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A Deduced Feynman Rule for Calculating Retarded and Advanced Green Function in Closed Time Path Formalism *

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Abstract Based on the closed time path formalism, a deduced Feynman rule for directly calculating the retarded and advanced Green functions is given. This Feynman rule is used to calculate the two-point self-energy and three-point vertex correction in ϕ^3 theory. The generalized fluctuation-dissipation theorem for three-point nonlinear response function is verified.

Key words thermal field theory, Feynman rule, retarded Green function, advanced Green function

The closed time path formalism (CTPF)[1-3] in real-time finite-temperature field theory has been widely used to investigate the equilibrium and non-equilibrium properties[4-10] of the thermal system. Different from the imaginary time formalism (ITF), the physical region can be reached without analytical continuation in the real time formalism. In the CTPF two kinds of fields are defined according to the closed time integral contour in the generating function of Green function. The closed time integral contour consists of the first branch which runs from negative infinity to positive infinity and the second branch which runs back from positive infinity to negative infinity. The fields located on the first and the second branch are defined as physical (type-1) and ghost (type-2) fields, respectively. The propagator is a 2×2 matrix with components G_{11}, G_{12}, G_{21}, G_{22}, their corresponding self-energies are denoted as $\sum_{11}, \sum_{12}, \sum_{21}, \sum_{22}$ according to the type-1 or type-2 external leg of the Feynhman diagram. It is known that the self-energy with physical interpretation is not $\sum_{11}$ but the retarded function $\sum^{ret}$ (or advanced function $\sum^{adv}$) which is the analytical continuation of the self-energy obtained in the ITF[9]. In order to calculate retarded (or advanced) self-energy in the real time formalism, the usual way is to calculate $\sum_{11}, \sum_{12}, \sum_{21}$ and $\sum_{22}$ first, and then to use the relation $\sum^{ret} = \sum_{11} + \sum_{12}$ (or $\sum^{adv} = \sum_{11} + \sum_{21}$). For the calculation of n-point ($n>2$) retarded/advanced vertex[11], for example, the three-point fully retarded vertex Γ_{raa} defined in Ref. [12], one needs to calculate eight components $\Gamma_{111}, \Gamma_{112}, \Gamma_{121}, \Gamma_{122}, \Gamma_{211}, \Gamma_{212}, \Gamma_{221}$ and Γ_{222} first and then to use the relation $\Gamma_{raa} = \frac{1}{2}(\Gamma_{111} - \Gamma_{112} - \Gamma_{121} + \Gamma_{122} + \Gamma_{211} - \Gamma_{212} - \Gamma_{221} + \Gamma_{222})$. Such calculation becomes tedious. In this paper we deduce a Feynman rule which can be used to calculate n-point retarded/advanced Green function directly, and then we verify the generalized Fluctuation-dissipation theorem for three-point nonlinear response function by calculating the vertex correction.

In the CTPF, for any field ϕ the four components of the matrix propagator in the single-time representation are defined as

$$G_{a_1a_2}(x_1,x_2) \equiv -\mathrm{i}\langle T_p(\phi_{a_1}(x_1)\phi_{a_2}(x_2))\rangle, \tag{1}$$

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where T_p represents the time ordering operator along the closed time path, it is normal and antichronological time ordering on the first and second branch of the closed time path, respectively, $a_1, a_2 \in \{1,2\}$ indicate on which of the two branches the ϕ fields are located. $\langle\cdots\rangle$ stands for the thermal expectation value. Following Ref. [3] we define

$$\phi_a(x) = \phi_1(x) - \phi_2(x),$$
$$\phi_r(x) = \frac{1}{2}(\phi_1(x) + \phi_2(x)), \qquad (2)$$

and the 2-point Green function in the above (r, a) basis

$$G_{\alpha_1\alpha_2}(x_1, x_2) \equiv - i2^{n_r-1}\langle T_p(\phi_{\alpha_1}(x_1)\phi_{\alpha_2}(x_2))\rangle, \qquad (3)$$

