Calculation of the Giant Monopole Resonance Using Relativistic Mean Field Theory *

LI Yan-Song[1,2] F. H. AL-Khudair[1,2] LONG Gui-Lu[1,2,3,4]
1(Department of Physics, Tsinghua University, Beijing 100084, China)
2(Key Laboratory For Quantum Information and Measurements, Beijing 100084, China)
3(Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China)
4(Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator of Lanzhou, Lanzhou 730000, China)

Abstract Giant monopole resonances (GMR) are studied using a recently developed relativistic mean field theory computer code. We calculated the GMR energies of ^{16}O, ^{40}Ca and ^{208}Pb for both the isoscalar and isovector cases. Good agreement with experimental data is obtained.

Key words relativistic mean field, giant monopole resonance, RECAPS

1 Introduction

Recently we have written a new relativistic mean field computer code. It is used for the relativistic consistent angular-momentum projected shell-model (RECAPS). The RECAPS is a new nuclear model that combines the advantage of relativistic mean field theory and the projected shell model[1,2]. RECAPS is a self-consistent microscopic model and it is a new attempt for studying nuclear structure of normal nuclei and nuclei far from the stability. The basic ideas and some applications of RECAPS are given in Refs. [1, 2]. In the RECAPS, the relativistic mean field has to be performed first to determine the shape and single particle energy level for a given nucleus, and later a small model space is chosen near the Fermi surface to calculate the excited structure. The new relativistic mean field computer code uses spherical harmonic function for expansion, and the new code RECAPS-RMF is more convenient for angular-momentum projection. It has been shown that RECAPS computer code is reliable in calculating the binding energies and single particle levels[1]. In this paper, we shall calculate the giant monopole resonance of ^{16}O, ^{40}Ca and ^{208}Pb nuclei, using the RECAPS-RMF code. The results agree with experimental data well.

2 Relativistic mean field

We will just give the basic idea about the calculation of GMR energy. Because GMR reflects the incompressibility of nuclei, calculating GMR energy is an important way to test whether the model used can describe the incompressibility of nuclei well. It is well known that RMF is an excellent model, which can reproduce the incompressibility of nuclei.

The Lagrangin density of RMF having a system with nucleons, mesons and photons has the form[3]

$$\mathscr{L}=\bar{\psi}\left[\mathrm{i}\gamma^{\mu}\partial_{\mu}-m-g_{\sigma}\sigma-g_{\omega}\gamma^{\mu}\omega_{\mu}-g_{\rho}\gamma^{\mu}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_{\mu}-e\gamma^{\mu}\frac{1-\tau_3}{2}A_{\mu}\right]\psi+\frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma-\frac{1}{2}m_{\sigma}^{2}-\frac{1}{3}g_{2}\sigma^{3}-\frac{1}{4}g_{3}\sigma^{4}-\frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu}+\frac{1}{2}m_{\omega}^{2}\omega^{\mu}\omega_{\mu}-\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}^{\mu\nu}\cdot\boldsymbol{\rho}_{\mu\nu}+\frac{1}{2}m_{\rho}^{2}\boldsymbol{\rho}^{\mu}\cdot\boldsymbol{\rho}_{\mu}-\boldsymbol{\rho}^{\mu\nu}A^{\mu\nu}A_{\mu\nu}, \quad (1)$$

where the vectors are the isospin vectors and

$$\omega^{\mu\nu}=\partial^{\mu}\omega^{\nu}-\partial^{\nu}\omega^{\mu}, A^{\mu\nu}=\partial^{\mu}A^{\nu}-\partial^{\nu}A^{\mu},$$
$$\boldsymbol{\rho}^{\mu\nu}=\partial^{\mu}\boldsymbol{\rho}^{\nu}-\partial^{\nu}\boldsymbol{\rho}^{\mu}-2g_{\rho}\boldsymbol{\rho}^{\mu}\times\boldsymbol{\rho}^{\nu},$$
$$\tau_3|n\rangle=|n\rangle, \tau_3|p\rangle=|p\rangle.$$

