

# Magnetic Moments of $J^P = \frac{3}{2}^+$ Pentaquarks<sup>\*</sup>

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**Abstract** If the  $J^P$  of  $\Theta_5^+$  and  $\Xi_5^-$  pentaquarks is really found to be  $\frac{1}{2}^+$  in future experiments, they will be accompanied by  $J^P = \frac{3}{2}^+$  partners in some models. It is reasonable to expect that these  $J^P = \frac{3}{2}^+$  states will also be discovered in the near future with the current intensive experimental and theoretical efforts. We estimate  $J^P = \frac{3}{2}^+$  pentaquark magnetic moments using different models.

**Key words** pentaquark, magnetic moment

## 1 Introduction

After LEPS Collaboration announced the discovery of the  $\Theta^+$  pentaquark<sup>[1]</sup>, several experimental groups have confirmed its existence in various reaction channels<sup>[2-7]</sup>. This state lies around 1540 MeV with strangeness  $S = +1$ , baryon number  $B = +1$  and a very narrow width. Such a state cannot be accommodated within the conventional quark model. Its minimum quark content is  $|uudds\rangle$ . NA49 Collaboration found a new narrow baryon resonance with  $B = +1$ ,  $Q = -2$ ,  $S = -2$ ,  $I = 3/2$  around  $(1.862 \pm 0.002)\text{GeV}$ <sup>[8]</sup>.

The  $\Theta^+$  pentaquark is likely an isoscalar from the lack of enough signals in the  $pK^+$  channel according to Refs. [3,4,6,7]. The  $J^P$  of  $\Theta^+$  has not yet been determined from experiments. Most theoretical papers postulated its angular momentum was  $J = \frac{1}{2}$ . But the possibility of  $J = \frac{3}{2}$  is not excluded<sup>[9]</sup>. Some models assume that the parity of  $\Theta^+$  is positive<sup>[10-15]</sup> while some other models favor negative parity<sup>[16,17]</sup>. The approaches of QCD sum rule<sup>[18,19]</sup> and lattice QCD<sup>[20]</sup> indicate that the parity of  $\Theta^+$  may be negative. There

are many theoretical papers proposing possible ways to determine its parity, among which Refs. [21,22] are two recent ones. A short review of the present status of the pentaquark quantum numbers can be found in Refs. [23,24].

The  $\Theta^+$  pentaquark mass was predicted to be around 1535 MeV in the chiral soliton model<sup>[10,11]</sup>. But the theoretical foundation of this model was challenged by<sup>[25]</sup> in the large  $N_c$  formalism. Recently, Karliner and Lipkin (KL) estimated the mass of  $\Theta^+$  with the assumption that  $\Theta^+$  is composed of one diquark and one triquark with one orbital excitation  $L = 1$  between them<sup>[12]</sup>. Jaffe and Wilczek (JW) assumed that the  $\Theta^+$  pentaquark is composed of two identical scalar diquarks and one anti-quark. Bose symmetry requires odd orbital angular momentum between the scalar diquark pair. In this way they estimated the masses of the antidecuplet, octet, and also some heavy flavor pentaquarks<sup>[13]</sup>. Shuryak and Zahed (SZ) suggested that  $\Theta^+$  mass might be lowered by replacing one scalar diquark with one tensor diquark in JW's model and hence avoiding the orbital excitation between the diquark pair<sup>[14]</sup>.

In Karliner and Lipkin's model the angular momentum of

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the triquark is  $\frac{1}{2}$ . The resulting pentaquark angular momentum is the sum of the orbital and triquark angular momentum,  $J = L + S_{\text{tri}}$ . Hence, one would expect  $J = \frac{1}{2}$  or  $\frac{3}{2}$ . In Ref. [12] only  $J = \frac{1}{2}$  is considered. Similarly, the scalar diquark pair in Jaffe and Wilczek's model carries one unit of angular momentum, which couples to the anti-quark to form  $J = \frac{1}{2}$  or  $\frac{3}{2}$  states. Only the case of  $J = \frac{1}{2}$  is considered in Ref. [13]. In Shuryak and Zahed's model there is no orbital excitation. But one diquark is the tensor diquark with  $S = 1$ . So one would also expect the resulting states to have  $J = \frac{1}{2}$  or  $\frac{3}{2}$ .

In general, the lower the angular momentum, the lower the mass. So the  $J = \frac{3}{2}$  pentaquark will be heavier than its  $J = \frac{1}{2}$  partner. But their mass difference is not expected to be larger than 300MeV if we could rely on the past experience with the  $\Delta$  and nucleon mass splitting. If the  $\Theta^+$  pentaquark does exist, then its  $J = \frac{3}{2}$  pentaquark partner should also be reachable by future experiments.

Dudek and Close estimated the  $J = \frac{3}{2} \Theta^+$  pentaquark mass in JW's model by considering the spin-orbital force and discussed the possible decay channels of these new states<sup>[9]</sup>.

In this work we shall calculate the magnetic moments of the  $J^P = \frac{3}{2}^+$  pentaquarks in the above three models. The magnetic moment is another intrinsic observable of particles which may encode important information of its quark gluon structure and underlying dynamics. In Ref. [24], we have calculated the magnetic moments of the  $J = \frac{1}{2}$  antidecuplet and octet pentaquarks in Strottman's model<sup>[26]</sup> and also in the above mentioned three models. The present work is a straightforward extension of our previous paper<sup>[24]</sup>.

Our paper is organized as follows: in Section 2, 3 and 4, we calculate the  $J^P = \frac{3}{2}^+$  pentaquark magnetic moments in JW's model, SZ's model and KL's model respectively. Finally we present a brief discussion of our results.

## 2 Pentaquark as a bound state of two scalar diquarks and one anti-quark

According to Jaffe and Wilczek's model, highly correlat-

ed up and down quarks form a scalar isoscalar diquark. The pentaquark  $\Theta^+$  is composed of two identical diquarks and one anti-strange quark<sup>[13]</sup>. There is one orbital angular momentum excitation  $L = 1$  between the two scalar diquarks. But there is no orbital excitation between up and down quarks inside the diquark. To obtain a color singlet pentaquark state  $\Theta^+$ , the

**Table 1. Magnetic moments  $\mu_P$  of  $\overline{10}$  pentaquarks in JW's**

**model (in unit of  $\mu_N$ ), where  $\mu_i = \frac{e_i}{2m_i}$  is the  $i$ -th quark magneton, and  $m_u = m_d = 0.36$  GeV,  $m_s = 0.54$  GeV for I and II. For set I we use  $m_{ud} = 0.72$  GeV and  $m_{us} = m_{ds} = 0.90$  GeV from Ref. [12]. For set II we use  $m_{ud} = 0.42$  GeV and  $m_{us} = m_{ds} = 0.60$  GeV from Ref. [14].**

