

# Symmetry and Mixed Symmetry Band Structures in Low-lying Levels of $^{76-84}\text{Kr}$ Isotopes \*

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**Abstract** The level structure of  $^{76-84}\text{Kr}$  isotopes is discussed within the framework of Interacting Boson Model (IBM-2). One-phonon mixed symmetry states  $J^+ = 2^+$  and two-phonon mixed symmetry states  $J^+ = 1^+, 2^+$  and  $3^+$  have been identified by analyzing the wave function and M1 transitions. Special attention is paid to the occurrence of  $0_2^+$  which is not reproduced well by other calculations. The study of the influence of the  $[d^+d]_{\pi}^L \cdot [d^+d]_{\nu}^L$  interactions on the nuclear structure of these nuclei are undertaken. The calculated results are compared with available experimental data; the results are in general good agreement.

**Key words** mixed symmetry states,  $^{76-84}\text{Kr}$  isotopes, IBM-2

## 1 Introduction

In recent years, many mixed symmetry states have been found for even-even nuclei. In a given mass region, the mixed symmetry states usually show similar properties in energy and electromagnetic transition. The occurrence of mixed symmetry states has been predicted in various models, such as the geometrical model<sup>[1,2]</sup> and the Interacting Boson Model<sup>[3-5]</sup>. The interacting boson model assumes that the low-lying collective levels of nuclei are composed primarily of  $J = 0^+$  and  $2^+$  coherent pairs of valence nucleons which are approximated by s and d boson respectively. In the original version (IBM-1), no distinction is made between proton boson and neutron boson, therefore all states are symmetric<sup>[6]</sup>. The second version the IBM-2, does distinguish between proton boson and neutron boson. The states in the new version include all symmetry states as well as mixed symmetry states belong-

ing to the  $U(6)$  representation  $[N-1, 1]$ . The different neutron-proton symmetries can be conveniently labelled by introducing a new quantum number called  $F$ -spin. A boson is an object with  $F$ -spin equal to  $1/2$  and with projections  $1/2$  and  $-1/2$  for a proton and neutron boson, respectively. The two kinds of bosons form a  $F$ -spin multiplet namely  $|\pi\rangle = |1/2, 1/2\rangle$  and  $|\nu\rangle = |1/2, -1/2\rangle$ . The states with,  $F_{\max} = (N_{\nu} + N_{\pi})/2$ , belong to the maximally symmetric representation  $[N]$  of  $U(6)$ . The mixed symmetric states characterized by decreasing  $F$ -spin values, such as  $F = F_{\max} - 1$  belong to the  $[N-1, 1]$  representation and so on<sup>[7]</sup>. The mixed symmetry states have the following signatures: weak collective  $E2$  transitions to the symmetric states and strong M1 transition to symmetric states with matrix elements of order  $\langle J_{FS} | M1 | J_{MS} \rangle \simeq 1\mu\text{N}$ . One example of the mixed symmetry states is the  $J = 1^+$ , which is called the scissors mode. In the IBM-2 picture, some  $1^+$  states arise from the proton-neu-

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tron mixed representation of the group  $U(6)$ . Classically, these states can be regarded as small amplitude oscillations of the angle between symmetry axes of the deformed valence neutrons and valence protons<sup>[8]</sup>. It is discovered in a high-resolution electron scattering experiment<sup>[9]</sup>. Many nuclei have been investigated within the framework of IBM, which gives a good agreement between the experimental and model result<sup>[10–14]</sup>.

Several experimental and theoretical investigations of even-mass Kr isotopes have been carried out:

(i) Kaup and Gelberg<sup>[15]</sup>, have performed systematic analysis of even-even Kr isotopes in the framework of the IBM-2, satisfactorily reproduced the excitation energy levels with the exception of the  $0_2^+$  states which were interpreted as *intruder states*.

(ii) Hellmeister *et al.*<sup>[16]</sup>, Wormann *et al.*<sup>[17]</sup>; and Barfield and Lieb<sup>[18]</sup> have considered the same set of Hamiltonian parameters which extended the calculations to include B(E2) and obtained a reasonable agreement with the experimental data.

(iii) Meyer *et al.*<sup>[19]</sup> have investigated the structure of  $^{82}\text{Kr}$  isotope using in-beam and decay spectroscopy studies. They also made a comparison between the experimental data and the IBM-2 results.

(iv) Brusserman *et al.*<sup>[20]</sup>, have performed the calculation using rotor, interacting boson and Gneuss-Greiner models for  $^{82}\text{Kr}$  isotope, and compared the obtained results with multiple coulomb excitation. The authors considered that the 1957 keV  $2_3^+$  state as a  $F = F_{\max} - 1$  state in this nucleus.

