# A Monte Carlo Study on Event-by-Event Transverse Momentum Fluctuation at RHIC<sup>\*</sup>

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**Abstract** The experimental observation on the multiplicity dependence of event-by-event transverse momentum fluctuation in relativistic heavy ion collisions is studied using Monte Carlo simulation. It is found that the Monte Carlo generator HIJING is unable to describe the experimental phenomenon well. A simple Monte Carlo model is proposed, which can recover the data and thus shed some light on the dynamical origin of the multiplicity dependence of event-by-event transverse momentum fluctuation.

**Key words** relativistic heavy ion collision, event by event fluctuation, transverse momentum, dynamical fluctuation, Monte Carlo model

The relativistic heavy ion collider RHIC at BNL has successfully finished its four running period. A large number of unexpected new results have been observed<sup>[1]</sup>. The discovery of a new state of matter — Quark Gluon Plasma (QGP) is near at hand<sup>[2]</sup>. To further confirm the observation of QGP and study its property are under intensive investigation.

It is expected that the appearance of quark gluon plasma (QGP) phase transition will result in an anomalous behavior in the event-by-event fluctuation of transverse momentum<sup>[3-5]</sup>. The central and</sup> peripheral heavy ion collisions have very different physics. The former one has a large number of participants and occupies a large volume, and QGP is more probably produced in such collisions. Therefore, the centrality dependence of the event-by-event transverse momentum fluctuation can provide important information on the appearance and disappearance of the QGP phase transition. Recently, STAR Collaboration<sup>[6]</sup> published the results from their experiment, using multiplicity to characterize centrality. The aim of the present letter is to discuss the dynamical origin behind the experimentally observed<sup>[6]</sup> multiplicity dependence of event-by-event transverse momentum fluctuation using Monte Carlo method.

Firstly, let us briefly discuss the measure of the event-by-event fluctuation of transverse momentum. A typical measure for this purpose is<sup>[7]</sup>

$$\sigma_{\bar{p}_{t \text{CERES}}}^{2} = \frac{\sum_{j=1}^{N} N_{j} (\bar{p}_{t}^{j} - \langle \bar{p}_{t} \rangle)^{2}}{\sum_{j=1}^{N} N_{j}} - \frac{\sigma_{p_{t} \text{incl}}^{2}}{\langle n \rangle}, \quad (1)$$

where  $\sigma_{p_{tincl}}^{2}$  is the inclusive variance of transverse momentum,  $\langle n \rangle$  the average (charged) multiplicity,  $\bar{p}_{t}$ the mean transverse momentum in a single event,  $N_{j}$ is the (charged) multiplicity in the event, and  $\mathcal{N}$  the total number of events. While studying the multiplicity dependence, we divide the event sample into multiplicity bins and the average multiplicity in each bin  $\langle n \rangle$ , denoted in the following simply by n, is taken as the characteristic of the bin. Thus the eventby-event fluctuation of transverse momentum in the multiplicity bin n is

$$\sigma_{\bar{p}_{t\,\text{CERES}}}^{(n)2} = \frac{1}{n} \left( \frac{1}{\mathcal{N}} \sum_{j=1}^{\mathcal{N}} N_j (\bar{p}_t^{\ j} - \langle \bar{p}_t \rangle)^2 - \sigma_{p_t\text{incl}}^2 \right). \quad (2)$$

STAR Collaboration presented their data using a related quantity —— "difference factor"  $\Delta \sigma_{p_t n}$ 

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$$\Delta \sigma_{p_t n} = \frac{n}{2\sigma_{p_t \text{incl}}} \sigma_{\vec{p}_t \text{ CERES}}^{(n)2} \quad . \tag{3}$$

The dependence of  $\Delta \sigma_{p_{\rm t}n}$  on *n* from STAR data is shown as solid circles in Fig. 1. In the abscissa of their plot,  $n_0$  is the half-max point at the end of the mini-bias distribution plotted as  $d\sigma/dN_{\rm ch}^{1/4}$ . In our calculation, we take  $n_0 = 630^{[8]}$ .