where $\alpha_1, \alpha_2 \in \{a, r\}$, and n_r is the number of r indices among $\{\alpha_1, \alpha_2\}$. Inserting Eq (2) into (3), we get

$$G_{rr}(x_1, x_2) = G_{12}(x_1, x_2) + G_{21}(x_1, x_2) = - i\langle[\phi(x_1), \phi(x_2)]_{\pm}\rangle, \qquad (4)$$

$$G_{ra}(x_1, x_2) = G_{11}(x_1, x_2) - G_{12}(x_1, x_2) = - i\theta(x_1^0 - x_2^0)\langle[\phi(x_1), \phi(x_2)]_{\mp}\rangle, \qquad (5)$$

$$G_{ar}(x_1, x_2) = G_{11}(x_1, x_2) - G_{21}(x_1, x_2) = i\theta(x_2^0 - x_1^0)\langle[\phi(x_1), \phi(x_2)]_{\mp}\rangle, \qquad (6)$$

$$G_{aa}(x_1, x_2) = 0. \qquad (7)$$

Here $[\phi(x_1), \phi(x_2)]_{\pm} = \phi(x_1)\phi(x_2) \pm \phi(x_2)\phi(x_1)$. The double sign $\pm$ or $\mp$ in Eqs. (4) – (6) corresponds to Boson and Fermion field for the upper and lower cases, respectively. Obviously $G_{ra}(x_1, x_2)$ and $G_{ar}(x_1, x_2)$ are the usual retarded and advanced Green functions.

Denote the 2×2 matrix propagator in the single-time representation as

$$G = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}, \qquad (8)$$

and the matrix propagator in the (r, a) representation as

$$\bar{G} = \begin{pmatrix} G_{aa} & G_{ar} \\ G_{ra} & G_{rr} \end{pmatrix} = \begin{pmatrix} 0 & G^{adv} \\ G^{ret} & G^{cor} \end{pmatrix}, \qquad (9)$$

where G^{adv}, G^{ret} are the advanced and retarded Green functions. In the momentum space the correlation function G^{cor} is related to advanced and retarded Green function by well-known fluctuation-dissipation theorem[13, 10] in linear response theory,

$$G^{cor}(k) = (1 + 2n(k^0))(G^{ret}(k) - G^{adv}(k)) \qquad (10)$$

where $n(k^0) = 1/(\exp(\beta k^0) - 1)$ is the Bose-Einstein distribution function.

Both the matrix propagators G and $\bar{G}$ satisfy the following transformation relation[3]:

$$G = Q^+ \bar{G} Q, \qquad (11)$$

where Q is orthogonal Keldysh transformation matrix

$$Q = \begin{pmatrix} Q_{a1} & Q_{a2} \\ Q_{r1} & Q_{r2} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \qquad (12)$$

$$Q^+ = \begin{pmatrix} Q^+_{1a} & Q^+_{1r} \\ Q^+_{2a} & Q^+_{2r} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}. \qquad (13)$$

From the transformation (11) we see that the left side and the right side of the propagator $\bar{G}$ associate with the transformation matrix Q^+ and Q, respectively. We can establish a deduced Feynman rule in the following way : as illustrated in Fig. 1, the matrix Q^+ can be absorbed into the left vertex with the outgoing momentum line, and Q can be absorbed into the right vertex with the incoming momentum line, we leave $\bar{G}_{\alpha\beta}$ in the (r, a) basis as a new propagator of the propagating line. After absorbing all Q or Q^+ from the propagating line, the new bare vertex with all incoming momentum (as illustrated in Fig. 2(a)) can be defined as

$$\gamma_{\beta\beta'\cdots\beta''}(p, q, \ldots r) = Q_{\beta a}Q_{\beta' b}\ldots Q_{\beta'' c}g_{ab\ldots c}, \qquad (14)$$