(v) Giannatiempo *et al.*<sup>[21]</sup>, have studied the lifetime measurement of the  $0_2^+$  state in the  $^{80}\text{Kr}$  isotope and compared with the calculated values of the IBM-2. This study also includes the IBM-2 calculation for  $^{78-82}\text{Kr}$  isotopes.

(vi) Dejbakhsh *et al.*<sup>[22]</sup>, have performed the IBM-2 calculation using two different approaches. The first investigation is based on ( $\in_v = \in_\pi$ ) and the second investigation is based on ( $\in_v \neq \in_\pi$ ). The agreement between the results obtained in this study and the experimental data is reasonable except for  $2_3^+$  in the two approaches.

(vii) Recently, Giannatiempo *et al.*<sup>[23]</sup>, have investigated the symmetry character of the bands in the  $^{72-84}\text{Kr}$  isotopes by calculating the  $F$ -spin and the  $n_d$  components of the wave function of the states of these

bands. The bands investigated are restricted to those built on the  $0_1^+$ ,  $2_2^+$  and  $3_1^+$  states.

The aims of the present paper are the following:

1) To carry out systematic IBM-2 calculation of the even mass  $^{76-84}\text{Kr}$  isotopes in the context of new experimental data.

2) To study the mixed symmetric characters of the eigenstates through a study of various quantities, for instance correlation in the energy levels, the wave functions, the  $F$ -spin values and the electromagnetic transition probabilities.

3) Identification of the one-phonon and two-phonon mixed symmetry states.

## 2 Interacting boson model-2

The IBM-2 Hamiltonian can be written as:

$$H = \in_d(\hat{n}_{dn} + \hat{n}_{db}) + \kappa_m \hat{Q}_\pi \cdot \hat{Q}_v + \sum_{\rho=\pi,v} \hat{V}_{\rho\rho} + \hat{M}_{\pi v} + \sum_{L=0}^4 G_m^{(L)} ([d^+ \tilde{d}]_\pi^{(L)} \cdot [d^+ \tilde{d}]_v^{(L)}), \quad (1)$$

where  $\in_d$  is the d boson excitation energy,  $n_{dn}$ ,  $n_{db}$  are the number of proton, neutron d-boson operators.  $\kappa_m \hat{Q}_\pi \cdot \hat{Q}_v$  is the quadruple interaction between proton and neutron boson, where  $\hat{Q}_\rho$  quadruple operator is given by

$$\hat{Q}_\rho = (s_\rho^+ \tilde{d}_\rho + s^+ d_\rho^+)^2 + \chi_\rho (d^+ \tilde{d}^2), \quad (2)$$

and

$$\hat{M}_{\pi v} = \xi_2 [(d_v^+ s_\pi^+ - d_\pi^+ s_v^+) \cdot (\tilde{d}_v s_\pi - \tilde{d}_\pi s_v)]^{(2)} + \frac{1}{2} \sum_{k=1,3} \xi_k [d_v^+ d_\pi^+]^{(k)} \cdot [\tilde{d}_v \tilde{d}_\pi]^{(k)} \quad (3)$$

is the Majorana operator, it only affects the position of the mixed symmetry states. The  $\hat{V}_{\rho\rho}$  represents the interaction between like-bosons, usually it is

$$\hat{V}_{\rho\rho} = 1/2 \sum_{L=0,2,4} [2L+1] C_\rho^{(L)} [d_\rho^+ d_\rho^+]^{(L)} \cdot [\tilde{d}_\rho \tilde{d}_\rho]^{(L)}, \quad (4)$$

where  $\rho = \pi, v$ .

In the IBM-2, E2 and M1 operators are expressed as

$$\hat{T}(E2) \equiv e_\pi \hat{T}_\pi(E2) + e_v \hat{T}_v(E2) = e_\pi \hat{Q}_\pi + e_v \hat{Q}_v, \quad (5)$$

where the quadruple operators  $\hat{Q}_\pi$  and  $\hat{Q}_v$  are defined in Eq.(2),  $e_\pi$  and  $e_v$  the proton and neutron boson effective charges.

The M1 operator is expressed as

$$\hat{T}(\text{M1}) = \sqrt{3/4\pi}(g_\pi \hat{L}_\pi + g_\nu \hat{L}_\nu), \quad (6)$$

where  $\hat{L}$  is the angular momentum operator,

$$\hat{L}_\rho = \sqrt{10}[d_\rho + \tilde{d}_\rho]^{(1)}, \quad (7)$$

$g_\pi$  and  $g_\nu$  are  $g$  factors for proton and neutron boson, respectively.

