{1}{16\pi} \left(1 - \frac{4m_\pi^2}{s}\right)^{1/2}. \quad (9)$$

The relation among the partial wave amplitude T_l^I with isospin I and the phase shift parameters δ_l^I and

η_l^I reads

$$T_l^I(s) = \frac{1}{2i\rho(s)} (\eta_l^I(s) e^{2i\delta_l^I(s)} - 1). \quad (10)$$

Then, taking on-shell approximation in relating $t_{\pi\pi, \pi\pi}^{I=0}$ in the decay amplitude to the physical values of phase shifts, the S wave isoscalar $\pi\pi$ scattering amplitude $t_{\pi\pi}^{I=0}$ becomes

$$t_{\pi\pi, \pi\pi}^{I=0}(s) = \frac{1}{2i\rho(s)} (e^{2i\delta_0^0(s)} - 1), \quad (11)$$

where the inelastic coefficient $\eta_0^0(s)$ is taken to be 1 in this elastic scattering process. From Eq. (??), one finds that $t_{\pi\pi, \pi\pi}^{I=0}$ is also independent of q , so that it can be factorized out from the loop integration. The $\pi\pi$ loop integration

$$G(s) = i \int \frac{dq^4}{(2\pi)^4} \frac{1}{q^2 - m_\pi^2 + i\epsilon} \frac{1}{(p_1 + p_2 - q)^2 - m_\pi^2 + i\epsilon}, \quad (12)$$

where $s = (p_1 + p_2)^2$ is the squared four-momentum of the dipion system, can be calculated by using a three-momentum cut-off parameter q_{\max} , and the analytic formula in the frame of the center of mass of dipion can be derived as

$$G(s) = \frac{1}{8\pi^2} \left\{ \sigma(s) \arctan \frac{1}{\lambda\sigma(s)} - 2 \ln \left[\frac{q_{\max}}{m_\pi} (1 + \lambda) \right] \right\}, \quad (13)$$

where $\sigma(s) = \sqrt{\frac{4m_\pi^2}{s} - 1}$, $\lambda = \sqrt{1 + \frac{m_\pi^2}{q_{\max}^2}}$, and q_{\max} takes a reasonable value around 1GeV^[3, 17].

Finally, we rewrite the decay amplitude as

$$T \equiv \varepsilon'^{(\lambda')} \cdot \varepsilon^{(\lambda)*} T_R(s) = -\frac{2g}{f_\pi^2} \varepsilon'^{(\lambda')} \cdot \varepsilon^{(\lambda)*} (s - 2m_\pi^2) (1 + 2G(s) t_{\pi\pi, \pi\pi}^{I=0}). \quad (14)$$

From Eq. (14), one finds that the angular dependence in the total decay amplitude comes from $\varepsilon'^{(\lambda')} \cdot \varepsilon^{(\lambda)*}$ only.

The differential decay width is given^[18] by

$$d\Gamma = \frac{1}{(2\pi)^5} \frac{1}{16M^2} \sum_\lambda \overline{\sum_{\lambda'}} |T|^2 |\mathbf{p}_1^*| |\mathbf{p}_3| dm_{\pi\pi} d\Omega_1^* d\Omega_3, \quad (15)$$

where M is the mass of ψ' , $m_{\pi\pi} = \sqrt{s}$ is the invariant mass of $\pi^+\pi^-$, $\sum_\lambda \overline{\sum_{\lambda'}}$ describes the average over initial states and the sum over final states, $(|\mathbf{p}_1^*|, \Omega_1^*)$ is the momentum of π^+ in the rest frame of the dipion, and $(|\mathbf{p}_3|, \Omega_3)$ is the momentum of J/ψ in the rest frame of ψ' . $|\mathbf{p}_1^*|$ and $|\mathbf{p}_3|$ are expressed, respectively, by

$$|\mathbf{p}_1^*| = \left(\frac{s}{4} - m_\pi^2 \right)^{1/2}, \quad (16)$$

and

$$|\mathbf{p}_3| = \frac{1}{2M} [(M^2 - (m_{\pi\pi} + m_3)^2)(M^2 - (m_{\pi\pi} - m_3)^2)]^{1/2}, \quad (17)$$

where m_3 is the mass of J/ψ . Taking into account the fact that ψ' produced in the e^+e^- collision experiments is transversely polarized with respect to the beam direction, the average over the initial state should be performed with respect to only two directions. With

$$\sum_\lambda \overline{\sum_{\lambda'}} |\varepsilon'^{(\lambda')} \cdot \varepsilon^{(\lambda)*}|^2 = \frac{1}{2} \sum_{\lambda=0, \pm 1} \sum_{\lambda'=\pm 1} |\varepsilon'^{(\lambda')} \cdot \varepsilon^{(\lambda)*}|^2 = 1 + \frac{\mathbf{p}_3^2}{2m_3^2} (1 - \cos^2\theta_3), \quad (18)$$

the differential decay width with respect to the $\pi\pi$ invariant mass can finally be written as

$$\begin{aligned} \frac{d\Gamma}{dm_{\pi\pi}} &= \frac{1}{32\pi^3 M^2} |T_R(s)|^2 |\mathbf{p}_1^*| |\mathbf{p}_3| \left(1 + \frac{\mathbf{p}_3^2}{3m_3^2}\right) = \\ &= \frac{4g^2}{32\pi^3 M^2 f_\pi^2} (s - 2m_\pi^2)^2 |\mathbf{p}_1^*| |\mathbf{p}_3| \left(1 + \frac{\mathbf{p}_3^2}{3m_3^2}\right) \times \\ &= \left| 1 + \frac{G(s)}{i\rho(s)} (e^{2i\delta_0^0(s)} - 1) \right|^2, \end{aligned} \quad (19)$$

where $s = m_{\pi\pi}^2$. By fitting experimental $\pi^+\pi^-$ invariant mass spectrum of the $\psi' \rightarrow J/\psi \pi^+\pi^-$ decay bin by bin, one can extract the S wave isoscalar $\pi\pi$ phase shifts.

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从 $\psi' \rightarrow J/\psi \pi^+ \pi^-$ 衰变中抽取 $\pi\pi$ 相移*

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摘要 $\psi' \rightarrow J/\psi \pi^+ \pi^-$ 提供了一个抽取低于 0.59 GeV 的 $\pi\pi$ S 波相移的新途径. 基于在树图衰变振幅中忽略 D 波的近似, 给出了从 $\psi' \rightarrow J/\psi \pi^+ \pi^-$ 的 $\pi\pi$ 不变质量谱中得到 $\pi\pi$ S 波相移的公式.

关键词 $\pi\pi$ S 波相移 $\psi' \rightarrow J/\psi \pi^+ \pi^-$