where $a, b, \ldots c \in 1,2$ and $\beta, \beta', \ldots \beta'' \in a, r$, $p + q + \ldots + r = 0$ because of energy-momentum conservation. A summation over repeated indices is understood in Eq. (14). In the single-time representation, $g_{11\cdots1} = - g_{22\cdots2} = g$ (coupling constant), all other components vanish. As shown in Fig. 2, we change the orientation of the incoming momentum r in Fig. 2(a) as the outgoing line in Fig. 2(b) with energy-momentum conservation $p + q + \ldots - r = 0$. After absorbing Q^+ associated with the outgoing momentum r and all Q associated with the incoming momentums into the vertex, this new bare vertex in Fig. 2(b) should be

$$\gamma_{\beta\beta'\cdots\beta''}(p, q, \ldots - r) = Q_{\beta a}Q_{\beta' b}\ldots Q^+_{\beta'' c}g_{ab\ldots c}. \qquad (15)$$

From Eqs. (12) and (13) we know $Q_{\beta'' c} = Q^+_{c\beta''}$.

Comparing Eq. (15) with Eq. (14), we see clearly that the bare vertex $\gamma_{\beta\beta'\ldots\beta''}(p, q, \ldots r)$ with $p+q+\ldots+r=0$ is the same as the bare vertex $\gamma_{\beta\beta'\ldots\beta''}(p, q, \ldots -r)$ with $p+q+\ldots-r=0$. This property indicates that the new bare vertex defined here is independent of the orientation of the momentum.

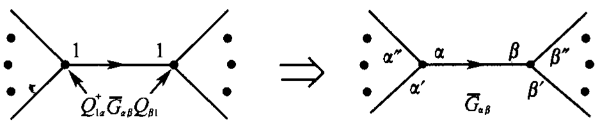

Fig. 1. The new bare vertex and propagator $\bar{G}_{\alpha\beta}$ after absorbing Q^+ and Q into the left and right vertex in our deduced Feynman rule.

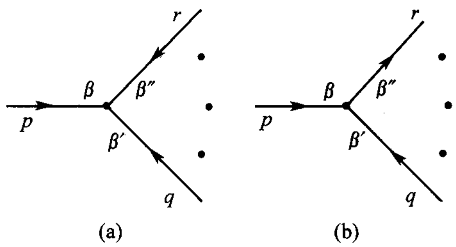

Fig. 2. The bare vertex in the (r, a) basis. The difference between (a) and (b) is that the orientation of momentum r is changed.

In the following we drop the momentum arguments for the bare vertex. Substituting Q_{a1}, Q_{a2}, Q_{r1} Q_{r2} in Eq. (12) into Eq. (14), we can express the n-point new bare vertex in the (r, a) basis as

$$\gamma_{\alpha_1\alpha_2\cdots\alpha_n} = \left(\frac{1}{\sqrt{2}}\right)^n g[1-(-1)^{n_a(\alpha_1,\alpha_2,\cdots\alpha_n)}], \tag{16}$$

where $n_a(\alpha_1, \alpha_2, \ldots, \alpha_n)$ is the number of a indices among $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$. As this number is even, the bare vertex is zero, so in the (r, a) basis half of the bare vertices vanish and the rest of them are only related to the coupling constant. This property will help us to simplify the calculation greatly. For example, the explicit form of three-point bare vertices can be expressed as

$$\gamma_{aaa} = \gamma_{arr} = \gamma_{rra} = \gamma_{rar} = \frac{g}{\sqrt{2}},$$
$$\gamma_{aar} = \gamma_{ara} = \gamma_{rrr} = \gamma_{raa} = 0. \tag{17}$$

As the first application we use this deduced Feynman rule to calculate the retarded (or advanced) self-energy in ϕ^3 theory. As shown in Fig. 3, all components of the self-energy matrix in the (r, a) basis can be expressed as

$$-i\Sigma_{\beta\beta'}(q) = \int\frac{d^4p}{(2\pi)^4}(-i\gamma_{\alpha\beta\delta})(i\Delta_{\alpha\alpha'}(p)) \times (-i\gamma_{\alpha'\beta'\delta'})(\Delta_{\delta'\delta}(r)), \tag{18}$$