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In the Lagrangian(1), nucleons (ψ) interact by the exchange of mesons: scalar mesons(σ) which produce a strong attractive force, isoscalar vector mesons (ω) which produce a repulsive force almost as strong as the attraction, isovector vector mesons(ρ) causes the difference of interaction between neutrons and protons. Photons (A) are also included and they produce the well-known electromagnetic interaction. From the Lagrangian (1), the equations of motion for the fields can be given through the variational principle. The equations are the Dirac equation for the nucleons

$$\Big[\mathrm{i}\gamma^{\mu}\partial_{\mu}-(m+g_{\sigma}\sigma)-g_{\omega}\gamma^{\mu}\omega_{\mu}-g_{\rho}\gamma^{\mu}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_{\mu}- e\frac{1-\tau_3}{2}\gamma^{\mu}A_{\mu}\Big]\psi=0 \quad (2)$$

and the Klein-Gordon equations for mesons

$$(\partial^{\mu}\partial_{\mu}+m_{\sigma}^2)\sigma=-g_{\sigma}\rho_s-g_2\sigma^2-g_3\sigma^3,\ \rho_s\equiv\bar{\psi}\psi, \quad (3)$$

$$\partial_{\mu}\omega^{\mu\nu}+m_{\omega}^2\omega^{\nu}=g_{\omega}j^{\nu},\ j^{\nu}\equiv\bar{\psi}\gamma^{\nu}\psi, \quad (4)$$

$$\partial_{\mu}\boldsymbol{\rho}^{\mu\nu}+m_{\rho}^2\boldsymbol{\rho}^{\nu}=g_{\rho}(\boldsymbol{j}^{\nu}+\boldsymbol{\rho}_{\mu}\times\boldsymbol{\rho}^{\mu\nu}),\ \boldsymbol{j}^{\nu}\equiv\bar{\psi}\boldsymbol{\tau}\gamma^{\nu}\psi, \quad (5)$$

$$\partial_{\mu}A^{\mu\nu}=ej_c^{\nu},\ j_c^{\nu}\equiv\bar{\psi}\gamma^{\nu}\frac{1-\tau_3}{2}\psi, \quad (6)$$

where ρ_s is the scalar density, j^{ν} is the mass current density, $\boldsymbol{j}^{\nu}$ is the isovector current density and j_c^{ν} is the electric current density. The Eqs. (2—6) can't be solved exactly, because they are complicated nonlinear equations about field4 operators. So some approximate treatments have to be adopted. The approximate treatments have been discussed in the Ref. [1], which include the no-sea approximation, mean-field approximation, neglecting the spacial components of vector fields and that only the 3rd-component of the isovector mesons is reserved. At last, we have the steady Dirac equation for nucleons as follows

$$[-\mathrm{i}\alpha\cdot\nabla+\beta m+V(\boldsymbol{r})]\varphi_k(\boldsymbol{r})=\varepsilon_k\varphi_k(\boldsymbol{r}), \quad (7)$$

where ε_k is the eigenvalue of single nucleon energy. And the motion equations of mesons become the forms

$$(-\nabla^2+m_{\sigma}^2)\sigma(\boldsymbol{r})=-g_{\sigma}\rho_s(\boldsymbol{r})-g_2\sigma^2(\boldsymbol{r})-g_3\sigma^3(\boldsymbol{r}), \quad (8)$$

$$(-\nabla^2+m_{\omega}^2)\omega(\boldsymbol{r})=-g_{\omega}(\rho_{\nu}^n(\boldsymbol{r})+\rho_{\nu}^p(\boldsymbol{r})), \quad (9)$$

$$(-\nabla^2+m_{\rho}^2)\rho(\boldsymbol{r})=-g_{\rho}(\rho_{\nu}^n(\boldsymbol{r})+\rho_{\nu}^p(\boldsymbol{r})), \quad (10)$$

$$-\nabla^2A(\boldsymbol{r})=e\rho_{\nu}^p(\boldsymbol{r}), \quad (11)$$

where $V(\boldsymbol{r})$, ρ_s, ρ_{ν}^n and ρ_{ν}^p are defined as follows

$$V(\boldsymbol{r})\equiv g_{\sigma}\sigma(\boldsymbol{r})+g_{\omega}\omega(\boldsymbol{r})+g_{\rho}\tau_3\rho(\boldsymbol{r})+e\frac{1-\tau_3}{2}A(\boldsymbol{r}), \quad (12)$$

$$\rho_s(\boldsymbol{r})\equiv\sum_i n_i\varphi_i(\boldsymbol{r})\beta\varphi_i(\boldsymbol{r}), \quad (13)$$

$$\rho_{\nu}^n(\boldsymbol{r})\equiv\sum_n n_n\varphi_n(\boldsymbol{r})\varphi_n(\boldsymbol{r}),\ \text{(summation for neutrons)}, \quad (14)$$

$$\rho_{\nu}^p(\boldsymbol{r})\equiv\sum_p n_p\varphi_p(\boldsymbol{r})\varphi_p(\boldsymbol{r}),\ \text{(sumation for protons)}, \quad (15)$$