$(Y, I, I_3)$	$\overline{10}$	set I	set II
$(2, 0, 0)$	$-\mu_s + \frac{e_{ud}}{2m_{ud}}$	1.01	1.32
$(1, \frac{1}{2}, \frac{1}{2})$	$-\frac{1}{3}\mu_d - \frac{2}{3}\mu_s + \frac{e_{ud}}{6m_{ud}}$ $+ \frac{1}{3(m_{ud} + m_{us})} \left( \frac{m_{us}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us}} e_{us} \right)$	1.08	1.36
$(1, \frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{3}\mu_u - \frac{2}{3}\mu_s + \frac{e_{ud}}{6m_{ud}}$ $+ \frac{1}{3(m_{ud} + m_{ds})} \left( \frac{m_{ds}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds}} e_{ds} \right)$	-0.093	0.061
$(0, 1, 1)$	$-\frac{1}{3}\mu_s - \frac{2}{3}\mu_d + \frac{e_{us}}{6m_{us}}$ $+ \frac{1}{3(m_{ud} + m_{us})} \left( \frac{m_{us}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us}} e_{us} \right)$	1.15	1.38
$(0, 1, 0)$	$-\frac{1}{3}\mu_u - \frac{1}{3}\mu_d - \frac{1}{3}\mu_s$ $+ \frac{1}{6(m_{ud} + m_{ds})} \left( \frac{m_{ds}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds}} e_{ds} \right)$ $+ \frac{1}{6(m_{ud} + m_{us})} \left( \frac{m_{us}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us}} e_{us} \right)$ $+ \frac{1}{6(m_{us} + m_{ds})} \left( \frac{m_{ds}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds}} e_{ds} \right)$	-0.045	0.037
$(0, 1, -1)$	$-\frac{1}{3}\mu_s - \frac{2}{3}\mu_u + \frac{e_{ds}}{6m_{ds}}$ $+ \frac{1}{3(m_{ud} + m_{ds})} \left( \frac{m_{ds}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds}} e_{ds} \right)$	-1.24	-1.31
$(-1, \frac{3}{2}, \frac{3}{2})$	$-\mu_d + \frac{e_{us}}{2m_{us}}$	1.22	1.39
$(-1, \frac{3}{2}, \frac{1}{2})$	$-\frac{1}{3}\mu_u - \frac{2}{3}\mu_d + \frac{e_{us}}{6m_{us}}$ $+ \frac{1}{3(m_{us} + m_{ds})} \left( \frac{m_{ds}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds}} e_{ds} \right)$	0	0
$(-1, \frac{3}{2}, -\frac{1}{2})$	$-\frac{1}{3}\mu_d - \frac{2}{3}\mu_u + \frac{e_{ds}}{6m_{ds}}$ $+ \frac{1}{3(m_{ds} + m_{us})} \left( \frac{m_{us}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{us}} e_{us} \right)$	-1.22	-1.39
$(-1, \frac{3}{2}, -\frac{3}{2})$	$-\mu_u + \frac{e_{ds}}{2m_{ds}}$	-2.43	-2.78

color wave function of the scalar diquark must be antisymmetric, i. e., in the  $\bar{\mathbf{3}}_c$  color representation. The diquark spin and space wave functions are antisymmetric and symmetric respectively. For the diquark pair, the color, the spin and the space wave functions are antisymmetric, symmetric, and antisymmetric respectively with  $L = 1$ . The spin of the anti-quark is  $\frac{1}{2}$ .

The total angular momentum of the pentaquark will be  $J = \frac{1}{2}$

or  $J = \frac{3}{2}$ . The magnetic moments of  $J^P = \frac{1}{2}^+$  have been calculated in Ref. [24]. Now we shall extend the same

**Table 2. Magnetic moments  $\mu_P$  of 8 pentaquarks in JW's model (in unit of  $\mu_N$ ). The same set of parameters are used as in Table 1.**

$(Y, I, I_3)$	<b>8</b>	set I	set II
$(1, \frac{1}{2}, \frac{1}{2})$	$-\frac{2}{3}\mu_d - \frac{1}{3}\mu_s + \frac{e_{ud}}{3m_{ud}}$ + $\frac{1}{6(m_{ud} + m_{us})} \left( \frac{m_{us}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us}} e_{us} \right)$	1.19	1.49
$(1, \frac{1}{2}, -\frac{1}{2})$	$-\frac{2}{3}\mu_u - \frac{1}{3}\mu_s + \frac{e_{ud}}{3m_{ud}}$ + $\frac{1}{6(m_{ud} + m_{ds})} \left( \frac{m_{ds}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds}} e_{ds} \right)$	-0.70	-0.47
$(0, 1, 1)$	$-\frac{1}{3}\mu_d - \frac{2}{3}\mu_s + \frac{e_{us}}{3m_{us}}$ + $\frac{1}{6(m_{ud} + m_{us})} \left( \frac{m_{us}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us}} e_{us} \right)$	1.04	1.24
$(0, 1, 0)$	$-\frac{1}{6}\mu_u - \frac{1}{6}\mu_d - \frac{2}{3}\mu_s$ + $\frac{1}{12(m_{ud} + m_{ds})} \left( \frac{m_{ds}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds}} e_{ds} \right)$ + $\frac{1}{12(m_{ud} + m_{us})} \left( \frac{m_{us}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us}} e_{us} \right)$ + $\frac{1}{3(m_{us} + m_{ds})} \left( \frac{m_{ds}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds}} e_{ds} \right)$	0.18	0.18
$(0, 1, -1)$	$-\frac{1}{3}\mu_u - \frac{2}{3}\mu_s + \frac{e_{ds}}{3m_{ds}}$ + $\frac{1}{6(m_{ud} + m_{ds})} \left( \frac{m_{ds}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds}} e_{ds} \right)$	-0.68	-0.88
$(-1, \frac{1}{2}, \frac{1}{2})$	$-\frac{2}{3}\mu_u - \frac{1}{3}\mu_d + \frac{e_{us}}{3m_{us}}$ + $\frac{1}{6(m_{us} + m_{ds})} \left( \frac{m_{ds}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds}} e_{ds} \right)$	-0.69	-0.61
$(-1, \frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{3}\mu_u - \frac{2}{3}\mu_d + \frac{e_{ds}}{3m_{ds}}$ + $\frac{1}{6(m_{ds} + m_{us})} \left( \frac{m_{us}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{us}} e_{us} \right)$	-0.52	-0.78
$(0, 0, 0)$	$-\frac{1}{2}\mu_u - \frac{1}{2}\mu_d$ + $\frac{1}{4(m_{ud} + m_{us})} \left( \frac{m_{us}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us}} e_{us} \right)$ + $\frac{1}{4(m_{ud} + m_{ds})} \left( \frac{m_{ds}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds}} e_{ds} \right)$	-0.27	-0.10

formalism and discuss the case of  $J = \frac{3}{2}$ .