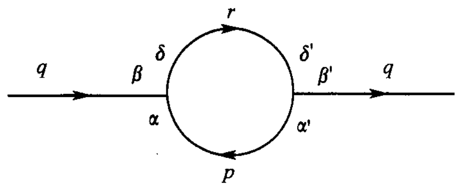

Fig. 3. The one-loop Feynman diagram for calculating the retarded and advanced self-energy in ϕ^3 theory.

where $r = p + q$. From Eq. (7) propagator $\Delta_{aa} = 0$. Noticing the poles of $\Delta_{ra}(p)\Delta_{ra}(r)$ and $\Delta_{ar}(p)\Delta_{ar}(r)$ are both on the same side of the real axis in the complex p^0 plane, we have

$$\int_{-\infty}^{+\infty}\frac{dp^0}{2\pi}\Delta_{ra}(p)\Delta_{ra}(r) = \int_{-\infty}^{+\infty}\frac{dp^0}{2\pi}\Delta_{ar}(p)\Delta_{ar}(r) = 0. \tag{19}$$

Using these relations, we derive the four components of the self-energy matrix as

$$-i\Sigma_{ra}(q) = g^2\int\frac{d^4p}{(2\pi)^4}\{n(r_0)\Delta_{ar}(p)[\Delta_{ra}(r) - \Delta_{ar}(r)] + n(p_0)\Delta_{ra}(r)[\Delta_{ra}(p) - \Delta_{ar}(p)]\}, \tag{20}$$

$$-i\Sigma_{ar}(q) = g^2\int\frac{d^4p}{(2\pi)^4}\{n(p_0)\Delta_{ar}(r)[\Delta_{ra}(p) - \Delta_{ar}(p)] + n(r_0)\Delta_{ra}(p)[\Delta_{ra}(r) - \Delta_{ar}(r)]\}, \tag{21}$$

$$-i\Sigma_{aa}(q) = [2n(q_0)+1][\Sigma_{ra}(q) - \Sigma_{ar}(q)], \tag{22}$$

$$-i\Sigma_{rr}(q) = 0. \tag{23}$$

It is easy to show that $\Sigma_{ar}(q)$ and $\Sigma_{ra}(q)$ are the retarded and advanced self-energies which correspond to the retarded and advanced analytical continuation of the self-energy calculated in the ITF[14]. Eq. (22) corresponds to the Fluctuation-Dissipation theorem in linear response theory[13].

The one-loop three-point vertex correction is illustrated in Fig. 4. The general expression for this vertex correction in the (r, a) basis can be expressed as

$$-i\Gamma_{\alpha\beta\delta}(p,q,r) = \int \frac{d^4 l_1}{(2\pi)^4}(-i\gamma_{\alpha\alpha'\alpha''})[i\Delta_{\alpha'\beta''}(l_1)] (-i\gamma_{\beta\beta'\beta''})\times[i\Delta_{\beta'\delta''}(l_2)] (-i\gamma_{\delta\delta'\delta''})[i\Delta_{\delta'\alpha''}(l_3)], \quad (24)$$

where $p+q+r=0$ and $l_1+l_2+l_3=0$.