Here n_i is the occupation number of the corresponding orbit.

3 Calculating GMR energies

The detail of solving the Eqs. (7—11) is shown in the Ref. [1]. Because RMF can reproduce the incompressibility of nuclei, it is supposed that RMF can give proper GMR energies. The Refs. [4,5] calculate the GMR energies by the generated coordinate method based on RMF. The treatments of Refs. [4,5] are calculations of complete quantum mechanics, whose results show that RMF can describe GMR of spherical nuclei well. In our calculations, we take the generated coordinate method, too. Indeed we take a semiclassical way to calculate GMR energies of spherical nuclei.

Firstly, it is obvious that the Eq. (7) is a steady Dirac equation for single particles. We can rewrite the equation as follows

$$\hat{h}\varphi_k=\varepsilon_k\varphi_k, \quad (16)$$

where $\hat{h}=-\mathrm{i}\alpha\cdot\nabla+\beta m+V(\boldsymbol{r})$ is single particle Harmiltonian. Then we can change the equation above to the form as follows

$$(\hat{h}-qr^2)\varphi_k=\varepsilon_k'\varphi_k, \quad (17)$$

where q is just the generated coordinate. Also we have the formula for the RMF energy (also the binding energy)

$$E_{\mathrm{RMF}}=E_{\mathrm{part}}+E_{\mathrm{pair}}+E_{\sigma}+E_{\omega}+E_{\rho}+E_c+E_{\mathrm{CM}}-AM, \quad (18)$$

where E_{part} is the energy of the nucleon, E_{pair} is the pairing energy and $E_{\mathrm{CM}}=-\frac{3}{4}41A^{-1/3}$ is the correction caused by motion of the mass center. The other terms in (18) are energies of the massive mesons and photons

$$E_{\sigma}=E_{\sigma L}+E_{\sigma NL}, \quad (19)$$

$$E_{\sigma L}=-\frac{g_{\sigma}}{2}\int\mathrm{d}^3\boldsymbol{r}\rho_s(\boldsymbol{r})\sigma(\boldsymbol{r}), \quad (20)$$

$$E_{\sigma NL}=-\frac{1}{2}\int\mathrm{d}^3\boldsymbol{r}\Big\{\frac{1}{3}g_2\sigma(\boldsymbol{r})^3+\frac{1}{2}g3\sigma(\boldsymbol{r})^4\Big\}, \quad (21)$$

$$E_{\omega}=-\frac{g_{\omega}}{2}\int\mathrm{d}^3\boldsymbol{r}\rho_{\nu}(\boldsymbol{r})\omega(\boldsymbol{r}), \quad (22)$$

$$E_{\rho}=-\frac{g_{\rho}}{2}\int\mathrm{d}^3\boldsymbol{r}[\rho_{\nu}^n(\boldsymbol{r})-\rho_{\nu}^p(\boldsymbol{r})]\rho(\boldsymbol{r}), \quad (23)$$

$$E_c=-\frac{e}{2}\int\mathrm{d}^3\boldsymbol{r}\rho_{\nu}^p(\boldsymbol{r})A(\boldsymbol{r}), \quad (24)$$