The flavor wave functions are the same as those given in Ref. [24]. There is no orbital excitation between the diquark pair and the anti-quark. We focus on the spin and space wave function of  $J = \frac{3}{2}$  pentaquarks, which reads:

$$\Psi_{\frac{3}{2}\frac{3}{2}} = \psi_{11} \chi_{00} \chi_{00} \chi_{\frac{1}{2}\frac{1}{2}}, \quad (1)$$

where  $\psi_{11}$  is the space wave function of the two scalar diquark systems,  $\chi_{00}$  is the scalar diquark spin wave function, and  $\chi_{\frac{1}{2}\frac{1}{2}}$  is the anti-quark spin wave function. The subscripts denote the angular momentum and the third component.

Here we briefly outline some useful equations. The magnetic moment of a compound system is:

$$\boldsymbol{\mu} = \sum_i \boldsymbol{\mu}_i = \sum_i (g_i \boldsymbol{s}_i + \boldsymbol{l}_i) \mu_i, \quad (2)$$

where  $g_i$  is the  $g$ -factor of  $i$ -th constituent and  $\mu_i$  is the magneton of the  $i$ -th constituent  $\mu_i = \frac{e_i}{2m_i}$ . Since there is no excitation between the two scalar diquark systems and the anti-quark, only the diquark pair system with  $L = 1$  contributes to the orbital magnetic moment:

$$\mu_l = \frac{m_2 \mu_1}{m_1 + m_2} + \frac{m_1 \mu_2}{m_1 + m_2}, \quad (3)$$

where  $m_i$  and  $\mu_i$  are the mass and magneton of  $i$ -th diquark respectively. Finally, we obtain the total magnetic moment of a pentaquark for the case  $J = \frac{3}{2}$ :

$$\begin{aligned} \mu &= \langle 2\mu_{\bar{q}} \frac{\mathbf{1}}{2} + \mu_l \mathbf{l} \rangle \left( J_z = \frac{3}{2} \right) = \\ &\langle 11 \frac{1}{2} \frac{1}{2} \left| \frac{3}{2} \frac{3}{2} \right\rangle^2 \mu_{\bar{q}} + \langle 11 \frac{1}{2} \frac{1}{2} \left| \frac{3}{2} \frac{3}{2} \right\rangle^2 \mu_l = \\ &\mu_{\bar{q}} + \mu_l. \end{aligned} \quad (4)$$

In Jaffe and Wilczek's model, we present pentaquark magnetic moments and their numerical values in Table 1 and Table 2 with the input parameters  $m_u = m_d = 0.36$  GeV and  $m_s = 0.54$  GeV.

### 3 Pentaquark as a bound state of one scalar diquark, one tensor diquark and one anti-quark

According to Shuryak and Zahed,  $\Theta^+$  pentaquark is a bound state composed of one scalar diquark, one tensor diquark and one anti-strange quark, without any relative angular momentum excitations among the three clusters<sup>[14]</sup>. Now both the scalar diquark and the tensor diquark are in the antisym-

metric  $\bar{\mathbf{3}}_c$  color representation and antisymmetric  $\bar{\mathbf{3}}_f$  flavor representation. For  $\Theta^+$  pentaquark of isospin  $I = 0$ , the scalar

diquark-tensor diquark system must be still in the symmetric  $\bar{\mathbf{6}}_f$  flavor representation.

**Table 3.** Magnetic moments  $\mu_p$  of  $\mathbf{10}$  pentaquarks in SZ's model (in unit of  $\mu_N$ ) with the parameters  $m_{ud}^T = 0.57$  GeV,  $m_{us}^T = m_{ds}^T = 0.72$  GeV,  $m_u = m_d = 0.36$  GeV,  $m_s = 0.54$  GeV from Ref [14].

$(Y, I, I_3)$	$\bar{\mathbf{10}}$	$\mu_p$
$(2, 0, 0)$	$\frac{1}{2} \mu_u + \frac{1}{2} \mu_d - \mu_s + \frac{1}{4(m_u + m_d)} \left( \frac{m_d}{m_u} e_u + \frac{m_u}{m_d} e_d \right)$	1.23
$\left(1, \frac{1}{2}, \frac{1}{2}\right)$	$\frac{1}{2} \mu_u - \frac{1}{2} \mu_s + \frac{1}{6(m_u + m_d)} \left( \frac{m_d}{m_u} e_u + \frac{m_u}{m_d} e_d \right) + \frac{1}{12(m_u + m_s)} \left( \frac{m_u}{m_s} e_s + \frac{m_s}{m_u} e_u \right)$	1.44
$\left(1, \frac{1}{2}, -\frac{1}{2}\right)$	$\frac{1}{2} \mu_d - \frac{1}{2} \mu_s + \frac{1}{6(m_u + m_d)} \left( \frac{m_d}{m_u} e_u + \frac{m_u}{m_d} e_d \right) + \frac{1}{12(m_d + m_s)} \left( \frac{m_d}{m_s} e_s + \frac{m_s}{m_d} e_d \right)$	-0.13
$(0, 1, 1)$	$\frac{1}{2} \mu_u - \frac{1}{2} \mu_d + \frac{1}{12(m_u + m_d)} \left( \frac{m_d}{m_u} e_u + \frac{m_u}{m_d} e_d \right) + \frac{1}{6(m_u + m_s)} \left( \frac{m_u}{m_s} e_s + \frac{m_s}{m_u} e_u \right)$	1.65
$(0, 1, 0)$	$\frac{1}{12(m_u + m_d)} \left( \frac{m_d}{m_u} e_u + \frac{m_u}{m_d} e_d \right) + \frac{1}{12(m_d + m_s)} \left( \frac{m_d}{m_s} e_s + \frac{m_s}{m_d} e_d \right) + \frac{1}{12(m_u + m_s)} \left( \frac{m_u}{m_s} e_s + \frac{m_s}{m_u} e_u \right)$	0.082
$(0, 1, -1)$	$-\frac{1}{2} \mu_u + \frac{1}{2} \mu_d + \frac{1}{12(m_u + m_d)} \left( \frac{m_u}{m_d} e_d + \frac{m_d}{m_u} e_u \right) + \frac{1}{6(m_d + m_s)} \left( \frac{m_d}{m_s} e_s + \frac{m_s}{m_d} e_d \right)$	-1.48
$\left(-1, \frac{3}{2}, \frac{3}{2}\right)$	$\frac{1}{2} \mu_u - \mu_d + \frac{1}{2} \mu_s + \frac{1}{4(m_u + m_s)} \left( \frac{m_u}{m_s} e_s + \frac{m_s}{m_u} e_u \right)$	1.85
$\left(-1, \frac{3}{2}, \frac{1}{2}\right)$	$-\frac{1}{2} \mu_d + \frac{1}{2} \mu_s + \frac{1}{6(m_u + m_s)} \left( \frac{m_u}{m_s} e_s + \frac{m_s}{m_u} e_u \right) + \frac{1}{12(m_d + m_s)} \left( \frac{m_d}{m_s} e_s + \frac{m_s}{m_d} e_d \right)$	0.29
$\left(-1, \frac{3}{2}, -\frac{1}{2}\right)$	$-\frac{1}{2} \mu_u + \frac{1}{2} \mu_s + \frac{1}{12(m_u + m_s)} \left( \frac{m_u}{m_s} e_s + \frac{m_s}{m_u} e_u \right) + \frac{1}{6(m_d + m_s)} \left( \frac{m_d}{m_s} e_s + \frac{m_s}{m_d} e_d \right)$	-1.27
$\left(-1, \frac{3}{2}, -\frac{3}{2}\right)$	$-\mu_u + \frac{1}{2} \mu_d + \frac{1}{2} \mu_s + \frac{1}{4(m_d + m_s)} \left( \frac{m_d}{m_s} e_s + \frac{m_s}{m_d} e_d \right)$	-2.84