### 3 Interaction parameters

The even-even Kr isotopes with  $Z = 36$  and  $40 \leq N \leq 48$  were studied systematically, taking  $Z = 28$  and  $N = 50$  as closed shells. According to this, the proton boson is of particle-type while the neutron boson is of hole-type. The best fitted parameters are summarized in Table 1. It can be seen from the table that the values of  $C_\pi^L = C_\nu^L$  ( $L = 0, 2, 4$ ) = 0.1 for all isotopes, and that the absence of these terms corresponds to the  $U(5)$  limit<sup>[6]</sup>. The adopted values of the parameter  $\epsilon_d$  show a smooth variation with the neutron number. Similar type of neutron number dependence has been observed for the parameter  $\epsilon_d$  in other calculation<sup>[23]</sup>. The values of  $\kappa_\pi$  increase with the increase of the neutron number. The parameters  $\chi_\pi$  and  $\chi_\nu$  have been kept constant in all Kr isotopes, and are taken as the same as those in Refs. [21, 23]. On the other hand, we choose  $\xi_3 = 0.1$  for all isotopes and vary  $\xi_1$  and  $\xi_2$  to best fit the spectrum. This selection of Majorana parameters will depress the  $2^+$  mixed symmetry states with respect to  $1^+$  states. The aim was to minimize the position of  $2^+$  mixed symmetry states in the Kr isotopes, and to monitor the effects of such a change on the calculated energy levels. In order to obtain a better agreement between the calculated spectra and the experimental results, we added the  $G_{\text{int}}^L$  ( $L = 0, 2$ ) terms. The parameters  $G_{\text{int}}^L$  ( $L = 0, 2$ ) are adjustable to put the  $0^+$  and  $3^+$  energies right, and they have very small effect on the ground state band. They are given in Table 1.

**Table 1. The parameters of the IBM-2 Hamiltonian.**

$\chi_\pi = -0.1$ ,  $\chi_\nu = -1.1$ ,  $C_\pi^L = C_\nu^L$  ( $L = 0, 2, 4$ ) = 0.1 and  $\xi_3 = 0.1$  have been chosen for  $^{76-84}\text{Kr}$ . All the parameters are in MeV except  $\chi_\pi$  and  $\chi_\nu$  which are dimensionless and

$e_\pi, e_\nu$ are in e. b. unit.									
$A$	$\kappa_\pi$	$\epsilon_d$	$\xi_1$	$\xi_2$	$G_{\text{int}}^0$	$G_{\text{int}}^2$	$e_\pi$	$e_\nu$	
76	-0.100	0.85	0.050	0.035	-0.40	-0.25	0.085	0.100	
78	-0.094	0.84	0.101	0.130	-0.25	-0.33	0.085	0.100	
80	-0.083	0.90	0.240	0.050	-0.14	-0.46	0.080	0.095	
82	-0.078	0.99	0.180	0.050	-0.14	-0.48	0.070	0.085	
84	-0.075	1.00	0.600	0.430	-0.14	-0.48	0.065	0.080	

### 4 Energy levels

In this section, we systematically show the results of the present calculation of energy levels of  $^{76-84}\text{Kr}$  nuclei. They are shown in Figs. 1—5. Reproduction of the trend in the experimental data can be seen, especially those of the  $0^+$  and  $2^+$  states. The data are quoted from compilation in Refs. [19, 20, 24]. The energy levels have been grouped according to bands and  $F$ -spin values, and they provide an opportunity to study the possible collective band structures that are predicted in these nuclei. Examining these figures, they agree very well with the experiment, in particular, all  $0_2^+$  and  $2_3^+$  states, except the  $0_2^+$  in the  $^{76}\text{Kr}$  where the deviation is 0.27 MeV higher than the experimental one. It is possible that this large difference is due to an intruder configuration. The  $0_2^+$  states in the  $^{82,84}\text{Kr}$  have boson seniority  $\tau = 2$ , and these nuclei are close to the  $U(5)$  limit. Comparing these results with the experimental levels, we find that the calculated ground state bands in the  $^{82,84}\text{Kr}$  isotopes are generally higher than the experimental ones. However, it should be emphasized that the closed shell  $N = 50$  plays an essential role in these parameter values. We should point out that the energy of  $3_1^+$  states predicted is in a good agreement with the experimental ones. This is a consequence of the presence of a  $G_{\text{int}}^L$  terms in the Hamiltonian. We have chosen these parameters in such a way that it pushes up  $3_1^+$  states higher than the  $2_2^+$  gamma band head state. The IMB-2 prediction of the gamma band of the Kr isotopes is satisfactory. The staggering of odd-even angular momentum levels in the gamma band, (i. e.  $(3_1^+, 4_2^+), \dots$ ) has been reproduced satisfactorily in the IBM-2 cal-