Because the steady Dirac equation is changed, the RMF energy E_{RMF} will be different. With the changing of q, E_{RMF} is changing, which is $E_{RMF} = E_{RMF}(q)$. Also the mean square root radius R is changing, which is $R = R(q)$. In another word, with the changing of R, E_{RMF} is changing, which is $E_{RMF} = E_{RMF}(R)$. $E_{RMF} = E_{RMF}(R)$ approximately gives a parabolic curve nearby $R = R_0$, where $R_0 = R(q)|_{q=0}$. So it can be approxiamately regarded as the potential of the one dimension harmonic oscillator $V(x) = \frac{1}{2}x^2$, where

$$K = \frac{d^2 E_{RMF}(q)}{dR^2}\Big|_{q=0}. \quad (25)$$

It is well known that the quantized energy of the one dimension harmonic oscillator is

$$\hbar\omega = \hbar\sqrt{\frac{K}{M}} = \sqrt{\frac{(\hbar c)^2 K}{M}},$$

where M is the mass of the harmonic oscillator and its unit is MeV here. In our case, the mass should be the total mass of the atomic nucleus, that is $M \cong Am$. So the energy of GMR can be estimated as

$$E_1 \cong \sqrt{\frac{(\hbar c)^2}{Am}\frac{d^2 E_{RMF}(q)}{dR^2}}\Big|_{q=0}. \quad (26)$$

In this case, because we have restricted neutrons and protons moving together, the Eq. (26) gives the energy of the isoscalar GMR. However, the real neutrons and protons can move contra, which makes the isovector GMR. In the new case, the model of the one dimension harmonic oscillator is also effective. Of course, to get the correct energy of the isovector GMR, we have to make some modifications. Firstly, the Eq. (17) should be rewritten as

$$(\hat{h} - q\tau_3 r^2)\varphi_k = \varepsilon_k{}'\varphi_k, \quad (27)$$

where an iso-operator τ_3 is added, which makes an opposite effect between neutrons and protons. Then the mean square root radius R should be replaced by $r = R_n - R_p$, where R_n and R_p are the mean square root radii of neutrons and protons respectively, because r describes the relative motion between neutrons and protons. For the same reason, the mass M should be replaced by the one of relative motion, too. This is just the effective mass μ. Because the neutrons' mass M_n is about Nm and the protons' mass M_p is about Pm, the effective mass will be

$$\mu = \frac{M_n M_p}{M_n + M_p} \cong \frac{NPm}{A}.$$

Hence, we can estimate the energy of isovector GMR as

$$E_1 \cong \sqrt{\frac{A(\hbar c)^2}{NPm}\frac{d^2 E_{RMF}(q)}{dr^2}}\Big|_{q=0}. \quad (28)$$

4 Results and discussions

With the Eqs. (26) and (28), we can calculate the GMR energies now. We calculate the GMR energies of ^{16}O, ^{40}Ca and ^{208}Pb in the two cases of isoscalar and isovector. To test the results, we also compare our calculations, the results of Ref. [5] and corresponding experiment data, which come from Ref. [5], too.

The Fig. 1—5 give the curves of RMF energy changing with the mean square root radii, from which the GMR energies can be got. The Table 1 gives the comparison among the calculations of Ref. [5], our results and the experiment data[5], in the case of the isoscalar GMR, where NL1, NL3, NL-SH and NL2 are the different parameter sets of RMF. The Table 2 gives the comparison in the case of the isovector GMR. It can be found that our calculations are coincident with the experiment data, also the results of Ref. [5] except the isovector GMR energy of ^{208}Pb. Our result is closer to experiments than the calculation of the reference. This is maybe caused by the difference of the selected potential. Our potential is just restricted nearby the point $q = 0$, while the calculations of complete quantum mechanics includes not only points near by $q = 0$ but also ones far from $q = 0$. The potential nearby $q = 0$

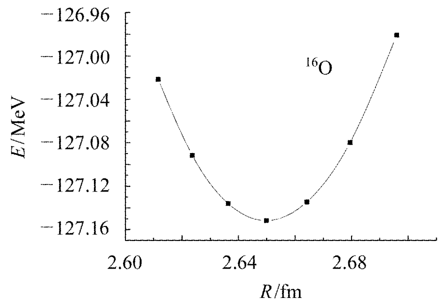

Fig. 1. ^{16}O's potential (Iso-scalar case).

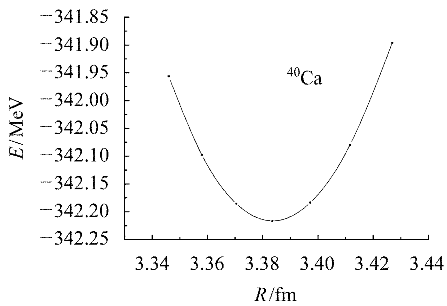

Fig. 2. ^{40}Ca's potential (Iso-scalar case).