The flavor wave functions of the pentaquarks remain the same as those in Ref. [24]. Here the total angular momentum of the tensor diquark is chosen to be  $J = 1$  [14]. The tensor diquark spin-space wave function reads:

$$\Psi_{11} = \frac{1}{\sqrt{2}} \chi_{11} \psi_{10} - \frac{1}{\sqrt{2}} \chi_{10} \psi_{11}, \quad (5)$$

where  $\chi_{11}$  and  $\chi_{10}$  are the tensor diquark spin wave functions,  $\psi_{10}$  and  $\psi_{11}$  are the orbital wave functions, and the anti-quark spin wave function is  $\chi_{\frac{1}{2} \frac{1}{2}}$ .

Diquarks are treated as point particles [13, 14]. There is no orbital magnetic moment. So the total magnetic moment of the pentaquark in Shuryak and Zahed's model comes from the sum of the spin magnetic moment of the anti-quark and the tensor diquark:

$$\begin{aligned} \boldsymbol{\mu} &= (g_1 \mathbf{0} + \mathbf{0}) \mu_1 + (g_2 \mathbf{1} + \mathbf{0}) \mu_2 + \left( g_3 \frac{\mathbf{1}}{2} + \mathbf{0} \right) \mu_3 = \\ &g_2 \mathbf{1} \mu_2 + g_3 \frac{\mathbf{1}}{2} \mu_3, \end{aligned} \quad (6)$$

where  $\mu_2$  is the magneton of the tensor diquark.  $\mu_2$  can be extracted from the following equation:

$$g_2 \mu_2 = \langle 1110 | 11 \rangle^2 (\mu_l + \mu_i + \mu_j), \quad (7)$$

where  $\mu_l$  has the similar expression as Eq. (3) by replacing one diquark with one quark,  $\mu_i$  and  $\mu_j$  are the quark magnetons inside the tensor diquark. Then we obtain the magnetic moment of a  $J^P = \frac{3}{2}^+$  pentaquark:

$$\begin{aligned} \mu &= \langle 2 \mu_{\bar{q}} \frac{\mathbf{1}}{2} + g_2 \mu_2 \mathbf{1} \rangle \left( J = \frac{3}{2} \right) = \\ &\langle 11 \frac{1}{2} \frac{1}{2} \left| \frac{3}{2} \frac{3}{2} \right\rangle^2 \mu_{\bar{q}} + \langle 11 \frac{1}{2} \frac{1}{2} \left| \frac{3}{2} \frac{3}{2} \right\rangle^2 g_2 \mu_2 = \\ &\mu_{\bar{q}} + g_2 \mu_2. \end{aligned} \quad (8)$$

We use parameters  $m_{ud}^T = 0.57$  GeV,  $m_{us}^T = m_{ds}^T = 0.72$  GeV,  $m_u = m_d = 0.36$  GeV and  $m_s = 0.54$  GeV to evaluate the magnetic moments. We list the expressions of magnetic moments and their numerical values in Table 3 and Table 4.

**Table 4.** Magnetic moments  $\mu_P$  of 8 pentaquarks in SZ's model (in unit of  $\mu_N$ ) with the same set of parameters as in Table 3.

$(Y, I, I_3)$	8	$\mu_P$
$(1, \frac{1}{2}, \frac{1}{2})$	$\frac{1}{2}\mu_u - \frac{1}{4}\mu_d - \frac{1}{4}\mu_s + \frac{5}{24(m_u + m_d)}\left(\frac{m_u}{m_d}e_d + \frac{m_d}{m_u}e_u\right) + \frac{1}{24(m_u + m_s)}\left(\frac{m_u}{m_s}e_s + \frac{m_s}{m_u}e_u\right)$	1.48
$(1, \frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{4}\mu_u + \frac{1}{2}\mu_d - \frac{1}{4}\mu_s + \frac{5}{24(m_u + m_d)}\left(\frac{m_u}{m_d}e_d + \frac{m_d}{m_u}e_u\right) + \frac{1}{24(m_d + m_s)}\left(\frac{m_d}{m_s}e_s + \frac{m_s}{m_d}e_d\right)$	-0.61
$(0, 1, 1)$	$\frac{1}{2}\mu_u - \frac{1}{4}\mu_d - \frac{1}{4}\mu_s + \frac{1}{24(m_u + m_d)}\left(\frac{m_u}{m_d}e_d + \frac{m_d}{m_u}e_u\right) + \frac{5}{24(m_u + m_s)}\left(\frac{m_u}{m_s}e_s + \frac{m_s}{m_u}e_u\right)$	1.60
$(0, 1, 0)$	$\frac{1}{8}\mu_u + \frac{1}{8}\mu_d - \frac{1}{4}\mu_s + \frac{1}{24(m_u + m_d)}\left(\frac{m_u}{m_d}e_d + \frac{m_d}{m_u}e_u\right) + \frac{5}{48(m_u + m_s)}\left(\frac{m_u}{m_s}e_s + \frac{m_s}{m_u}e_u\right) + \frac{5}{48(m_d + m_s)}\left(\frac{m_d}{m_s}e_s + \frac{m_s}{m_d}e_d\right)$	0.30
$(0, 1, -1)$	$-\frac{1}{4}\mu_u + \frac{1}{2}\mu_d - \frac{1}{4}\mu_s + \frac{1}{24(m_u + m_d)}\left(\frac{m_u}{m_d}e_d + \frac{m_d}{m_u}e_u\right) + \frac{5}{24(m_d + m_s)}\left(\frac{m_d}{m_s}e_s + \frac{m_s}{m_d}e_d\right)$	-1.00
$(-1, \frac{1}{2}, \frac{1}{2})$	$-\frac{1}{4}\mu_u - \frac{1}{4}\mu_d + \frac{1}{2}\mu_s + \frac{5}{24(m_u + m_s)}\left(\frac{m_u}{m_s}e_s + \frac{m_s}{m_u}e_u\right) + \frac{1}{24(m_d + m_s)}\left(\frac{m_d}{m_s}e_s + \frac{m_s}{m_d}e_d\right)$	-0.23
$(-1, \frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{4}\mu_u - \frac{1}{4}\mu_d + \frac{1}{2}\mu_s + \frac{1}{24(m_u + m_s)}\left(\frac{m_u}{m_s}e_s + \frac{m_s}{m_u}e_u\right) + \frac{5}{24(m_d + m_s)}\left(\frac{m_d}{m_s}e_s + \frac{m_s}{m_d}e_d\right)$	-0.75
$(0, 0, 0)$	$-\frac{1}{8}\mu_u - \frac{1}{8}\mu_d + \frac{1}{4}\mu_s + \frac{1}{8(m_u + m_d)}\left(\frac{m_u}{m_d}e_d + \frac{m_d}{m_u}e_u\right) + \frac{1}{16(m_u + m_s)}\left(\frac{m_u}{m_s}e_s + \frac{m_s}{m_u}e_u\right) + \frac{1}{16(m_d + m_s)}\left(\frac{m_d}{m_s}e_s + \frac{m_s}{m_d}e_d\right)$	-0.14