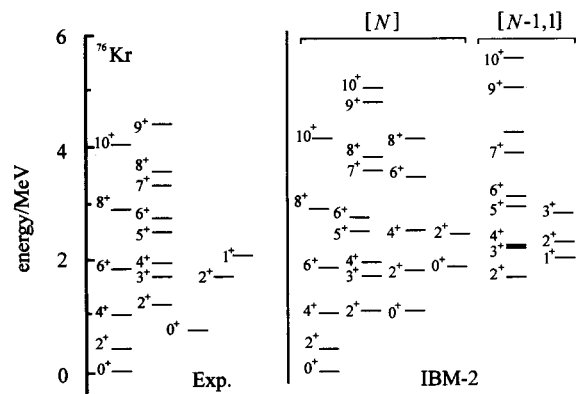


Fig. 1 Comparison between energy levels in the IBM-2 calculation and experimental data in  $^{76}\text{Kr}$ .

ulation. Though the predicted  $0_3^+$  levels around 1.8 MeV in the  $^{76-80}\text{Kr}$  have not been observed, the calculated  $0_3^+$  state in the  $^{84}\text{Kr}$  is very close to the experimental one at 2.170 MeV. The calculations are done using the IBM-2 computer code NPBOS<sup>[5]1)</sup>.

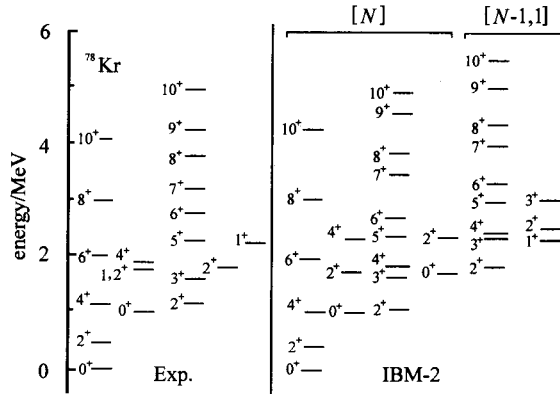


Fig. 2 Comparison between energy levels in the IBM-2 calculation and experimental data in  $^{78}\text{Kr}$ .

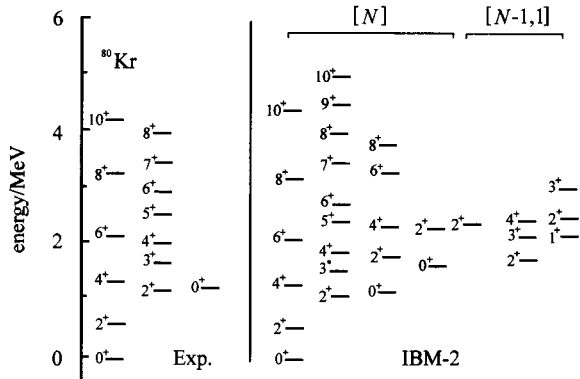


Fig. 3 Comparison between energy levels in the IBM-2 calculation and experimental data in  $^{80}\text{Kr}$ .

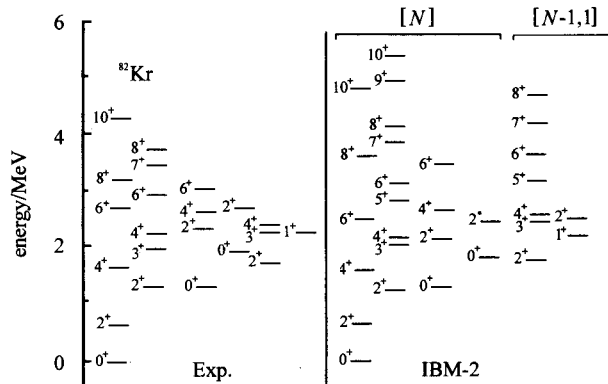


Fig. 4 Comparison between energy levels in the IBM-2 calculation and experimental data in  $^{82}\text{Kr}$ .

