Final State Interactions in D→PP Decays*

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Abstract The two-body nonleptonic charmed meson decays into two pseudoscalar mesons are studied using one-particle-exchange method. The effects of the final state interactions are analyzed through the strong phases extracted from the experimental data.

Key words factorization, decays of charmed mesons, flavor symmetries

1 Introduction

The study of the two-body nonleptonic weak decays of particles containing heavy (c, b) quarks appears to offer a unique opportunity to determine the basic parameters of quark mixing, and to investigate the mechanism of CP violation. However, the quarks are not free, they are bound in hadrons by strong interactions which are described by nonperturbative QCD. Solving the problem of nonperturbative QCD needs efforts in both experiment and theory. In the near future BESIII and CLEO-c detectors will provide high precision data in charm physics including data on D meson decays, which will provide the possibility for understanding the physics in charm sector.

It is interesting to study the weak decays of charmed mesons beyond the factorization approach[1]. In general, if a process happens in an energy scale where there are many resonance states, this process must be seriously affected by these resonances[2]. This is a highly nonperturbative effect. Near the scale of D meson mass many resonance states exist. D meson decays must be affected seriously by these resonances. After weak decays the final state particles rescatter into other particle states through nonperturbative strong interactions[2,3]. Different D decay channels can contribute to one another through final state interactions (FSIs). One can model this rescattering effect as one-particle-exchange process[4,5], namely the final state particles be scattered into other particle states by exchanging one resonance state existing near the mass scale of the decaying meson, or alternatively a Regge trajectory[6]. There are also other ways to treat the nonperturbative and FSI effects in nonleptonic D decays. One approach is that in which the FSIs are expressed by the phase shifts of the decaying amplitudes[7]. The other method is a flavor topology approach[8,9], where the relative phases between various quark-diagram[10] amplitudes arise from the final state rescattering.

The final state rescattering effects for charmed meson decays into two pions have been studied using the one-particle-exchange method[11], where the magnitudes of hadronic cou-plings are extracted from experimental data on the measured branching fractions of resonance decays. In addition, a strong phase is introduced for the hadronic coupling which is important for obtaining the correct branching ratios in these decays. A similar analysis has been applied to D → PV decays[12], where P is a pseudoscalar meson, and V is a vector meson. The decay $D_s \to \phi\pi$ has been analyzed[13] beyond naive factorization.

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In the present work, we extend the study of the final state interactions in $D\to\pi\pi$ decays to $D\to PP$ decays. The coupling constants extracted from experimental data are small for s-channel contribution and large for t-channel contribution. Therefore the s-channel contribution is numerically negligible in $D\to PP$ decays. We safely drop the s-channel contribution in our discussion. In Sec. 2, we present the calculation within the naive factorization approach. The main scheme of one-particle-exchange method is described in Sec. 3. We give the numerical calculations and discussions in Sec. 4. The final section is reserved for summary.

2 Calculations in the factorization approach

The charmed meson decay can be described by the low energy effective Hamiltonian[14]

$$\mathscr{H} = \frac{G_F}{\sqrt{2}}\Big[\sum_{q=d,s} v_q(C_1 Q_1^q + C_2 Q_2^q)\Big], \tag{1}$$

where C_1 and C_2 are the Wilson coefficients at m_c scale, v_q is the product of Cabibbo-Kobayashi Maskawa (CKM) matrix elements and defined as

$$v_q = V_{uq} V_{cq}^*, \tag{2}$$

and the current-current operators are given by

$$\begin{aligned} Q_1^q &= (\bar{u}q)_{V-A}(\bar{q}c)_{V-A}, \\ Q_2^q &= (\bar{u}_\alpha q_\beta)_{V-A}(\bar{q}_\beta c_\alpha)_{V-A}. \end{aligned} \tag{3}$$

We do not consider the contributions of QCD and electroweak penguin operators in the decays of $D\to PP$ because their contributions are negligible in D decays. QCD factorization approach[15] is inapplicable to these decay modes, as the charmed meson is not heavy enough. The values of C_1 and C_2 at m_c scale are taken to be[14]

$$C_1 = 1.216, \quad C_2 = -0.415.$$

In the naive factorization approach, the decay amplitude can be generally factorized into a product of two current matrix elements and can be obtained from Eq. (1),