#### 4 Pentaquark as a bound state of a diquark and triquark

According to Karliner and Lipkin<sup>[12]</sup>, the  $\Theta^+$  pentaquark is composed of two color nonsinglet clusters, namely a scalar diquark and a triquark. In this picture, the scalar diquark-triquark system carries one unit of orbital excitation, namely  $L = 1$ . For the scalar diquark, it is still in the antisymmetric  $\bar{\mathbf{3}}_c$  color representation and antisymmetric  $\bar{\mathbf{3}}_f$  flavor representation. Two quarks within the triquark form  $\mathbf{6}_c$  color representation and  $\bar{\mathbf{3}}_f$  flavor representation. Then they couple with the anti-quark to form  $\mathbf{3}_c$  color representation and  $\bar{\mathbf{6}}_f$  flavor representation. Finally the direct product of the  $\bar{\mathbf{3}}_f$  flavor representation of the scalar diquark and the  $\bar{\mathbf{6}}_f$  flavor representation of the triquark leads to  $\bar{\mathbf{10}}_f$  representation and  $\mathbf{8}_f$  representation. Thus the spin wave function of the two quarks among the triquark is symmetric while the triquark spin is  $S = \frac{1}{2}$ . The flavor wave function is the same as that in Ref. [24]. The spin-space wave function is

$$\Psi_{\frac{3}{2} \frac{3}{2}} = \psi_{11} \chi_{\frac{1}{2} \frac{1}{2}}. \quad (9)$$

Here  $\psi_{11} \sim e^{i\mathbf{P}\cdot\mathbf{R}}\psi_{11}(\mathbf{r})$  is the two-body space wave function in the center of mass frame,  $\chi_{\frac{1}{2} \frac{1}{2}} = \chi_{di} \chi_{tri}$  is the spin wave function, and

$$\chi_{tri} = \frac{1}{\sqrt{6}}(2|\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle - |\uparrow\downarrow\uparrow\rangle) \quad (10)$$

is the triquark spin wave function.

Since the diquark is a scalar, the total magnetic moment is the sum of the angular magnetic moment of the diquark-triquark system and the spin magnetic moment of the triquark:

$$\mu = \langle 11 \frac{1}{2} \frac{1}{2} \left| \frac{3}{2} \frac{3}{2} \right\rangle^2 \mu_l + \langle 11 \frac{1}{2} \frac{1}{2} \left| \frac{3}{2} \frac{3}{2} \right\rangle^2 \frac{1}{2} g_{tri} \mu_{tri} = \mu_l + \frac{1}{2} g_{tri} \mu_{tri}, \quad (11)$$

where  $g_{tri}$  and  $\mu_{tri}$  are the triquark's  $g$  factor and magneton respectively<sup>[24]</sup>. The intrinsic magnetic moment of the triquark is:

$$\frac{1}{2} g_{tri} \mu_{tri} = \langle 11 \frac{1}{2} - \frac{1}{2} \left| \frac{1}{2} \frac{1}{2} \right\rangle^2 (\mu_{q1} + \mu_{q2}) + \left( \langle 10 \frac{1}{2} \frac{1}{2} \left| \frac{1}{2} \frac{1}{2} \right\rangle^2 - \langle 11 \frac{1}{2} - \frac{1}{2} \left| \frac{1}{2} \frac{1}{2} \right\rangle^2 \right) \mu_{\bar{q}}. \quad (12)$$

The orbital part is

$$\mu_l = \frac{m_{tri} \cdot \mu_{di} + m_{di} \cdot \mu_{tri}}{m_{tri} + m_{di}}, \quad (13)$$

where  $m_{di}$  is the mass of the diquark,  $m_{tri}$  is the mass of the triquark.

The parameters are  $m_m = m_d = 0.36$  GeV,  $m_s = 0.54$  GeV,  $m_{ud} = 0.72$  GeV and  $m_{us} = m_{ds} = 0.90$  GeV from Ref. [12]. The triquark mass is the sum of its constituent mass, e. g.  $m_{ud\bar{u}} = m_u + m_d + m_{\bar{u}} = 1.08$  GeV<sup>[12]</sup>. The resulting pen-

taquark magnetic moments and their numerical values are presented in Table 5 and 6.

**Table 5. Magnetic moments  $\mu_P$  of  $\overline{10}$  pentaquarks in KL's model (in unit of  $\mu_N$ ).**

We follow Ref. [12] and use  $m_u = m_d = 0.36$  GeV,  $m_s = 0.54$  GeV,  $m_{ud} = 0.72$  GeV,

$m_{us} = m_{ds} = 0.90$  GeV. Triquark mass is the sum of its constituent mass, e. g.  $m_{ud\bar{u}} = m_u + m_d + m_{\bar{u}} = 1.08$  GeV.