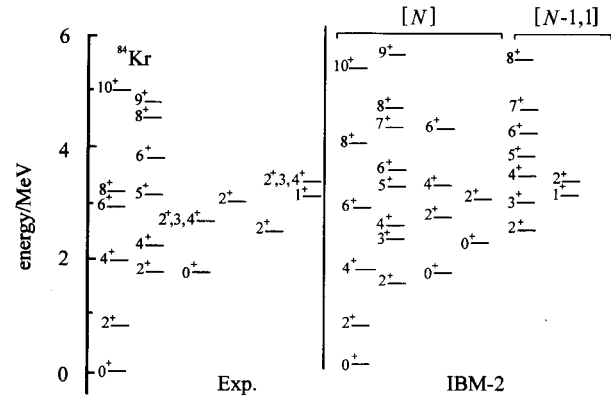


Fig. 5 Comparison between energy levels in the IBM-2 calculation and experimental data in  $^{84}\text{Kr}$ .

## 5 Mixed symmetry states

The excitation energies of the mixed symmetry state are related to the Majorana interactions. We have also calculated the ratio

$$R = \langle J | F^2 | J \rangle / F_{\max}(F_{\max} + 1), \quad (8)$$

as a measure of the mixing  $F$ -spin states. Since we are interested mainly in the  $F = F_{\max}$  and  $F = F_{\max} - 1$  states, we can assume a state has the following form

$$| J \rangle = \alpha | F_{\max} \rangle + \beta | F_{\max} - 1 \rangle, \quad (9)$$

$$\alpha^2 + \beta^2 = 1,$$

and it is easy to calculate

$$\langle J | F^2 | J \rangle = \alpha^2 F_{\max}(F_{\max} + 1) + \beta^2 (F_{\max} - 1) F_{\max}. \quad (10)$$

The values of the  $\alpha$  and  $\beta$  are important as they are a measure of the amount symmetry mixing in each state. We found that the excitation energy of states such as  $2_3^+$  and  $3_2^+$  states have predominately mixed symmetry character and are strongly affected by the  $\xi_2$  value. The  $2_2^+$ ,  $3_1^+$  and  $4_2^+$  states have the similar behaviors and these levels are members of the gamma band. In order to identify the lowest mixed symmetry state, the value of  $\alpha^2$  of the  $2_3^+$  and  $2_4^+$  as a function of  $\xi_2$  is plotted in Fig. 6, and the  $\xi_2$  varies around the best fitted value. Among the lowest  $2^+$  states, the result shows generally that the  $2_3^+$  is the lowest mixed symmetry state and is from the  $s^{N-1}d$  configuration. What is noteworthy is the  $\alpha^2$  around 0.5, and the crossing which occurs so that the  $2^+$  state of predominant

full symmetry becomes yrast. The larger component of the mixed symmetry in the  $2_4^+$  of  $^{78}\text{Kr}$  compared with other Kr isotopes, could be due to the very small energy separation between  $2_3^+$  and  $2_4^+$  in  $^{78}\text{Kr}$ . In  $^{76}\text{Kr}$  a first and second scissor mode state at 2.067 and 2.870 MeV are close to the experimental ones with spin  $(1, 2^+)$  at 2.091 and 2.816 MeV respectively. In  $^{78}\text{Kr}$  the calculation strongly suggests the  $1_2^+$  state at 2.245 MeV be close to the observed one at 2.240 MeV with possible  $J = 1^+, 2^+$ . In  $^{84}\text{Kr}$  the energy levels with spin  $(2^+, 3, 4^+)$  at 3.185 and 3.426 MeV are calculated by the IBM both with spin  $2^+$  at 3.241 ( $F = F_{\max}$ ) and 3.451 MeV ( $F = F_{\max} - 1$ ) respectively. The IBM analysis gives a first and

second scissor state at 2.352 and 3.890 MeV in this nucleus which could correspond to the observed levels with spin  $(1, 2^+)$  at 3.365 and 4.084 MeV respectively. For the  $3^+$  states, one should clearly distinguish between the two lowest  $3^+$  states, however, they are not so directly related to the mixed symmetry structure, since it is possible to have a  $3^+$  with  $d^3$  configuration which is of full symmetry as well as the lowest mixed symmetry with  $d^2$  configuration state. We conclude that the  $3_2^+$  states to be the lowest  $J^\pi = 3^+$  mixed symmetry states with two-phonon excitation in all these isotopes. The low-lying states with  $J^\pi > 3^+$  with a large mixed symmetry component 100 also predicted by this IBM calculation, and they are drawn in the figures.

