## Resummation Study on Decay $\rho \rightarrow \pi\pi$ in $U(2)_L \times U(2)_R$ Chiral Theory of Mesons \*

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Abstract We improve  $O(p^4)$  calculation in  $U(2)_L \times U(2)_R$  chiral theory of mesons by resummation calculation for vector mesons physics and restudy decay  $\rho \rightarrow \pi\pi$ . We obtain a complete and compact expression for  $f_{\rho\pi\pi}(p^2)$  (up to  $O(p^{\infty})$ ), from which an important non-perturbative conclusion is drawn from convergence and unitarity consideration.

Key words resummation, chiral theory, non-perturbative calculation

The Nambu-Jona-Lasinio (NJL) model<sup>[1]</sup> and its extensions are widely used to understand hadron physics (for reviews see, e.g., Ref. [2]). There are various methods to parametrize the NJL version models, among which is the  $U(N_f)_L \times U(N_f)_R$  chiral theory of mesons<sup>[3]</sup> with  $N_f = 2$  or 3 (proposed by Li and called Li's model hereafter). This model can be regarded as a realization of chiral symmetry, current algebra and vector meson dominance (VMD). It provides a unified description of pseudoscalar, vector, and axial-vector mesons, which are introduced as bound states of quark fields. Its basic inputs are the cutoff  $\Lambda$  (or g in Ref. [3]) and constituent quark mass m (related to quark condensate). This theory has been studied extensively [4-10], in particular it has been used recently [11] to analyze the data of g-2 of muon reported by CMD-2 group<sup>[12]</sup>. Hence Li's model is a good phenomenological model.

So far Li's model has been only investigated up to  $O(p^4)^{[3]}$  in perturbation manner. This treatment is reasonable for chiral perturbation theory  $(ChPT)^{[13]}$ , because the typical energy there is much less than the energy scale of chiral symmetry spontaneously breaking

(CSSB)  $\Lambda_{\rm CSSB} \approx 2\pi F_{\pi} \approx 1 {\rm GeV}$ . This is not the case here because the typical energy p of vector mesons is comparable with  $\Lambda_{\rm CSSB}$ . For example, at  $\rho$  energy scale,  $p \approx m_{\rho}$  and the perturbation parameter  $p/\Lambda_{\rm CSSB} \approx 0.7$ . Therefore, studies in Ref. [3] should be improved by going beyond  $O(p^4)$ . But we know that, in usual perturbation treatment, it is difficult to deduce effective action of mesons at higher order. This is a paradox. As a solution to it, we develop a method to obtain the sum of all terms of the p-expansion<sup>[14]</sup>. We call it resummation study. In this paper, we want to use it to restudy typical decay  $\rho \rightarrow \pi \pi$  in Li's model. Other processes can be restudied in a similar way.

Before this study, let us outline procedures of perturbation calculation and our resummation calculation. Given a quark model which describes interactions of constituent quarks q and mesons M and satisfies requirement of Chiral symmetry, we can write the Lagrangian as  $\mathcal{Z} = \mathcal{Z}(q,\bar{q},M) = \bar{q}\,\mathcal{D}q$ , where  $\mathcal{D} = \mathcal{D}(M)$  is characteristic of the model. The first step of perturbation calculation is using the method of path integral to integrate out the quark fields,

Received 1 July 2003

<sup>\*</sup> Supported by NSFC (90103002)

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$$e^{i \int d^4 x \mathscr{L}_{eff}(M)} = \int [dq] [d\bar{q}] e^{i \int d^4 x \bar{q} \mathscr{D}} q. \qquad (1)$$

After functional integration, we can formally obtain the effective action  $S_{\rm eff}$  (M) of mesons as  $S_{\rm eff}$  (M) =  $\ln \det \mathcal{D}(M)$ . To regularize this determinant, Schwinger's proper time method<sup>[15]</sup> or heat kernel method<sup>[16]</sup> should be employed. This directly results in an expansion in p, and usually up to  $O(p^4)$ . One will encounter great difficulty when he attempts to calculate terms of higher orders.