$(Y, I, I_3)$	$\overline{10}$	$\mu_P$
$(2, 0, 0)$	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_s + \frac{1}{2(m_{ud} - m_{u\bar{s}})} \left( \frac{m_{u\bar{d}s}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{u\bar{s}}} e_{u\bar{d}s} \right)$	0.84
$\left(1, \frac{1}{2}, \frac{1}{2}\right)$	$\frac{2}{3}\mu_u + \frac{4}{9}\mu_d + \frac{2}{9}\mu_s - \frac{1}{9}\mu_{\bar{d}} - \frac{2}{9}\mu_s + \frac{1}{6(m_{ud} + m_{u\bar{d}\bar{l}})} \left( \frac{m_{u\bar{d}\bar{l}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{u\bar{d}\bar{l}}} e_{u\bar{d}\bar{l}} \right) + \frac{1}{6(m_{ud} + m_{us\bar{s}})} \left( \frac{m_{us\bar{s}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us\bar{s}}} e_{us\bar{s}} \right) + \frac{1}{6(m_{us} + m_{u\bar{d}\bar{s}})} \left( \frac{m_{u\bar{d}\bar{s}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{u\bar{d}\bar{s}}} e_{u\bar{d}\bar{s}} \right)$	0.86
$\left(1, \frac{1}{2}, -\frac{1}{2}\right)$	$\frac{4}{9}\mu_u + \frac{2}{3}\mu_d + \frac{2}{9}\mu_s - \frac{1}{9}\mu_{\bar{u}} - \frac{2}{9}\mu_s + \frac{1}{6(m_{ud} + m_{u\bar{d}\bar{u}})} \left( \frac{m_{u\bar{d}\bar{u}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{u\bar{d}\bar{u}}} e_{u\bar{d}\bar{u}} \right) + \frac{1}{6(m_{ud} + m_{ds\bar{s}})} \left( \frac{m_{ds\bar{s}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds\bar{s}}} e_{ds\bar{s}} \right) + \frac{1}{6(m_{ds} + m_{u\bar{d}\bar{s}})} \left( \frac{m_{u\bar{d}\bar{s}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{u\bar{d}\bar{s}}} e_{u\bar{d}\bar{s}} \right)$	0.18
$(0, 1, 1)$	$\frac{2}{3}\mu_u + \frac{2}{9}\mu_d + \frac{4}{9}\mu_s - \frac{2}{9}\mu_{\bar{d}} - \frac{1}{9}\mu_s + \frac{1}{6(m_{us} + m_{u\bar{d}\bar{l}})} \left( \frac{m_{u\bar{d}\bar{l}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{u\bar{d}\bar{l}}} e_{u\bar{d}\bar{l}} \right) + \frac{1}{6(m_{us} + m_{us\bar{s}})} \left( \frac{m_{us\bar{s}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{us\bar{s}}} e_{us\bar{s}} \right) + \frac{1}{6(m_{ud} + m_{us\bar{l}})} \left( \frac{m_{us\bar{l}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us\bar{l}}} e_{us\bar{l}} \right)$	0.88
$(0, 1, 0)$	$\frac{4}{9}\mu_u + \frac{4}{9}\mu_d + \frac{4}{9}\mu_s - \frac{1}{9}\mu_{\bar{u}} - \frac{1}{9}\mu_{\bar{d}} - \frac{1}{9}\mu_s + \frac{1}{12(m_{us} + m_{u\bar{d}\bar{u}})} \left( \frac{m_{u\bar{d}\bar{u}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{u\bar{d}\bar{u}}} e_{u\bar{d}\bar{u}} \right) + \frac{1}{12(m_{us} + m_{ds\bar{s}})} \left( \frac{m_{ds\bar{s}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds\bar{s}}} e_{ds\bar{s}} \right) + \frac{1}{12(m_{ds} + m_{u\bar{d}\bar{l}})} \left( \frac{m_{u\bar{d}\bar{l}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{u\bar{d}\bar{l}}} e_{u\bar{d}\bar{l}} \right) + \frac{1}{12(m_{ds} + m_{us\bar{s}})} \left( \frac{m_{us\bar{s}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{us\bar{s}}} e_{us\bar{s}} \right) + \frac{1}{12(m_{ud} + m_{us\bar{l}})} \left( \frac{m_{us\bar{l}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us\bar{l}}} e_{us\bar{l}} \right)$	0.19
$(0, 1, -1)$	$\frac{2}{9}\mu_u + \frac{2}{3}\mu_d + \frac{4}{9}\mu_s - \frac{2}{9}\mu_{\bar{u}} - \frac{1}{9}\mu_s + \frac{1}{6(m_{ds} + m_{u\bar{d}\bar{u}})} \left( \frac{m_{u\bar{d}\bar{u}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{u\bar{d}\bar{u}}} e_{u\bar{d}\bar{u}} \right) + \frac{1}{6(m_{ds} + m_{ds\bar{s}})} \left( \frac{m_{ds\bar{s}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{ds\bar{s}}} e_{ds\bar{s}} \right) + \frac{1}{6(m_{ud} + m_{ds\bar{l}})} \left( \frac{m_{ds\bar{l}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds\bar{l}}} e_{ds\bar{l}} \right)$	-0.50
$\left(-1, \frac{3}{2}, \frac{3}{2}\right)$	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_s - \frac{1}{3}\mu_{\bar{d}} + \frac{1}{2(m_{us} + m_{us\bar{l}})} \left( \frac{m_{us\bar{l}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{us\bar{l}}} e_{us\bar{l}} \right)$	0.89
$\left(-1, \frac{3}{2}, \frac{1}{2}\right)$	$\frac{4}{9}\mu_u + \frac{2}{9}\mu_d + \frac{2}{3}\mu_s - \frac{1}{9}\mu_{\bar{u}} - \frac{2}{9}\mu_{\bar{d}} + \frac{1}{6(m_{ds} + m_{us\bar{l}})} \left( \frac{m_{us\bar{l}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{us\bar{l}}} e_{us\bar{l}} \right) + \frac{1}{6(m_{us} + m_{us\bar{s}})} \left( \frac{m_{us\bar{s}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{us\bar{s}}} e_{us\bar{s}} \right) + \frac{1}{6(m_{us} + m_{ds\bar{l}})} \left( \frac{m_{ds\bar{l}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds\bar{l}}} e_{ds\bar{l}} \right)$	0.19
$\left(-1, \frac{3}{2}, -\frac{1}{2}\right)$	$\frac{2}{9}\mu_u + \frac{4}{9}\mu_d + \frac{2}{3}\mu_s - \frac{2}{9}\mu_{\bar{u}} - \frac{1}{9}\mu_{\bar{d}} + \frac{1}{6(m_{ds} + m_{us\bar{s}})} \left( \frac{m_{us\bar{s}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{us\bar{s}}} e_{us\bar{s}} \right) + \frac{1}{6(m_{ds} + m_{ds\bar{l}})} \left( \frac{m_{ds\bar{l}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{ds\bar{l}}} e_{ds\bar{l}} \right) + \frac{1}{6(m_{us} + m_{ds\bar{s}})} \left( \frac{m_{ds\bar{s}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds\bar{s}}} e_{ds\bar{s}} \right)$	-0.51
$\left(-1, \frac{3}{2}, -\frac{3}{2}\right)$	$\frac{2}{3}\mu_d + \frac{2}{3}\mu_s - \frac{1}{3}\mu_{\bar{u}} + \frac{1}{2(m_{ds} + m_{ds\bar{l}})} \left( \frac{m_{ds\bar{l}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{ds\bar{l}}} e_{ds\bar{l}} \right)$	-1.20

Table 6. Magnetic moments  $\mu_P$  of 8 pentaquarks in KL's model(in unit of  $\mu_N$ ). The same parameters are used as in Table 5.