$$\begin{aligned}
A(D^+\to\pi^+\pi^0) &= -\frac{G_F}{2}V_{ud}V_{cd}^*(a_1+a_2)if_\pi(m_D^2-m_\pi^2)\cdot F^{D\pi}(m_\pi^2),\\
A(D^0\to\pi^+\pi^-) &= \frac{G_F}{\sqrt{2}}V_{ud}V_{cd}^*a_1 if_\pi(m_D^2-m_\pi^2)F^{D\pi}(m_\pi^2),\\
A(D^0\to\pi^0\pi^0) &= -\frac{G_F}{2}V_{ud}V_{cd}^*a_2 if_\pi(m_D^2-m_\pi^2)F^{D\pi}(m_\pi^2),\\
A(D^+\to\bar{K}^0\pi^+) &= \frac{G_F}{\sqrt{2}}V_{ud}V_{cs}^*[a_1 if_\pi(m_D^2-m_K^2)F^{DK}(m_\pi^2)+ a_2 if_K(m_D^2-m_\pi^2)F^{D\pi}(m_K^2)],\\
A(D^0\to K^-\pi^+) &= \frac{G_F}{\sqrt{2}}V_{ud}V_{cs}^*a_1 if_\pi(m_D^2-m_K^2)F^{DK}(m_\pi^2),\\
A(D^0\to\bar{K}^0\pi^0) &= \frac{G_F}{2}V_{ud}V_{cs}^*a_2 if_K(m_D^2-m_\pi^2)F^{D\pi}(m_K^2),\\
A(D^0\to K^+\pi^-) &= \frac{G_F}{\sqrt{2}}V_{us}V_{cd}^*a_1 if_K(m_D^2-m_\pi^2)F^{D\pi}(m_K^2),\\
A(D^+\to K^+\pi^0) &= -\frac{G_F}{2}V_{us}V_{cd}^*a_1 if_K(m_D^2-m_\pi^2)F^{D\pi}(m_K^2),\\
A(D^+\to K^0\pi^+) &= -\frac{G_F}{\sqrt{2}}V_{us}V_{cd}^*a_2 if_K(m_D^2-m_\pi^2)F^{D\pi}(m_K^2),\\
A(D^0\to K^0\pi^0) &= \frac{G_F}{2}V_{us}V_{cd}^*a_2 if_K(m_D^2-m_\pi^2)F^{D\pi}(m_K^2),\\
A(D^+\to K^+\bar{K}^0) &= \frac{G_F}{\sqrt{2}}V_{us}V_{cs}^*a_1 if_K(m_D^2-m_K^2)F^{DK}(m_K^2),\\
A(D^0\to K^+K^-) &= \frac{G_F}{\sqrt{2}}V_{us}V_{cs}^*a_1 if_K(m_D^2-m_K^2)F^{DK}(m_K^2),\\
A(D^0\to K^0\bar{K}^0) &= 0,
\end{aligned} \tag{4}$$

where the parameters a_1 and a_2 are defined as[9]

$$\begin{aligned} a_1 &= C_1 + C_2\left(\frac{1}{N_c}+\chi\right), \\ a_2 &= C_2 + C_1\left(\frac{1}{N_c}+\chi\right), \end{aligned} \tag{5}$$

with the color number $N_c = 3$, and χ is the phenomenological parameter which takes into account nonfactorizable correction. For q^2 dependence of the form factors, we take the BSW model[1], i.e., the monopole dominance assumption:

$$F(q^2) = \frac{F(0)}{1-q^2/m_*^2}, \tag{6}$$

where m_* is the relevant pole mass.

The decay width of a D meson at rest decaying into PP is

$$\Gamma(D\to PP) = \frac{1}{8\pi}|A(D\to PP)|^2\frac{|\boldsymbol{p}|}{m_D^2}, \tag{7}$$

where $|\boldsymbol{p}|$ is the 3-momentum of each final meson. The corresponding branching ratio is

$$Br(D\to PP) = \frac{\Gamma(D\to PP)}{\Gamma_{tot}}. \tag{8}$$

A comparison of the branching ratios of the naive factorization result with the experimental data is presented in Table 1. The second column gives the pure factorization result, where the nonfactorization effect is zero, while the third column represents the branching ratio with small nonfactorization correction. One can notice that the results are not in agreement with the experimental data. For doubly Cabibbo-suppressed decay modes, the experimental measurements of their decay rates are unavailable, except for the channel $D^0 \to K^+\pi^-$. We shall predict their branching ratios in section 4. The ratio for Cabibbo-suppressed decay mode $D^0 \to K^0\bar{K}^0$ vanishes in the naive factorization approach. This decay seems to be induced through final state rescattering.

Table 1. The branching ratios of D→PP obtained in the naive factorization approach and compared with the experimental results.