Table 1.　Comparison of the isoscalar GMR energies(MeV).

	other's results[5]				our results	exp.[5]
	NL1	NL3	NL-SH	NL2		
^{16}O	20.2	22.6	25.0	27.1	20.86^{NL1}	
^{40}Ca	16.6	19.6	22.0	24.4	19.1^{NL1}	
^{208}Pb	11.0	13.0	15.0	16.0	14.03^{NL3}	13.7 ± 0.3

Table 2.　Comparison of the isoscalar GMR energies(MeV).

	other's results[5]				our results	exp.[5]
	NL1	NL3	NL-SH	NL2	NL3	
^{40}Ca	29.0	28.6	28.5	30.3	29.85	31.1 ± 2.2
^{208}Pb	16.5	18.0	18.4	16.9	25.53	26.0 ± 3.0

agrees with experiments well, but the potential from RMF calculations may deviate experiments when q is far from the point $q = 0$. That will make the difference between our calculations and the calculations of complete quantum mechanics.

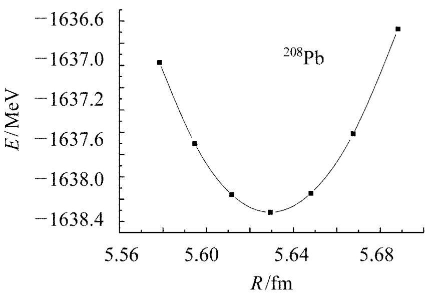

Fig.3.　^{208}Pb's potential(Iso-scalar case).

From the comparisons and discussions above, we see that the relativistic mean field theory can describe giant monopole resonances well. As the detailed computation is done using the newly written RECAPS-RMF computer code, it also lends a further support for the computer code.

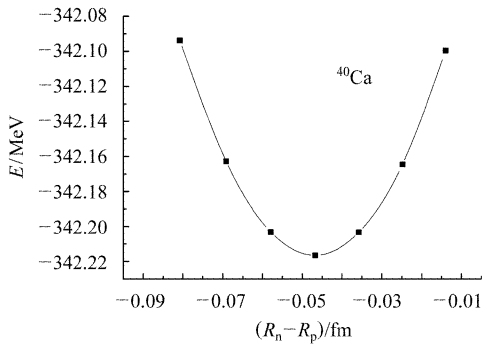

Fig.4.　^{40}Ca's potential(Iso-vector case).

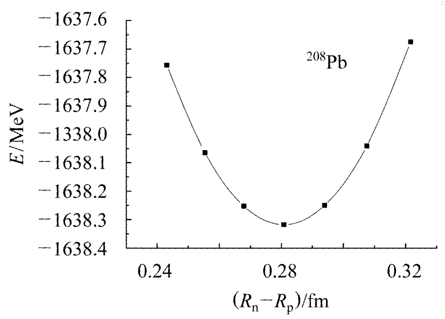

Fig.5.　^{208}Pb's potential(Iso-vector case).

References

1 LI Y S, LONG G L. Commun. Theor. Phys., 2004, **41**: 429

2 LI Y S, LONG G L. Commun. Theor. Phys., 2004, **41**: 579

3 Gambhir Y K, Ring P, Thimet A. Ann. Phys., 1990, **198**: 132

4 Ring P, Stoitsov M V, Sharma M M. Phys. Rev., 1994, **C50**: 1445

5 Behnsch R et al. Nucl. Phys., 1997, **A621**: 853

单极巨共振的相对论平均场计算*

李岩松[1,2] 法列[1,2] 龙桂鲁[1,2,3,4]

1(清华大学物理系 北京 100084)

2(量子信息与测量教育部重点实验室 北京 100084)

3(中国科学院理论物理研究所 北京 100080)

4(兰州重离子加速器国家实验室原子核理论中心 兰州 730000)

摘要 利用最近完成的相对论平均场理论的计算程序,研究了原子核单极巨共振.计算了^{16}O,^{40}Ca和^{208}Pb的同位旋标量和同位旋矢量单极巨共振.得到的结果与实验符合较好.

关键词 相对论平均场 单极巨共振 相对论自恰角动量投影壳模型

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