Fig. 1. The experimental (solid circles) and dynamical (stars)  $\Delta \sigma_{ptn}$ , together with those obtained from HIJING (solid triangles) and from the model of present letter (open circles). The experimental data are taken from Ref. [6].

The HIJING Monte Carlo generator has been very successful in explaining a large number of data from relativistic heavy ion collision experiments at RHIC energies. Therefore, we first compare the experimentally observed *n*-dependence of  $\Delta \sigma_{p_t n}$  with HIJING.

In total 1,000,000 mini-bias events are generated using HIJING Monte Carlo. The number of events in each impact-parameter bin is obtained by Glauber theory and the  $\Delta \sigma_{p_t n}$  in each multiplicity bin is then calculated. The results, however, turn out to be smaller than the experimental data for about a factor of three, *cf.* the solid triangles in Fig. 1. This shows that there should be some other dynamical reasons for the experimentally observed event-by-event fluctuation of transverse momentum, which has not been included in the HIJING model.

In order to discuss the dynamical origin of the experimental phenomenon we propose a simple Monte Carlo model. The basic assumption of this model is that, in each single event there exists a dynamical transverse momentum distribution, which fluctuates event-by-event.

The dynamical transverse momentum distribution in each single event is assumed to be a  $\Gamma$  distribution  $P(p_t|\bar{p}_t)$ , with the average value equal to  $\bar{p}_t$ .

$$\int P(p_t|\bar{p}_t) dp_t = 1, \qquad (4)$$

$$\int p_{\rm t} P(p_{\rm t}|\bar{p}_{\rm t}) \mathrm{d}p_{\rm t} = \bar{p}_{\rm t}.$$
(5)

Due to these conditions, the distribution has only one parameter  $\alpha$  and can be written as

$$P(p_{t}|\bar{p}_{t}) = \frac{\alpha^{\alpha \bar{p}_{t}}}{\Gamma(\alpha \bar{p}_{t})} p_{t}^{\alpha \bar{p}_{t}-1} e^{-\alpha p_{t}}.$$
 (6)

We further assume that the parameter  $\alpha$  keeps constant and the event-by-event fluctuation of the distribution  $P(p_t|\bar{p}_t)$  is simplified as the fluctuation of  $\bar{p}_t$ . In a multiplicity bin *n* the fluctuation of  $\bar{p}_t$  is described by a distribution  $Q_n(\bar{p}_t)$  with the normalization conditions

$$\int Q_n(\bar{p}_t) \mathrm{d}\bar{p}_t = 1, \qquad (7)$$

$$\left[\bar{p}_{t}Q_{n}(\bar{p}_{t})\mathrm{d}\bar{p}_{t}=\langle p_{t}
angle _{n},$$
(8)

where  $\langle p_t \rangle_n$  is the average transverse momentum in the corresponding multiplicity bin. The  $\langle p_t \rangle_n$  versus *n* used in the following calculation is shown in Fig. 2<sup>[9]</sup>.



Fig. 2. Average transverse momentum in various multiplicity bins.

$$Q_{n}(\bar{p}_{t}) = \begin{cases} A, & \text{if } \bar{p}_{t} \in [0.2, a], \\ B, & \text{if } \bar{p}_{t} \in (a, a+b], \\ 0, & \text{else}, \end{cases}$$
(9)

with the boundary a of the two regions fixed at the average transverse momentum in the corresponding multiplicity bin:  $a = \langle p_t \rangle_n$ . The normalizations (7) and (8) express the parameters b and B in terms of A. Therefore, A is the only adjustable parameter of the distribution.

In the following calculation we take  $\alpha = 10$  and adjust the parameter A to fit the experimental data of  $\Delta \sigma_{p_t n}$  versus n. The results are shown in Fig. 1 as open circles. It can be seen from the figure that the fit between the open and solid circles is very well.