On the other hand, to perform resummation study, we calculate effective meson action via loop effects of constituent quarks. This method is equivalent to integration over quarks in path integral, but via this method we can obtain effects from all orders of p-expansion. Specifically, we divide  $\mathscr L$  into free-field part  $\mathscr L_0$  and interaction part  $\mathscr L_1$ , and turn to interaction picture. The effective action can be obtained as

$$\begin{split} \mathrm{e}^{\mathrm{i} S_{\mathrm{eff}}} &= \langle 0 \mid T_{\mathrm{q}} \mathrm{e}^{\mathrm{i} \int \mathrm{d}^{4} x \mathscr{L}_{1}} \mid 0 \rangle = \\ &1 + \sum_{n=1}^{\infty} \mathrm{i} \int \mathrm{d}^{4} p_{1} \, \frac{\mathrm{d}^{4} p_{2}}{(2\pi)^{4}} \cdots \, \frac{\mathrm{d}^{4} p_{n}}{(2\pi)^{4}} \times \\ &\delta^{4} (p_{1} + p_{2} + \cdots + p_{n}) \, \Pi_{n} (p_{1}, \cdots, p_{n}), (2) \end{split}$$

where  $T_q$  is time-order product of constituent quark fields,  $\Pi_n\left(p_1,\cdots,p_n\right)$  is one-loop effects of constituent quarks with n external fields,  $p_1,p_2,\cdots,p_n$  are their four-momentum. Getting rid of all disconnected diagrams, we have

$$S_{\text{eff}} = \sum_{n=1}^{\infty} S_n,$$

$$S_n = \int d^4 p_1 \frac{d^4 p_2}{(2\pi)^4} \cdots \frac{d^4 p_n}{(2\pi)^4} \delta^4 (p_1 + p_2 + \cdots + p_n) \prod_{n=1}^{\infty} (p_1, \cdots, p_n),$$
 (3)

where c denotes connected part. Obviously, in Eq. (3), the effective action  $S_{\rm eff}$  is expanded in number of external vertices and expressed as integral over external momenta. Hereafter we shall call this method proper vertex expansion, and call  $S_n$  n-point effective action. Using proper vertex expansion, we will obtain effective actions which include informations from all orders of chiral expansion. That is, we can do resummation of momentum expansion by this method. That is what we need.

It is instructive to compare these two kinds of calculations. The perturbation calculation can deal with all

kinds of meson interactions at low orders, while resummation calculation deals with specific process and includes informations of all orders. In the sense of *p*-expansion, resummation calculation is non-perturbative. As we can see later, this character will give important result.

Having showed outlines of resummation calculation, we turn to Li's model. Because we only focus on  $\rho$  physics, the  $N_f=2$  part of Li's model, i.e.,  $U(2)_L \times U(2)_R$  chiral theory of mesons suffices to study decay  $\rho \to \pi\pi$ . This Model is constructed by  $U(2)_L \times U(2)_R$  chiral symmetry and minimum coupling principle, and its ingredients are quarks (u and d), pseudoscalar mesons ( $\pi$  and  $\pi$  (u and d component)), vector mesons ( $\rho$  and  $\omega$ ), axial-vector mesons ( $\pi$  and  $\pi$  (1285)), lepton, photon and W bosons. For our purpose, we only write the relevant Lagrangian as

$$\mathcal{L} = \bar{q} \left( i \partial + V + A \gamma_5 - mu(x) \right) q + \frac{1}{4} m_0^2 \left( \langle V_\mu V^\mu \rangle + \langle A_\mu A^\mu \rangle \right), \tag{4}$$

where  $A_{\mu}=\tau_{i}a_{\mu}^{i}+f_{\mu}$ ,  $V_{\mu}=\tau_{i}\rho_{\mu}^{i}+\omega_{\mu}$ ,  $u=\mathrm{e}^{\mathrm{i}\Phi\gamma_{5}}$ ,  $\Phi=\tau_{i}\pi^{i}+\eta$ , and  $\langle\cdots\rangle$  denotes trace in flavor space. The quark part of this Lagrangian can be divided into two parts. The free-field part is  $\mathscr{L}_{0}^{q}=\bar{q}\left(\mathrm{i}\,\mathfrak{F}-m\right)q$ , and the interaction part is

$$\mathcal{L}_{1}^{q} = \bar{q} \left( \mathbf{V} + \mathbf{A} \gamma_{5} - i m \Phi \gamma_{5} + \frac{1}{2} m \Phi^{2} \right) q, \quad (5)$$

where terms involving more  $\Phi_s$  has been omitted, because they have nothing to do with decay  $\rho \rightarrow \pi\pi$ . All subsequent calculations are performed at chiral limit.