It is similar to Eq. (19), the l_1^0-integral over terms $\Delta_{ra}(l_1)\Delta_{ra}(l_2)\Delta_{ra}(l_3)$ and $\Delta_{ar}(l_1)\Delta_{ar}(l_2)\Delta_{ar}(l_3)$ vanish because the poles of this two terms are both on the same side of the real axis in the complex l_1^0 plane. Neglecting these vanishing terms and using $\Delta_{aa}=0$, we deduce eight components of one-loop vertex correction in ϕ^3 theory as

$$-i\Gamma_{rra}(p,q,r) = \frac{2g^3}{(\sqrt{2})^3}\int \frac{d^4 l_1}{(2\pi)^4}\{n(l_{01})[\Delta_{ra}(l_1) - \Delta_{ar}(l_1)]\Delta_{ar}(l_2)\Delta_{ra}(l_3) + n(l_{02})[\Delta_{ra}(l_2) - \Delta_{ar}(l_2)]\Delta_{ra}(l_1)\Delta_{ra}(l_3) + n(l_{03})[\Delta_{ra}(l_3) - \Delta_{ar}(l_3)]\Delta_{ar}(l_1)\Delta_{ar}(l_2)\}, \quad (25)$$

$$-i\Gamma_{rar}(p,q,r) = \frac{2g^3}{(\sqrt{2})^3}\int \frac{d^4 l_1}{(2\pi)^4}\{n(l_{01})[\Delta_{ra}(l_1) - \Delta_{ar}(l_1)]\Delta_{ra}(l_2)\Delta_{ra}(l_3) + n(l_{02})[\Delta_{ra}(l_2) - \Delta_{ar}(l_2)]\Delta_{ar}(l_1)\Delta_{ar}(l_3) + n(l_{03})[\Delta_{ra}(l_3) - \Delta_{ar}(l_3)]\Delta_{ar}(l_1)\Delta_{ra}(l_2)\}, \quad (26)$$

$$-i\Gamma_{arr}(p,q,r) = \frac{2g^3}{(\sqrt{2})^3}\int \frac{d^4 l_1}{(2\pi)^4}\{n(l_{01})[\Delta_{ra}(l_1) - \Delta_{ar}(l_1)]\Delta_{ar}(l_2)\Delta_{ar}(l_3) + n(l_{02})[\Delta_{ra}(l_2) - \Delta_{ar}(l_2)]\Delta_{ra}(l_1)\Delta_{ar}(l_3) + n(l_{03})[\Delta_{ra}(l_3) - \Delta_{ar}(l_3)]\Delta_{ra}(l_1)\Delta_{ra}(l_2)\}, \quad (27)$$

$$-i\Gamma_{aar}(p,q,r) = \frac{g^3}{(\sqrt{2})^3}\int \frac{d^4 l_1}{(2\pi)^4}\{[\Delta_{ra}(l_1) - \Delta_{ar}(l_1)]\Delta_{ra}(l_2)\Delta_{ar}(l_3)[N_2 - N_3]N_1 + [\Delta_{ra}(l_1) - \Delta_{ar}(l_1)]\Delta_{ar}(l_2)\Delta_{ar}(l_3)[1 - N_1N_2] + [\Delta_{ra}(l_1) - \Delta_{ar}(l_1)]\Delta_{ra}(l_2)\Delta_{ra}(l_3)[N_1N_3 - 1]\}, \quad (28)$$

$$-i\Gamma_{raa}(p,q,r) = \frac{g^3}{(\sqrt{2})^3}\int \frac{d^4 l_1}{(2\pi)^4}\{[\Delta_{ra}(l_2) - \Delta_{ar}(l_2)]\Delta_{ra}(l_1)\Delta_{ra}(l_3)[N_1N_2 - 1] + [\Delta_{ra}(l_2) - \Delta_{ar}(l_2)]\Delta_{ar}(l_1)\Delta_{ra}(l_3)[N_2N_3 - N_2N_1] + [\Delta_{ra}(l_2) - \Delta_{ar}(l_2)]\Delta_{ar}(l_1)\Delta_{ar}(l_3)[1 - N_2N_3]\}, \quad (29)$$