Note that the  $\Delta \sigma_{p_t n}$  used by STAR Collaboration as defined in Eq. (3) is based on the "non-statistical" variance  $\sigma_{\bar{p}_t \text{CERES}}^2$  of  $\bar{p}_t$  proposed in Ref. [7], as shown in Eq. (1). It is expected that in this "non-statistical" variance the statistical fluctuations have already been subtracted<sup>[10]</sup>, *cf.* the second term in the right hand side of Eq. (1). However, this subtraction is imperfect<sup>[11,12]</sup> and the  $\sigma_{\bar{p}_t \text{CERES}}^2$  is not identical to the real dynamical variance  $\sigma_{\bar{p}_t \text{dyn}}^2 = \langle \bar{p}_t^2 \rangle - \langle \bar{p}_t \rangle^2$ .

Similar to the definition Eq. (3) used by STAR Collaboration we define the dynamical "difference factor" in the multiplicity bin n as

$$\Delta \sigma_{p_{\rm t}n}^{\rm dyn} = \frac{n}{2\sigma_{p_{\rm t}{\rm incl}}} \sigma_{\bar{p}_{\rm t}{\rm dyn}}^{(n)2}, \quad \sigma_{\bar{p}_{\rm t}{\rm dyn}}^{(n)2} = \langle \bar{p}_{\rm t}^2 \rangle_n - \langle \bar{p}_{\rm t} \rangle_n^2. \tag{10}$$

The results of  $\Delta \sigma_{p_t n}^{\rm dyn}$  from our model are also shown in Fig.1 as stars.

Some discussions follow:

1. The dependence of  $\Delta \sigma_{p_t n}$  on *n* observed by STAR Collaboration is an important new phenomenon. HIJING is unable to describe this phenomenon well, showing that the dynamics behind this phenomenon is not included in HIJING.

2. This phenomenon can be simply explained under the assumption that in each single event there exists a dynamical transverse momentum distribution, which fluctuates event-by-event. 3. The measure  $\Delta \sigma_{p_t n}$  used by STAR Collaboration is based on the "non-statistical" variance  $\sigma_{\bar{p}_t CERES}^2$  of  $\bar{p}_t$  proposed in Ref. [7], in which the subtraction of statistical fluctuations is imperfect<sup>[11,12]</sup>. In particular, at low multiplicity the statistical fluctuations are "over-subtracted", so the measured values are smaller than the dynamical ones, *cf.* Fig. 1 of this letter and that of Ref. [12]. At very high multiplicity (n > 300), the subtraction is insufficient and the measured values are larger than the dynamical ones. The difference is highly pronounced by the square of  $\sigma$  and the multiplication of n, *cf.* Eq. (3), *cf.* Fig. 1 and Fig. 3.



Fig. 3. A comparison of  $\sigma_{\bar{p}_t \text{CERES}}$  and  $\sigma_{\bar{p}_t \text{Dynamics}}$ .

The good fit of the simple model in this letter with experimental data shows the validity of the basic model assumption — the event-by-event fluctuation of the dynamical single-event transverse momentum distribution. The details of the model are unimportant. For example, the simple shape of the distributions, the fixed value of the parameter ( $\alpha = 10$ ) for different multiplicity bins, are oversimplified. Further study of the problem is worthwhile.

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## 相对论重离子碰撞中逐事件横动量起伏的蒙特卡罗研究\*

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摘要 用蒙特卡罗模型研究了相对论重离子碰撞中逐事件横动量起伏与多重数的关联,发现蒙特卡罗产生器 HIJING不能描述实验现象.提出了一个简单的蒙特卡罗模型,可以与实验一致,有助于理解逐事件横动量起伏 与多重数关联的动力学起源.

关键词 相对论重离子碰撞 逐事件起伏 横动量 动力学起伏 蒙特卡罗模型

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