Before studying decay  $\rho \rightarrow \pi\pi$ , we have to do something to obtain physical fields. We should calculate effective actions for quadratic terms  $\langle VV \rangle$ ,  $\langle AA \rangle$ ,  $\langle \Phi\Phi \rangle$ ,  $\langle VA \rangle$ ,  $\langle V\Phi \rangle$  and  $\langle A\Phi \rangle$  through integration of two-point quark loops like Fig.1(a) (where C, D = V, A,  $\Phi$ ) and one-point quark loop Fig.1(b). These actions are calcu-

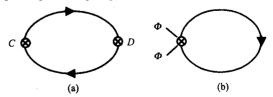


Fig. 1. Calculation of  $\langle CD \rangle$  terms  $(C, D = V, A, \Phi)$  from quark loops.

(a) Two-point diagram of quark loop; (b) One-point diagram of quark loop for  $\langle \Phi\Phi \rangle$  term.

lated as

$$S_{\text{eff}}^{q} = \frac{iN_{c}}{2} \int \frac{d^{4}p d^{4}k}{(2\pi 2\pi)^{4}} (\langle V_{\mu}(p) V_{\nu}(-p) \rangle \times$$

$$\text{Tr}[\gamma^{\mu} S_{F}(k-p) \gamma^{\nu} S_{F}(k)] + \langle A_{\mu}(p) A_{\nu}(-p) \rangle \times$$

$$\text{Tr}[\gamma^{\mu} \gamma_{5} S_{F}(k-p) \gamma^{\nu} \gamma_{5} S_{F}(k)] - \langle \Phi(p) \Phi(-p) \rangle \times$$

$$(m^{2} \text{Tr}[\gamma_{5} S_{F}(k-p) \gamma_{5} S_{F}(k)] + m \text{Tr}[S_{F}(k)]) -$$

$$2i m \langle V_{\mu}(p) \Phi(-p) \rangle \text{Tr}[\gamma^{\mu} S_{F}(k-p) \gamma_{5} S_{F}(k)] -$$

$$2i m \langle A_{\mu}(p) \Phi(-p) \rangle \text{Tr}[\gamma^{\mu} \gamma_{5} S_{F}(k-p) \times$$

$$\gamma_{5} S_{F}(k)] + 2 \langle V_{\mu}(p) A_{\nu}(-p) \rangle \times$$

$$\text{Tr}[\gamma^{\mu} S_{F}(k-p) \gamma^{\nu} \gamma_{5} S_{F}(k)]),$$

$$(6)$$

where, for  $\langle \Phi\Phi \rangle$  term, we have added contribution from one-point diagram (Fig.1(b)) of quark loop. This is so because  $\Phi$  is realized nonlinearly. After integrating over momentum k of quark loops, we get expressions including all terms up to  $O(p^{\infty})$ . But we are not interested in high order terms now. Discarding them and adding the non-quark part of Eq. (4), we obtain

$$S_{\text{eff}} = \frac{1}{4} \int \frac{d^4 p}{(2\pi)^4} (\langle V_{\mu}(p) V_{\nu}(-p) \rangle (g^2 (p^{\mu}p^{\nu} - g^{\mu\nu}p^2) + m_0^2) + \langle A_{\mu}(p) A_{\nu}(-p) \rangle \times$$

$$\left[ -g^2 \left( 1 - \frac{N_c}{6\pi^2 g^2} \right) g^{\mu\nu}p^2 + g^2 p^{\mu}p^{\nu} + (6g^2 m^2 + m_0^2) g^{\mu\nu} \right] + \langle \Phi(p) \Phi(-p) \rangle \times$$

$$\frac{3}{2} g^2 m^2 p^2 + \langle A_{\mu}(p) \Phi(-p) \rangle 6i g^2 m^2 p^{\mu} ),$$

where the logarithmic divergence is absorbed by

$$\frac{3}{8}g^{2} = \frac{N_{c}}{(4\pi)^{d/2}} \left(\frac{\mu^{2}}{m^{2}}\right)^{\epsilon/2} \Gamma\left(2 - \frac{d}{2}\right), (d = 4 - \epsilon)$$
(8)