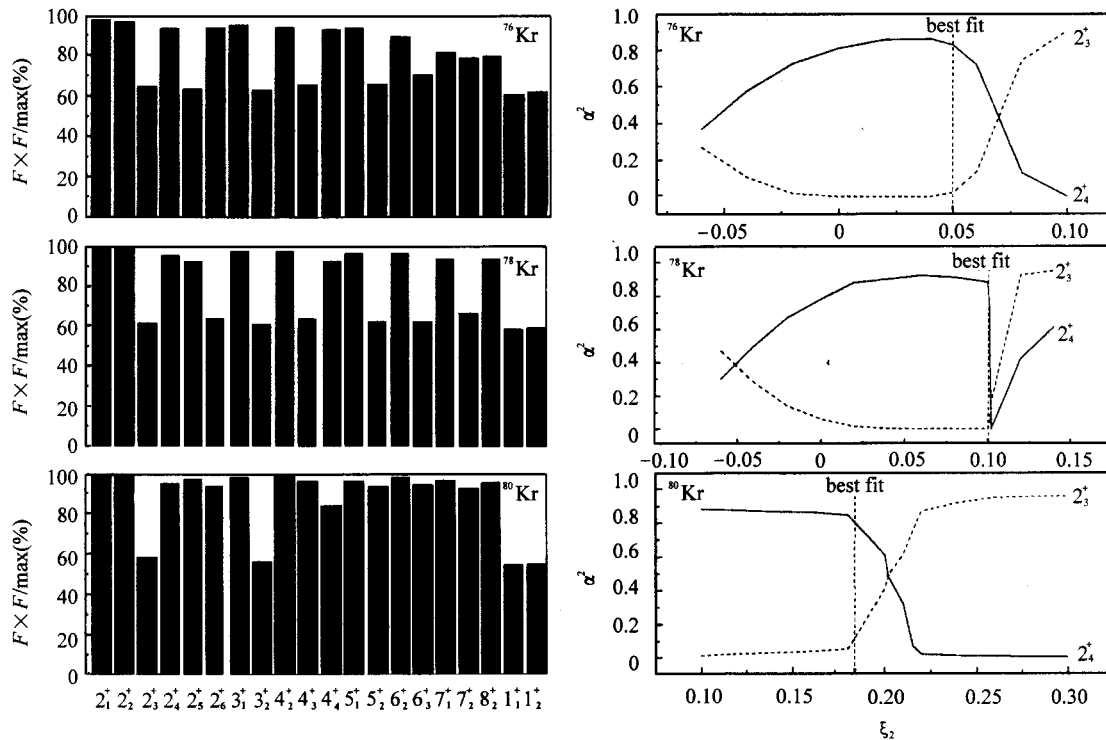


Fig. 6 The  $\langle J | F^2 | J \rangle / F_{\max}(F_{\max} + 1)$  and the full symmetry component of the  $2_3^+$  and  $2_4^+$  states in the  $^{76-80}\text{Kr}$  isotopes.

## 6 Electromagnetic Transitions

In our calculation the adopted values of the effective boson charge in the  $T(E2)$  operator have been determined by normalizing the calculated  $B(E2)$  value to the corresponding experimental results for the  $2_1^+ \rightarrow 0_1^+$  transition. The E2 matrix elements are very sensitive to the differ-

ence between neutron boson and proton boson effective charge, and they have kept constant at 0.015 for all  $^{76-84}\text{Kr}$  isotopes. For illustration we list the calculated and the available experimental data for  $^{76-80}\text{Kr}$  in Table 2, and those for  $^{82,84}\text{Kr}$  in Table 3. The fit of  $B(E2; J^+ \rightarrow J^+ - 2)$  values in the yrast band is satisfactory except  $B(E2; 8_1^+ \rightarrow 6_1^+)$  in the  $^{84}\text{Kr}$  isotope, where the experimental value of this transition is surprisingly very small

compared to the neighboring Kr isotopes; and cannot be explained on the basis of the model according to the position of  $8_1^+$  (see Fig. 5). The transitions from the second  $2^+$  to the ground state are very weak in comparison with the yrast ones. This feature is well described by the IBM. It is interesting to note that the M1 transition can also be used to prove the symmetric properties due to the predominantly isovector nature of  $\hat{T}$  (M1). The M1 transitions have been calculated by using the  $g_\pi = 0.8\mu_n$  and  $g_\nu = 0.3\mu_n$ . Since the reduced M1 transition probabilities de-

pend on the  $\epsilon_\pi$ ,  $\epsilon_\nu$ ,  $\chi_\pi$  and  $\chi_\nu$  as well as on  $g_\pi$  and  $g_\nu$ , we do not have sufficient experimental data at our disposal, and there could hardly be a unique fit to so many parameters. Therefore, we kept the boson  $g$ -factor values constant for all Kr isotopes. Reasonable good agreement is obtained and they are given in the tables. The  $2_3^+ \rightarrow 2_1^+$  transition is dominated by its M1 component and the rest of transitions are dominated by E2 transitions. The M1 matrix elements gave more information on the structure of the mixed symmetry states than the E2 matrix elements alone.

