SD-Pair Shell Model and Proton-Neutron Interacting Boson Model*

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Abstract The typical spectra corresponding to the $U(5)$, $SO(6)$ and $SU(3)$-limiting cases in the interacting boson model are studied within the framework of nucleon-pair shell model truncated to SD-subspace. It is found that they can all be reproduced approximately in the SD-pair shell model.

Key words nucleon-pair shell model, interacting boson model

1 Introduction

The discovery of collective motions, such as collective vibration, collective rotation, giant resonances, etc. in the medium and heavy nuclei is one of the great wonders in nuclear physics. How to describe these collective motions in terms of fermion degree of freedom is an interesting and challenging problem in theory of nuclear structure.

Since the modern computing tool is still out of reach for a direct large—space shell model calculation, one has to use some kinds of truncation schemes. By using the generalized Wick theorem for fermion clusters[1], the nucleon-pair shell model (NPSM) has been proposed for nuclear collective motion[2]. Because the computing time increases drastically with the size of the subspace, for applying this model to medium and heavy nuclei, we have to truncate the shell model space to the collective SD subspace, which is called the SD-pair shell model(SDPSM).

It is the aim of this paper to see if the SDPSM can reproduce the vibrational, rotational and γ-soft spectra corresponding to those of $U(5)$, $SU(3)$ and $SO(6)$-limit spectra shown in the IBM[3].

2 A Brief review of the model

The Hamiltonian is chosen to be

$$H = H_\pi + H_\nu - \kappa Q_\pi^2 Q_\nu^2,$$
$$H_\sigma = \sum_{\sigma a} \varepsilon_{\sigma a} n_{\sigma a} - G S^+(\sigma) S(\sigma),$$
$$Q_\mu^2 = \sqrt{16\pi/5}\sum_{i=1}^{n} r_i^2 Y_{2\mu}(\vartheta_i \varphi_i),$$

where the $\varepsilon_{\sigma a}$, G_σ and κ is the single particle energy for orbit a, pairing interaction strength and quadrupole-quadrupole interaction strength, respectively.

The E2 transition operator is

$$T(\mathrm{E2}) = e_\pi Q_\pi^2 + e_\nu Q_\nu^2$$

where e_ν and e_π are effective charges of neutron and proton, respectively. The collective pairs A_μ^{r+} of angular momentum $r=0,2$ with projection μ are $A_\mu^{r+} = \sum_{ab} y(abr)(C_a^+ \times C_b^+)_\mu^r$, where $y(abr)$ is the structure coefficients. We shall restrict ourselves in this paper to the case of degenerate j-orbits to simplify the treatment. Thus the S-pair structure coef-

*Supported by U.S. National Science Foundation (9970769, 0140300), China Education Department of Liaoning Province (202122024) and National Natural Science Foundation of China (10305006)

ficients are

$$y(aa0) = \widehat{a}\sqrt{\left(\frac{N}{\Omega_a - N}\right)^{1/2}} \quad ,$$

with $\Omega_a = a + \frac{1}{2}$, N is the number of the pairs for like-nucleon. The D pair[4] is obtained by using the commutator, $D^+ = \frac{1}{2}[Q^2, S^+] = \sum_{ab} y(ab2)(C_a^+ \times C_b^+)^2$.

3 Results

To see if the vibrational spectrum in the IBM[3] can be produced within the SDPSM, the proton-neutron coupled system with $N_\pi = N_\nu = 2$ is studied. We restricted ourselves to 50—82 shell. The pairing interaction strengths for proton and neutron are assumed for simplicity to be the same, $G_\pi = G_\nu \equiv G$. By fitting $E_{4_1^+}/E_{2_1^+} = 2.0$, $G/\kappa = 30r_0^4$ is fixed. Some low-lying states divided by $E_{2_1^+}$ are presented in Fig. 1. It is seen that the $U_\pi(5) \times U_\nu(5)$ limiting spectrum is produced within SDPSM. In addition to the spectrum, E2 transition can also be used to identify the collectivity of low-lying state. The relative B(E2) ratios for some low-lying states are presented in Fig. 1. It can be seen that the 4_1^+, 2_3^+ and 0_2^+ mainly de-excite to 2_1^+ state, a typical feature of the vibrational limit.

From above analysis one can see that the $U_\pi(5) \times U_\nu(5)$ symmetry in IBM can be realized in the SDPSM.

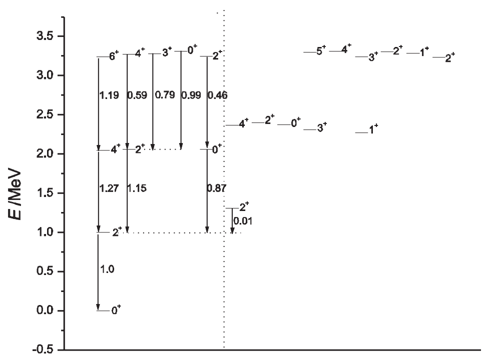

Fig. 1. The vibrational spectrum in the SDPSM. The relative B(E2) ratios are also shown with effective charges fixed as $e_\pi = 3e_\nu = 1.5e$.