and the quadratic divergence in  $\langle \Phi\Phi \rangle$  term has been reduced to logarithmic divergence by the identity  $\Gamma(1-d/2)+\Gamma(2-d/2)=-1+O(\varepsilon)$ . The first and second terms of Eq. (7) indicate needs for rescalings of  $V_\mu$  and  $A_\mu$ . (We see that axial-vector  $A_\mu$  does not satisfy gauge invariance. Because  $\partial_\mu A^\mu=0$  when  $A_\mu$  is on-shell, we shall ignore the  $p^\mu p^\nu$  term when redefining  $A_\mu$ .) The non-vanishing  $\langle A\Phi \rangle$  term indicates that there is a mixing between  $A_\mu$  and  $\partial_\mu \Phi$ , which should be erased by shift of  $A_\mu$ . Therefore, the redefinitions of  $V_\mu$  and  $A_\mu$  are

$$V_{\mu} \rightarrow \frac{V_{\mu}}{g}, A_{\mu} \rightarrow \frac{A_{\mu}}{g\sqrt{1 - N_{c}/6\pi^{2}g^{2}}} - \frac{c}{g} \partial_{\mu}\Phi,$$

$$(9)$$

and we can obtain in passing the mass formula for them

$$m_{\rm V}^2 = \frac{m_0^2}{\sigma^2}, \left(1 - \frac{N_c}{6\pi^2\sigma^2}\right) m_{\rm A}^2 = 6m^2 + m_{\rm V}^2.$$
 (10)

After redefining  $A_{\mu}$  and cancelling the mixing, and then making the kinetic term of  $\Phi$  in standard form, we obtain

$$c = \frac{3gm^2}{6m^2 + m_V^2}, \frac{1}{4}c^2m_V^2 + \frac{3}{8}\left(1 - \frac{2c}{g}\right)^2g^2m^2 = \frac{F_{\phi}^2}{16},$$

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$$c = \frac{F_{\phi}^2}{2gm_V^2}, \ 6\left(1 - \frac{2c}{g}\right)g^2m^2 = F_{\phi}^2,$$
 (11)

and rescaling for  $\Phi \rightarrow 2\Phi/F_{\phi}$ . Eqs. (9)—(11) are exactly those in Ref. [3].

Now we can study decay  $\rho \rightarrow \pi\pi$ . For vertex  $\rho$ - $\pi\pi$ , we need to calculate two Feynman diagrams of quark loop (see Fig. 2). The relevant Lagrangians (after redefini-

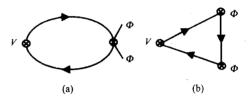


Fig. 2. Quark loops for vertex  $\rho \rightarrow \pi\pi$ .

(a) Two-point diagram; (b) Three-point diagram.

tions) for the first and second ones are

$$\mathcal{Z}_{1}^{(2)} = \bar{q} \left( \frac{1}{g} V + \frac{2m}{F_{\phi}^{2}} \Phi^{2} \right) q,$$

$$\mathcal{Z}_{1}^{(3)} = \bar{q} \left( \frac{1}{g} V - \frac{2c}{gF_{\phi}} \partial \Phi \gamma_{5} - \frac{2im}{F_{\phi}} \Phi \gamma_{5} \right) q, \quad (12)$$

respectively. For the first one,  $\mathscr{L}_{1}^{(2)}$  gives integration of the quark loop as  $\int \! \mathrm{d}^4 \, k {\rm Tr} [\, \gamma^\mu S_{\rm F} (\, k - p \,) \, S_{\rm F} (\, k \,) \,] \propto p^\mu \, (\, p \,)$  is momentum of vector field V). Because  $\partial_\mu V^\mu = 0$  when  $V_\mu$  is on-shell, contribution from the first diagram vanishes,  $S_2 = 0$ . For the second diagram, the Lagrangian  $\mathscr{L}_{1}^{(3)}$  determines that the effective action for this three-point diagram is calculated as