$(Y, I, I_3)$	8	$\mu_P$
$(1, \frac{1}{2}, \frac{1}{2})$	$\frac{2}{3} \mu_u + \frac{5}{9} \mu_d + \frac{1}{9} \mu_s - \frac{1}{18} \mu_{\bar{d}} - \frac{5}{18} \mu_{\bar{s}} + \frac{1}{12(m_{ud} + m_{ud\bar{d}})} \left( \frac{m_{ud\bar{d}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ud\bar{d}}} e_{ud\bar{d}} \right) + \frac{1}{12(m_{ud} + m_{us\bar{s}})} \left( \frac{m_{us\bar{s}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us\bar{s}}} e_{us\bar{s}} \right) + \frac{1}{3(m_{us} + m_{ud\bar{s}})} \left( \frac{m_{ud\bar{s}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ud\bar{s}}} e_{ud\bar{s}} \right)$	0.83
$(1, \frac{1}{2}, -\frac{1}{2})$	$\frac{5}{9} \mu_u + \frac{2}{3} \mu_d + \frac{1}{9} \mu_s - \frac{1}{18} \mu_{\bar{u}} - \frac{5}{18} \mu_{\bar{s}} + \frac{1}{12(m_{ud} + m_{ud\bar{u}})} \left( \frac{m_{ud\bar{u}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ud\bar{u}}} e_{ud\bar{u}} \right) + \frac{1}{12(m_{ud} + m_{ds\bar{s}})} \left( \frac{m_{ds\bar{s}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds\bar{s}}} e_{ds\bar{s}} \right) + \frac{1}{3(m_{ds} + m_{ud\bar{s}})} \left( \frac{m_{ud\bar{s}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{ud\bar{s}}} e_{ud\bar{s}} \right)$	0.19
$(0, 1, 1)$	$\frac{2}{3} \mu_u + \frac{1}{9} \mu_d + \frac{5}{9} \mu_s - \frac{5}{18} \mu_{\bar{d}} - \frac{1}{18} \mu_{\bar{s}} + \frac{1}{12(m_{us} + m_{ud\bar{d}})} \left( \frac{m_{ud\bar{d}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ud\bar{d}}} e_{ud\bar{d}} \right) + \frac{1}{12(m_{us} + m_{us\bar{s}})} \left( \frac{m_{us\bar{s}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{us\bar{s}}} e_{us\bar{s}} \right) + \frac{1}{3(m_{ud} + m_{us\bar{d}})} \left( \frac{m_{us\bar{d}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us\bar{d}}} e_{us\bar{d}} \right)$	0.91
$(0, 1, 0)$	$\frac{7}{18} \mu_u + \frac{7}{18} \mu_d + \frac{5}{9} \mu_s - \frac{5}{36} \mu_{\bar{u}} - \frac{5}{36} \mu_{\bar{d}} - \frac{1}{18} \mu_{\bar{s}} + \frac{1}{24(m_{us} + m_{ud\bar{u}})} \left( \frac{m_{ud\bar{u}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ud\bar{u}}} e_{ud\bar{u}} \right) + \frac{1}{24(m_{us} + m_{ds\bar{s}})} \left( \frac{m_{ds\bar{s}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds\bar{s}}} e_{ds\bar{s}} \right) + \frac{1}{24(m_{ds} + m_{ud\bar{d}})} \left( \frac{m_{ud\bar{d}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{ud\bar{d}}} e_{ud\bar{d}} \right) + \frac{1}{24(m_{ds} + m_{us\bar{s}})} \left( \frac{m_{us\bar{s}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{us\bar{s}}} e_{us\bar{s}} \right) + \frac{1}{6(m_{ud} + m_{us\bar{u}})} \left( \frac{m_{us\bar{u}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{us\bar{u}}} e_{us\bar{u}} \right) + \frac{1}{6(m_{ud} + m_{ds\bar{d}})} \left( \frac{m_{ds\bar{d}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds\bar{d}}} e_{ds\bar{d}} \right)$	0.21
$(0, 1, -1)$	$\frac{1}{9} \mu_u + \frac{2}{3} \mu_d + \frac{5}{9} \mu_s - \frac{5}{18} \mu_{\bar{u}} - \frac{1}{18} \mu_{\bar{s}} + \frac{1}{12(m_{ds} + m_{ud\bar{u}})} \left( \frac{m_{ud\bar{u}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{ud\bar{u}}} e_{ud\bar{u}} \right) + \frac{1}{12(m_{ds} + m_{ds\bar{s}})} \left( \frac{m_{ds\bar{s}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{ds\bar{s}}} e_{ds\bar{s}} \right) + \frac{1}{3(m_{ud} + m_{ds\bar{u}})} \left( \frac{m_{ds\bar{u}}}{m_{ud}} e_{ud} + \frac{m_{ud}}{m_{ds\bar{u}}} e_{ds\bar{u}} \right)$	-0.48
$(-1, \frac{1}{2}, \frac{1}{2})$	$\frac{5}{9} \mu_u + \frac{1}{9} \mu_d + \frac{2}{3} \mu_s - \frac{1}{18} \mu_{\bar{u}} - \frac{5}{18} \mu_{\bar{d}} + \frac{1}{12(m_{us} + m_{us\bar{u}})} \left( \frac{m_{us\bar{u}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{us\bar{u}}} e_{us\bar{u}} \right) + \frac{1}{12(m_{us} + m_{ds\bar{d}})} \left( \frac{m_{ds\bar{d}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds\bar{d}}} e_{ds\bar{d}} \right) + \frac{1}{3(m_{ds} + m_{us\bar{d}})} \left( \frac{m_{us\bar{d}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{us\bar{d}}} e_{us\bar{d}} \right)$	0.24
$(-1, \frac{1}{2}, -\frac{1}{2})$	$\frac{1}{9} \mu_u + \frac{5}{9} \mu_d + \frac{2}{3} \mu_s - \frac{5}{18} \mu_{\bar{u}} - \frac{1}{18} \mu_{\bar{d}} + \frac{1}{12(m_{ds} + m_{ds\bar{d}})} \left( \frac{m_{ds\bar{d}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{ds\bar{d}}} e_{ds\bar{d}} \right) + \frac{1}{12(m_{ds} + m_{us\bar{s}})} \left( \frac{m_{us\bar{s}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{us\bar{s}}} e_{us\bar{s}} \right) + \frac{1}{3(m_{us} + m_{ds\bar{u}})} \left( \frac{m_{ds\bar{u}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds\bar{u}}} e_{ds\bar{u}} \right)$	-0.55
$(0, 0, 0)$	$\frac{1}{2} \mu_u + \frac{1}{2} \mu_d + \frac{1}{3} \mu_s - \frac{1}{12} \mu_{\bar{u}} - \frac{1}{12} \mu_{\bar{d}} - \frac{1}{6} \mu_{\bar{s}} + \frac{1}{8(m_{us} + m_{ud\bar{u}})} \left( \frac{m_{ud\bar{u}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ud\bar{u}}} e_{ud\bar{u}} \right) + \frac{1}{8(m_{us} + m_{ds\bar{s}})} \left( \frac{m_{ds\bar{s}}}{m_{us}} e_{us} + \frac{m_{us}}{m_{ds\bar{s}}} e_{ds\bar{s}} \right) + \frac{1}{8(m_{ds} + m_{ud\bar{d}})} \left( \frac{m_{ud\bar{d}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{ud\bar{d}}} e_{ud\bar{d}} \right) + \frac{1}{8(m_{ds} + m_{us\bar{s}})} \left( \frac{m_{us\bar{s}}}{m_{ds}} e_{ds} + \frac{m_{ds}}{m_{us\bar{s}}} e_{us\bar{s}} \right)$	0.17