Table 2. Experimental and calculated  $B(E2)$  (in unit  $e^2b^2$ ) and  $B(M1)$  (in unit  $\mu_N^2$ ) for  $^{76-80}\text{Kr}$  isotopes.

$J_i^+ \rightarrow J_f^+$	$^{76}\text{Kr}$				$^{78}\text{Kr}$				$^{80}\text{Kr}$			
	$B(E2)$		$B(M1)$		$B(E2)$		$B(M1)$		$B(E2)$		$B(M1)$	
	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.
$2_1^+ \rightarrow 0_1^+$	0.1640(57)	0.1623			0.1206(79)	0.1213			0.0727(43)	0.0748		
$2_2^+ \rightarrow 0_1^+$	0.0090	0.0046			0.0030(4)	0.0038			0.0038	0.0018		
$2_2^+ \rightarrow 2_1^+$	0.0038	0.1035	0.0429	0.0029	0.0118(39)	0.0943	0.0157(21)	0.0021	0.0511(102)	0.0815	0.0004(1)	0.0019
$2_3^+ \rightarrow 2_1^+$		0.0003		0.0987			0.0001	0.0898		0.0001		0.0855
$0_2^+ \rightarrow 2_1^+$		0.1145				0.1180				0.0881		
$1_1^+ \rightarrow 2_1^+$		0.0005		0.0354		0.0005		0.0268		0.0004		0.0142
$1_1^+ \rightarrow 2_2^+$		0.0010		0.1091		0.0002		0.1142		0.0001		0.1273
$1_1^+ \rightarrow 2_3^+$		0.1405		0.0051		0.1053		0.0332		0.0694		0.0014
$3_1^+ \rightarrow 2_1^+$	0.0019	0.0078	0.0154	0.0012		0.1353		0.0029	0.0011(3)	0.0025	0.0007(2)	0.0037
$3_1^+ \rightarrow 2_2^+$		0.1685		0.0036		0.0058		0.0006	0.0695(102)	0.0935	0.0015(4)	0.0028
$3_2^+ \rightarrow 2_1^+$		0.0002		0.0209		0.0001		0.0143		0.0001		0.0079
$4_1^+ \rightarrow 2_1^+$	0.1982(190)	0.2561			0.1740(138)	0.1974			0.0899(122)	0.1245		
$4_2^+ \rightarrow 2_1^+$	0.0011(4)	0.0003				0.0011			0.0005(3)	0.0016		
$4_2^+ \rightarrow 2_2^+$	0.0858(286)	0.1211			0.1147(158)	0.1010			0.1021(613)	0.0653		
$4_2^+ \rightarrow 4_1^+$	0.0209(76)	0.0613	0.0172(62)	0.0163	0.0474(118)	0.0508	0.0046	0.0078		0.0395	0.0054(35)	0.0043
$5_1^+ \rightarrow 3_1^+$	0.1907(760)	0.1462			0.1523(223)	0.1145			0.1021(347)	0.0731		
$5_1^+ \rightarrow 4_1^+$	0.0057(3)	0.0033	0.0018(9)	0.0021	0.0051(13)	0.0034	0.0013(10)	0.0015	0.0024(14)	0.0022	0.0039(18)	0.0012
$6_1^+ \rightarrow 4_1^+$	0.1773(150)	0.2912			0.2017(336)	0.2278			0.1267(326)	0.1468		
$7_1^+ \rightarrow 5_1^+$	0.1506(476)	0.1641				0.1408			$\leq 0.0919$	0.0877		
$8_1^+ \rightarrow 6_1^+$	0.2326(228)	0.2906			0.1938(296)	0.2282			0.1839 $^{+}_{(919)}$ $^{(1839)}$	0.1471		
$9_1^+ \rightarrow 7_1^+$		0.1779			0.1246(257)	0.1285				0.0721		
$10_1^+ \rightarrow 8_1^+$	0.2286(305)	0.2581			0.1385(295)	0.2023			0.0960(490)	0.1283		

Table 3. Experimental and calculated  $B(E2)$  (in unit  $e^2b^2$ ) and  $B(M1)$  (in unit  $\mu_N^2$ ) for  $^{82,84}\text{Kr}$  isotopes.