$$\begin{split} S_{3} &= \frac{4\mathrm{i}\,N_{e}}{g\,F_{\Phi}^{2}} \int \frac{\mathrm{d}^{4}\,p\,\mathrm{d}^{4}\,q_{1}\,\mathrm{d}^{4}\,q_{2}}{(2\pi2\pi)^{4}} \delta^{4} \left(p + q_{1} + q_{2}\right) \times \\ &\quad \left\langle V_{\mu}(p)\,\Phi(q_{1})\,\Phi(q_{2})\right\rangle \! \int \frac{\mathrm{d}^{4}\,k}{(2\pi)^{4}} \! \left(\frac{c^{2}}{g^{2}}q_{1\nu}q_{2\rho}\mathrm{Tr} \times \right. \\ &\left. \left[\,\gamma^{\mu}S_{F}(k + q_{1})\,\gamma^{\nu}\gamma_{5}\,S_{F}(k)\,\gamma^{\rho}\gamma_{5}^{*}\,S_{F}(k - q_{2})\,\right] + \\ &\left. m^{2}\mathrm{Tr} \! \left[\,\gamma^{\mu}S_{F}(k + q_{1})\,\gamma_{5}\,S_{F}(k)\,\gamma_{5}\,S_{F}(k - q_{2})\,\right] - \\ &\left. \frac{cm}{g}q_{1\nu}\mathrm{Tr} \! \left[\,\gamma^{\mu}S_{F}(k + q_{1})\,\gamma^{\nu}\gamma_{5}\,S_{F}(k)\,\gamma_{5}\,S_{F}(k - q_{2})\,\right] - \\ \end{split}$$

$$q_{2})] = \frac{cm}{g} g_{2} \operatorname{Tr} [ \gamma^{\mu} S_{F} (k + q_{1}) \times$$

$$\gamma_{5} S_{F} (k) \gamma^{\nu} \gamma_{5} S_{F} (k - q_{2}) ] ).$$
(13)

After integrations of quark loops and considerations of  $\partial_{\mu} V^{\mu} = 0$  and chiral limit  $q_1^2 = q_2^2 = m_{\pi}^2 = 0$ , we obtain

the effective action for vertex  $\rho$ - $\pi\pi$  as

$$S_{\rho\pi\pi} = \int \frac{d^4 p d^4 q}{(2\pi 2\pi)^4} \epsilon_{ijk} \rho_{\mu}^{i}(p) \pi^{j}(-p-q) \times \pi^{k}(q)(-iq^{\mu}) f_{\rho\pi\pi}(p^2), \qquad (14)$$

where

$$f_{\rho\kappa\pi}(p^{2}) = \frac{2}{3\pi^{2}g} \int_{\pi}^{2} \left[ m^{2} (18(1 - 2c/g)\pi^{2}g^{2} - (2 - 3cIg)N_{c}) - p^{2}(c/g)^{2} (6\pi^{2}g^{2} - N_{c}) \right] + \frac{2N_{c}}{\pi^{2}gf_{\pi}^{2}} \int_{0}^{1} x dx \int_{0}^{1} dy \left\{ \frac{m^{2}}{m^{2} - p^{2}x(1 - x)(1 - y)} \left[ m^{2} (1 - 2c/g + xy) + p^{2} (x(1 - x)(1 - y)) \times (1 + xy) - c/g(1 + 2x - 2x^{2} - 3xy + 2x^{2}y) + c^{2}/g^{2}(1 - xy)) \right] - \ln\left(1 - \frac{p^{2}}{m^{2}}x(1 - x)\right) \times (1 - y) \left( m^{2} (1 - 4c/g + 3xy) - c^{2}p^{2}/g^{2}(1 - xy)) \right\} = \frac{12(N_{c} + 3g^{2}\pi^{2}) - (24c/g)(2N_{c} + 3g^{2}\pi^{2}) + 40N_{c}(c/g)^{2}}{3\pi^{2}gf_{\pi}^{2}} m^{2} - \frac{c^{2}(10N_{c} + 36g^{2}\pi^{2})}{9\pi^{2}g^{3}f_{\pi}^{2}} p^{2} - \frac{4N_{c}(3 - 12c/g + 10(c/g)^{2})m^{2} - 4N_{c}(c/g)^{2}p^{2}}{3\pi^{2}gf_{\pi}^{2}} \sqrt{\frac{4m^{2} - p^{2}}{p^{2}}} \operatorname{arctg} \sqrt{\frac{p^{2}}{4m^{2} - p^{2}}}.$$
(15)

The factor  $f_{\rho\pi\pi}(p^2)$  is complicated. Its expansion in p up to  $O(p^6)$  (corresponding to expansion of  $S_{\rho\pi\pi}$  up to  $O(p^8)$ ) is

$$f_{\rho\pi\pi}(p^{2}) = \frac{2}{g} \left[ 1 + \frac{N_{c}(1 - 2c/g)^{2} - 12c^{2}\pi^{2}}{6\pi^{2}f_{\pi}^{2}} p^{2} + \frac{(1 - 4c/g)N_{c}}{60\pi^{2}m^{2}f_{\pi}^{2}} p^{4} + \frac{(1 - 4c/g + (c/g)^{2})N_{c}}{420\pi^{2}m^{4}f_{\pi}^{2}} p^{6} + O(p^{8}) \right],$$
(16)

where condition (11) has been used. As we can see, the sum of the first two terms in this series is just  $f_{\rho\pi\pi}$  in Ref.[3] when  $N_c=3$  and  $p^2=m_\rho^2$  is set.