$$-i\Gamma_{ara}(p,q,r) = \frac{g^3}{(\sqrt{2})^3}\int \frac{d^4 l_1}{(2\pi)^4}\{[\Delta_{ra}(l_3) - \Delta_{ar}(l_3)]\Delta_{ar}(l_1)\Delta_{ar}(l_2)[1 - N_1N_3] + [\Delta_{ra}(l_3) - \Delta_{ar}(l_3)]\Delta_{ra}(l_1)\Delta_{ar}(l_2)[N_1N_3 - N_2N_3] + [\Delta_{ra}(l_3) - \Delta_{ar}(l_3)]\Delta_{ra}(l_1)\Delta_{ra}(l_2)[N_2N_3 - 1]\}, \quad (30)$$

$$-i\Gamma_{aaa}(p,q,r) = \frac{g^3}{(\sqrt{2})^3}\int \frac{d^4 l_1}{(2\pi)^4}\{N_1N_2N_3[\Delta_{ra}(l_1)\Delta_{ra}(l_2)\Delta_{ra}(l_3) - \Delta_{ar}(l_1)\Delta_{ar}(l_2)\Delta_{ar}(l_3)] + (N_1 - N_1N_2N_3)[\Delta_{ra}(l_1) - \Delta_{ar}(l_1)]\Delta_{ra}(l_2)\Delta_{ar}(l_3) + (N_2 - N_1N_2N_3)[\Delta_{ra}(l_2) - \Delta_{ar}(l_2)]\Delta_{ar}(l_1)\Delta_{ar}(l_3) + (N_3 - N_1N_2N_3)[\Delta_{ra}(l_3) - \Delta_{ar}(l_3)]\Delta_{ra}(l_1)\Delta_{ar}(l_2)\}, \quad (31)$$

$$-i\Gamma_{rrr}(p,q,r) = 0. \quad (32)$$

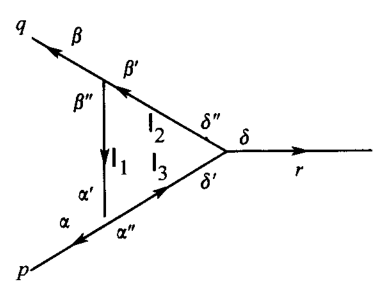

Fig. 4. The Feynman diagram for calculating one-loop vertex correction in the (r,a) basis in ϕ^3 theory.

Here $N_i = 1 + 2n(l_{0i})$. Eqs. (23) and (32) indicate that the two-point self-energy and three-point vertex with all indices being r vanish. This property is also true for n-point ($n>3$) vertex[11].

Denote $N_{k^0} = 1 + 2n(k^0)$ for $k^0 = p^0, q^0, r^0$. As $p+q+r=0$ and $l_1+l_2+l_3=0$, we have

$$N_{p^0}[N_3 - N_1] = N_3N_1 - 1, \quad (33)$$

$$N_{q^0}[N_1 - N_2] = N_1N_2 - 1, \quad (34)$$

$$N_{r^0}[N_2 - N_3] = N_2N_3 - 1. \quad (35)$$

Using Eqs. (33) - (35) we can verify that three-point vertex corrections in Eqs. (25)—(32) satisfy the following relations:

$$\Gamma_{ara} = N_p^0(\Gamma_{rar}^* - \Gamma_{rra}) + N_r^0(\Gamma_{rar}^* - \Gamma_{arr}), \quad (36)$$

$$\Gamma_{raa} = N_q^0(\Gamma_{arr}^* - \Gamma_{rra}) + N_r^0(\Gamma_{arr}^* - \Gamma_{rar}), \quad (37)$$

$$\Gamma_{aar} = N_p^0(\Gamma_{rra}^* - \Gamma_{rar}) + N_q^0(\Gamma_{rra}^* - \Gamma_{arr}), \quad (38)$$

$$\Gamma_{aaa} = \Gamma_{arr}^* + \Gamma_{rar}^* + \Gamma_{rra}^* + N_q^0 N_r^0(\Gamma_{arr} + \Gamma_{arr}^*) + N_p^0 N_r^0(\Gamma_{rar} + \Gamma_{rar}^*) + N_p^0 N_q^0(\Gamma_{rra} + \Gamma_{rra}^*). \quad (39)$$

These relations are just the generalized Fluctuation-Dissipation theorem obtained in our previous work[11] for three-point nolinear response function. Eqs. (36)—(39) together with Eq. (32) indicate that, among these vertex functions $\{\Gamma_{rra}, \Gamma_{rar}, \Gamma_{arr}, \Gamma_{raa}, \Gamma_{ara}, \Gamma_{aar}, \Gamma_{aaa}, \Gamma_{rrr}\}$ only three components are independent.