## 5 Discussions

The magnetic moments of  $J = \frac{1}{2}$  pentaquarks have been discussed by several groups recently. Within the chiral soliton model, Kim and Praszalowicz<sup>[27]</sup> derived relations for the  $\overline{10}_f$  magnetic moments and found that the magnetic moment of the  $\Theta^+$  pentaquark is between  $(0.2-0.3)\mu_N$ .

Using Jaffe and Wilczek's scalar diquark picture of  $\Theta^+$ , ~~Not~~ treated the pentaquark as the sum of  $(u\bar{s})$  and  $(udd)$  clusters.  $\Theta^+$  pentaquark from the neutron and estimated the anomalous magnetic moment to be  $-0.7$  (positive parity) and  $-0.2$  (negative parity) (in units of  $\Theta^+$  magneton  $\frac{e_0}{2m_{\Theta^+}}$ ).

In Ref. [29] a quark model calculation is also performed using JW's diquark picture, where Zhao got  $\mu_{\Theta^+} = 0.13 \frac{e_0}{2m_{\Theta^+}}$  for positive parity  $\Theta^+$ . In the case of negative parity,

and got  $\mu_{\Theta^+} = \frac{e_0}{6m_s}$ .

The magnetic moment of the  $\Theta^+$  pentaquark is also calculated using the method of light cone QCD sum rules. In Ref. [23] the authors arrived at  $\mu_{\Theta^+} = (0.12 \pm 0.06) \mu_N$ . The magnetic moments of all members of  $\overline{\mathbf{10}}_F$  and  $\mathbf{8}_F$  pentaquarks have recently been calculated within four models: Jaffe and Wilczek's model, Shuryak and Zahed's model, Karliner and Lipkin's model and Strottman's model in Ref. [24].

In this work we have extended the same formalism in

Ref. [24] and calculated the magnetic moments of the  $J^P = \frac{3}{2}^+$  pentaquark states in three different models. We have collected the numerical results for  $J = \frac{1}{2}$   $\overline{\mathbf{10}}$  members  $\Theta^+$ ,  $\Xi_5^{--}$ ,  $\Xi_5^+$  and  $J = \frac{3}{2}$   $\overline{\mathbf{10}}$  members  $\Theta^{*+}$ ,  $\Xi_5^{*-}$ ,  $\Xi_5^{*+}$  in Table 7. These states lie on the corners of the anti-decuplet triangle and have no mixing with octet pentaquarks. Hence their interpretation and identification should be relatively clean, at least theoretically.

**Table 7.** Comparison of magnetic moments of  $\Theta^+$ ,  $\Xi_5^{--}$ ,  $\Xi_5^+$  and  $\Theta^{*+}$ ,  $\Xi_5^{*-}$ ,  $\Xi_5^{*+}$  in different pentaquark models in literature. The numbers are in unit of  $\mu_N$ .

	$J = 1/2$			$J = 3/2$		
	$\Theta^+$	$\Xi_5^{--}$	$\Xi_5^+$	$\Theta^{*+}$	$\Xi_5^{*-}$	$\Xi_5^{*+}$
Ref. [27]	0.2—0.3	-0.4	0.2	—	—	—
Ref. [28]	0.2—0.5	—	—	—	—	—
Ref. [29]	0.08—0.6	—	—	—	—	—
Ref. [23]	$0.12 \pm 0.06$	—	—	—	—	—
present work (JW's model)	0.08	0.12	-0.06	1.01	-2.43	1.22
present work (SZ's model)	0.23	-0.11	0.33	1.23	-2.84	1.85
present work (KL's model)	0.19	-0.43	0.13	0.84	-1.20	0.89

We want to emphasize that we are not arguing that these models are correct. Instead, we may be able to judge these models through comparison with experimental data. For example, these models have definite predictions of magnetic moments for the  $J^P = \frac{1}{2}^+$  or  $J^P = \frac{3}{2}^+$  pentaquarks. Different magnetic moments will affect both the total and differential cross sections in the photo- or electro-production of pentaquarks. Hence, knowledge of the pentaquark magnetic moments will help us unveil the mysterious curtain over the pen-

taquarks at present and deepen our understanding of the underlying quark structure and dynamics.

The experimental evidence of the possible existence of the  $\Theta^+$  pentaquark is gradually increasing. If its  $J^P$  is really found to be  $\frac{1}{2}^+$  by future experiments, it is reasonable to expect that its  $J^P = \frac{3}{2}^+$  partners will also be discovered in the near future with the current intensive experimental and theoretical efforts.

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## 宇称为正角动量为 $\frac{3}{2}$ 的五夸克态的磁矩\*

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**摘要** 如果将来的实验证实五夸克态  $\Theta_5^+$  与  $\Xi_5^-$  的  $J^P = \frac{1}{2}^+$ , 在一些理论模型中,  $J^P = \frac{3}{2}^+$  与  $J^P = \frac{1}{2}^+$  的五夸克态必然同时出现. 这些态有可能很快会发现. 我们在不同理论模型中计算了  $J^P = \frac{3}{2}^+$  的五夸克态的磁矩.

**关键词** 五夸克态 磁矩

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