Moreover, expression (15) shows that series (16) is in fact an expansion in dimensionless quantity  $\tilde{p}^2 \equiv p^2/(4m^2)$ :  $f_{\rho\pi\pi}(p^2) = 2/g + c_1\tilde{p}^2 + c_2\tilde{p}^4 + c_3\tilde{p}^6 + \cdots$ . Convergence of this series means  $\tilde{p}^2 < 1$  or  $p^2 < 4m^2$ , which at  $\rho$  energy scale means the constituent quark mass  $m > m_\rho/2 \approx 385 \, \text{MeV}$ . The same conclusion can also be drawn from unitarity consideration. Unitarity of the theory demands that, at leading order of  $1/N_c$  expansion, there should be no imaginary parts in transition amplitude of meson decay<sup>[17]</sup>. The expression (15) for  $f_{\rho\pi\pi}$  is just at the leading order of  $1/N_c$  expansion, therefore, it must be real. Thus  $p^2 < 4m^2$  must be ensured in Eq. (15),

which means  $m > m_{\rho}/2$  at  $\rho$  energy scale. We should point out that this conclusion is drawn from non-perturbation expression (15) which is characteristic of resummation study.

The difference between our result of m > 385 MeVand the one of m = 300 MeV in Ref. [3] is understandable. The parametrization m = 300 MeV in Ref. [3] is consistent with its  $O(p^4)$  calculation, and the phenomenology studies are good there. Now that we have resummed all terms in p-expansion, we naturally obtain that m > 385 MeV using convergence or unitarity analysis. In fact, such high constituent quark mass is also reasonable from another aspect. When fitting the hadron spectra, we have to adopt high constituent quark mass, or we fail to account for the observed masses of scalar nonet [18]. It is argued in Ref. [18] that high constituent quark mass might be a consequence of lack of confinement of the model and could be avoided if we knew how to add confining forces to the model. This is suggested by the results of constituent quark models which use confining interactions[19].

We like to mention that only the lower limit of constituent quark mass is determined in this paper. To determine further the value of constituent quark mass in this model needs more phenomenological considerations besides the unitarity condition indicated in this paper. We

also like to mention that the model studied here is a QCD-inspired chiral constituent quark model with vector mesons. How to deduce chiral constituent quark model rigorously from the fundamental QCD theory remains to be open. And we only consider the effects of constituent quark, which dominates.

To conclude, we improve perturbation calculation in Li's model by resummation study. We first illustrate proper vertex expansion method in chiral quark model,

and then use it to perform resummation study on decay  $\rho \to \pi\pi$ . We derive the complete and compact expression of  $f_{\rho\pi\pi}(p^2)$  (up to  $O(p^\infty)$ ), from which the explicit expression of effective action  $S_{\rho\pi\pi}$  up to  $O(p^8)$  is easily obtained. We can see that this method of resummation study is a powerful method to catch informations from all orders of p-expansion. Using this method, we draw a non-perturbative conclusion that  $m > m_o/2$ .

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## $U(2)_{L} \times U(2)_{R}$ 介子手征理论中对衰变 $\rho \rightarrow \pi\pi$ 的重求和研究\*

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摘要 把  $U(2)_L \times U(2)_R$  介子手征理论中对矢量介子的  $O(p^4)$  计算用重求和方法进行了改进,并重新研究了衰变  $\rho \to \pi\pi$ . 由此得到了  $f_{\rho\pi\pi}(p^2)$ 直到  $O(p^{\infty})$ 的完整表达式,并基于收敛性条件和么正性条件给出了一个重要的非微扰结论.

关键词 重求和方法 手征理论 非微扰计算

<sup>2003 - 07 - 01</sup> 收稿

<sup>\*</sup> 国家自然科学基金(90103002)资助

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