From periodical table, we notice that the nuclei which show $SO(6)$ feature are all close to the end of the shell, at least for neutron sector. Hence, to see if the Υ-soft spectrum can be realized in the SDPSM, we still restricted ourselves to 50—82 shell, and the same system as those used in vibrational limit is still used here. As discussed in Ref. [5], since the $SO(6)$ nuclei in 50—82 shell are those with neutron number close to the end of the shell, the 2-neutron pairs are treated as 2-neutron-hole pairs, thus a negative κ is used. The parameter $G/\kappa = 3r_0^4$ was adopted by fitting $E_{4_1^+}/E_{2_1^+} = 2.5$. The excitation energies divided by $E_{2_1^+}$ are given in Fig. 2. We can see that the typical feature of the Υ-soft spectrum for the $SO_\pi(6) \times SO_\nu(6)$ symmetry in the IBM-Ⅱ[3] can be realized in the SDPSM. The relative E2 transitions for some important low-lying states are also shown in Fig. 2. It is seen that 4_1^+ and 2_2^+ states mainly de-excite to 2_1^+. What's more, the E2 transition between the 0_2^+ state and 2_2^+ state is dominant over that between 0_2^+ and 2_1^+ states, another typical feature of Υ-soft limit. In one word, the $SO_\pi(6)\times SO_\nu(6)$ symmetry in the IBM-Ⅱ can be reproduced rather well in the SDPSM.

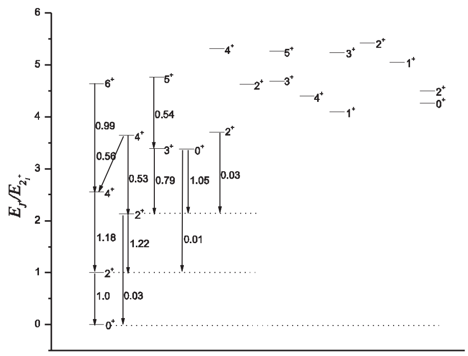

Fig. 2. The Υ-unstable spectrum. The relative B(E2) ratios are also given with the effective charges fixed as $e_\pi = 3e_\nu = 1.5e$.

To see if the rotational limit can be realized in the SDPSM, a Hamiltonian we used is

$$H = -\frac{1}{2}\kappa(Q_\pi^2 + Q_\nu^2)(Q_\pi^2 + Q_\nu^2)$$

Similar discussions as those in the vibration and Υ-soft limit is performed. We choose gds shell for both proton and neutron sectors as an example. With κ fixed as $0.01\text{MeV}/r_0^4$, the rotational limit for coupled proton-neutron system is shown in Fig. 3, from which one can see that similar to that shown in Ref. [6], the $SU(3)$ limit spectrum[3] can be reproduced very well in the SDPSM. In Fig. 3, all of the states are grouped into bands according to their B(E2) values. To show the B(E2) values explicitly, some of the relative B(E2) ratios are listed in Table 1, from which one can easily see that the inter-band transition are very strong, while they are much larger than those of the intra-band transitions. The three typical E2 transitions for rotational limit are realized. From above analysis, one can see that the $SU_\pi(3)\times SU_\nu(3)$ symmetry[3] in the IBM-Ⅱ can be produced in the SDPSM.

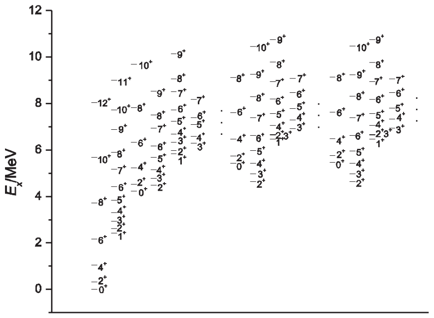

Fig. 3. The rotational spectrum for the coupled proton-neutron system in SDPSM.

Table 1. A part of relative B(E2) ratios for the rotational spectrum. The system with $N_\pi = N_\nu = 3$ was studied. The effective charges were fixed as $e_\pi = 3e_\nu = 1.5e$.

$J_i->J_f/2_1->0_1$		$J_i->J_f/3_1->1_1$	
$4_1->2_1$	1.346	$4_2->2_2$	1.112
$6_1->4_1$	1.319	$6_2->4_2$	1.163
$8_1->6_1$	1.138	$5_1->3_1$	1.439
$2_2->2_1$	0.009	$7_1->5_1$	1.356
$0_2->2_1$	0	$8_2->6_2$	0.933
$J_i->J_f/2_6->0_2$		$J_i->J_f/4_5->2_5$	
$4_6->2_6$	1.129	$5_4->3_4$	1.003
$6_6->4_6$	1.026	$6_5->4_5$	1.556
$8_6->6_6$	0.707	$7_4->5_4$	1.049
		$8_5->6_5$	1.323

4 Summary

In this paper, within the framework of nucleon-pair shell model truncated to SD subspace, the typical limiting spectra in the IBM are reproduced. Our study confirms that IBM has a sound shell model foundation, which also encourages us to further explore the SDPSM.

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SD对壳模型与质子中子相互作用玻色子模型*

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摘要　在SD对壳模型理论框架下, 讨论了相互作用玻色子模型下$U(5)$, $SO(6)$以及$SU(3)$的经典极限谱. 结果发现SD对壳模型可以很好地再现相互作用玻色子模型中的三种极限谱.

关键词　原子核配对壳模型　相互作用玻色子模型

* 美国国家自然基金(9970769, 0140300), 辽宁省教育部基金(202122024)和中国国家自然科学基金(10305006)资助