In Ref. [9] Aurenche and Becherrawy developed a Feynman rule labelled by R, A to calculate retarded and advanced vertex function. The main differences between their and our Feynman rule are the follo wing: in the (R, A) Feynman rule, the matrix propagator is diagonal and independent of the thermal distribution function; the bare vertex is related with the thermal distribution function and relies on the relative orientation of the momenta; only two of the vertex components vanish. In our (r, a) Feynman rule, the matrix propagator is non-diagonal, one element Δ_{rr} is related to the thermal distribution function; the bare vertex is independent of the thermal distribution function and doesn't rely on the orientation of the momenta; one half of the bare vertex components vanish. In the practical calculation both (R, A) and (r, a) Feynman rule have their own advantage and disadvantage. However, we should emphasize that, in our (r, a) basis the Green function is defined with time-ordering and have explicit physical interpretation (see Eqs. (4) – (7) and Ref. [11]), it works in both coordinate and momentum space. The (R, A) Feynman rule, as stated by the authors themselves in Ref. [9], only works in the momentum space, the (R, A) Green function has no explicit reference to a definite time-ordering.

By suitable combination we can get the (R, A) vertex from the (r, a) vertex. For three-point vertex the relation between them can be expressed as

$$\widetilde{\Gamma}_{RRR} = \widetilde{\Gamma}_{AAA} = 0, \quad (40)$$

$$\widetilde{\Gamma}_{RRA} = \Gamma_{rra}, \quad (41)$$

$$\widetilde{\Gamma}_{RAR} = \Gamma_{rar}, \quad (42)$$

$$\widetilde{\Gamma}_{ARR} = \Gamma_{arr}, \quad (43)$$

$$\widetilde{\Gamma}_{AAR} = -\frac{1}{2}[N_p^0 + N_q^0]\Gamma_{rra}^*, \quad (44)$$

$$\widetilde{\Gamma}_{ARA} = -\frac{1}{2}[N_p^0 + N_r^0]\Gamma_{rar}^*, \quad (45)$$

$$\widetilde{\Gamma}_{RAA} = -\frac{1}{2}[N_p^0 + N_r^0]\Gamma_{arr}^*. \quad (46)$$

In summary, a deduced Feynman rule in the (r, a) basis is given from the Keldysh transformation in the CTPF. This Feynman rule can be used to calculate the retarded and advanced Green function directly. In this deduced Feynman rule, the bare vertex is only related with the coupling constant and independent of the orientation of the momenta, one half of the bare vertex components vanish; the new propagator depends on the retarded, advanced propagator and the thermal distribution function. As applications, the one-loop self-energy and three-point vertex correction in ϕ^3 theory are calculated in the (r, a) basis, and the Fluctuation-Dissipation theorem is verified in the two-point and three-point cases. The diffierence between our (r, a) Feynman rule and the (R, A) Feynman rule introduced by Aurenche and Becherrawy[9] is discussed.

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在闭合时间路径温度场论中直接计算推迟和超前格林函数的费曼规则*

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摘要　在闭合时间路径温度场论基础上,导出了直接计算推迟和超前格林函数的一种费曼规则. 利用该费曼规则计算了 ϕ^3 理论中的两点自能和三点顶角函数修正. 验证了三点非线响应函数满足的推广的涨落-耗散定理.

关键词　热场理论　费曼规则　推迟格林函数　超前格林函数

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