$J_i^+ \rightarrow J_f^+$	$^{82}\text{Kr}$				$^{84}\text{Kr}$			
	$B(E2)$		$B(M1)$		$B(E2)$		$B(M1)$	
	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.
$2_1^+ \rightarrow 0_1^+$	0.0450(14)	0.0428			0.0244(11)	0.0264		
$2_2^+ \rightarrow 0_1^+$	0.0002	0.0004			0.0052(13)	0.0001		
$2_2^+ \rightarrow 2_1^+$	0.0053	0.0566	0.0010	0.0036	0.0239(87)	0.0386	0.0256(53)	0.0012
$2_3^+ \rightarrow 2_1^+$	0.0014(8)	0.0001		0.0825		0.0002		0.0509

续表

$J_i^+ \rightarrow J_f^+$	$^{82}\text{Kr}$				$^{84}\text{Kr}$			
	B(E2)		B(M1)		B(E2)		B(M1)	
	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.
$0_2^+ \rightarrow 2_1^+$	0.0317(105)	0.0521				0.0344		
$1_1^+ \rightarrow 2_1^+$		0.0003		0.0062		0.0001		0.0018
$1_1^+ \rightarrow 2_2^+$		0.0001		0.1318		0.0002		0.1120
$1_1^+ \rightarrow 2_3^+$		0.0418		0.0034		0.0197		0.0002
$3_1^+ \rightarrow 2_1^+$		0.0006		0.0005		0.0001		0.0002
$3_1^+ \rightarrow 2_2^+$		0.0554		0.0065		0.0325		0.0053
$3_2^+ \rightarrow 2_1^+$		0.0001		0.0028		0.0001		0.0006
$4_1^+ \rightarrow 2_1^+$	0.0676(253)	0.0693			0.0479(65)	0.0411		
$4_2^+ \rightarrow 2_1^+$	0.0024(4)	0.0005			0.0003	0.0001		
$4_2^+ \rightarrow 2_2^+$	0.0195(4)	0.0375			0.0035(5)	0.0216		
$4_2^+ \rightarrow 4_1^+$	0.0812(13)	0.0277	0.1414(208)	0.0047		0.0182		0.0003
$5_1^+ \rightarrow 3_1^+$		0.0407				0.0211		
$5_1^+ \rightarrow 4_1^+$		0.0004		0.0015		0.0001		0.0005
$6_1^+ \rightarrow 4_1^+$	0.0116	0.0801			0.0152(39)	0.0451		
$7_1^+ \rightarrow 5_1^+$		0.0444				0.0193		
$8_1^+ \rightarrow 6_1^+$	0.0126(17)	0.0771			0.0049(2)	0.0392		
$9_1^+ \rightarrow 7_1^+$		0.0305				0.0077		
$10_1^+ \rightarrow 8_1^+$	$0.0232^{+}_{(63)}(148)$	0.0623			0.0174(5)	0.0244		

## 7 Conclusions

We have calculated the energy levels and electromagnetic transition of  $^{76-84}\text{Kr}$  isotopes using the interacting boson model-2. The IBM-2 calculation well reproduce experimental data for these isotopes. We have found that the  $0_2^+$  state in the  $^{76}\text{Kr}$  alone is outside the IBM - 2 space. The

results of this work show that when the  $2_2^+$ ,  $2_4^+$  and  $3_1^+$  eigenstates are strongly dominated by the  $F = F_{\max}$ , the strongest contribution to the  $2_3^+$  and  $3_2^+$  states is the one with  $F = F_{\max} - 1$ . We can describe the  $2_3^+$  and  $3_2^+$  states as mixed symmetric states in the  $^{76-84}\text{Kr}$  isotopes. The IBM predicted more mixed symmetric states, and further experimental investigations are expected.

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## $^{76-84}\text{Kr}$ 原子核的低能级对称态能带和混合对称能带 \*

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**摘要** 在相互作用玻色子模型 2 中讨论了  $^{76-84}\text{Kr}$  的能级结构。通过对波函数和 M1 跃迁的分析, 确认了单声子混合对称态和双声子  $1^+, 2^+, 3^+$  混合对称态。特别注意了其他计算中与实验数据没有很好符合的  $0_2^+$  态的研究, 我们的计算在多数情况下改善了与实验的符合。研究了  $[d^+\bar{d}]_0^L \cdot [d^+\bar{d}]_0^L$  相互作用对这些核的结构的影响。计算结果与已有实验数据进行了比较, 计算结果和实验符合。

**关键词** 混合对称态  $^{76-84}\text{Kr}$  同位素 相互作用玻色子模型 2

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