Decay mode	Br(Theory) $\chi=0$	Br(Theory) $\chi=-8.6\times10^{-2}$	Br(Experiment)
$D^+\to\pi^+\pi^0$	3.1×10^{-3}	2.71×10^{-3}	$(2.5\pm0.7)\times10^{-3}$
$D^0\to\pi^+\pi^-$	2.48×10^{-3}	2.65×10^{-3}	$(1.43\pm0.07)\times10^{-3}$
$D^0\to\pi^0\pi^0$	9.98×10^{-8}	1.39×10^{-5}	$(8.4\pm2.2)\times10^{-4}$
$D^+\to\bar{K}^0\pi^+$	1.20×10^{-1}	9.98×10^{-2}	$(2.77\pm0.18)\times10^{-2}$
$D^0\to K^-\pi^+$	4.81×10^{-2}	5.14×10^{-2}	$(3.80\pm0.09)\times10^{-2}$
$D^0\to K^0\pi^0$	2.93×10^{-6}	4.1×10^{-4}	$(2.28\pm0.22)\times10^{-2}$
$D^0\to K^+\pi^-$	1.88×10^{-4}	2.01×10^{-4}	$(1.48\pm0.21)\times10^{-4}$
$D^+\to K^+\pi^0$	2.4×10^{-4}	2.56×10^{-4}	-
$D^+\to K^0\pi^+$	3.86×10^{-8}	5.39×10^{-6}	-
$D^0\to K^0\pi^0$	7.58×10^{-9}	1.06×10^{-6}	-
$D^0\to K^+\bar{K}^0$	9.15×10^{-3}	9.76×10^{-3}	$(5.8\pm0.6)\times10^{-3}$
$D^0\to K^+K^-$	3.59×10^{-3}	3.83×10^{-3}	$(4.12\pm0.14)\times10^{-3}$
$D^0\to K^0K^0$	0	0	$(7.1\pm1.9)\times10^{-4}$

3 The one particle exchange method for FSI

As we have seen above, the experimental results for the branching ratios are mostly in disagreement with the calculation from the naive factorization approach. The reason is that the physical picture of naive factorization is too simple, nonperturbative strong interactions are restricted in a single hadron, or between the initial and final hadrons which share the same spectator quark. If the mass of the initial particle is large, such as the case of B meson decay, the effect of nonperturbative strong interactions between the final hadrons is most probably small because the momentum transfer is large. However, in the case of D meson, its mass is not so large. The energy scale of D decays is not very high . Nonperturbative effects may give large contributions. Because there exist many resonances near the mass scale of D meson, it is possible that nonperturbative interactions propagate through these resonance states, such as $K^*(892)$, $K^*(1430)$, $f_0(1710)$, $\rho(770)$, $\phi(1020)$ etc.

The diagrams of these nonperturbative rescattering effects can be depicted in Figs. 1 and 2. The first part $D\to P_1P_2$ or $D\to V_1V_2$ represents the direct decay where the decay amplitudes can be obtained by using naive factorization method. The second part repre-sents the rescattering process where the effective hadronic couplings are needed in numerical calculation, which can be extracted from experimental data on the relevant resonance decays.

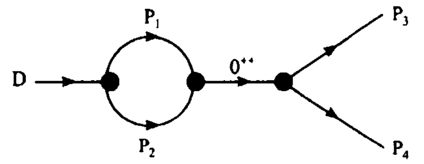
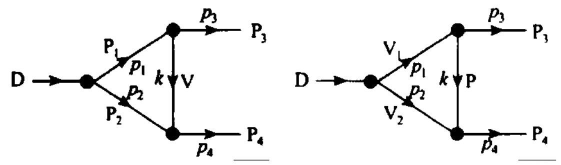


Fig. 1. s-channel contributions to final-state interactions in D→PP due to one particle exchange.



(a)　　(b)

Fig. 2. t-channel contributions to final-state interactions in D→PP due to one particle exchange. (a) Exchange a single vector meson; (b) Exchange a single pseudoscalar meson.

Fig. 1 is the s-channel contributions to the final state interactions. Here P_1 and P_2 are the intermediate pseudoscalar mesons. The resonance state has the quantum number $J^{PC}=0^{++}$ derived from the final state particles P_3 and P_4. From Particle Data Group[16], one can only choose $f_0(1710)$ as the resonance state to evaluate the s-channel contribution. However, the coupling of $f_0(1710)$ with two final mesons P_3 and P_4 is too small[11], we drop the s-channel contribution in the numerical calculation.

Fig. 2 shows the t-channel contribution to the final state interactions. P_1, P_2 and V_1, V_2 are the intermediate states. They rescatter into the final state P_3P_4 by exchanging one resonance state V or P. In this paper the intermediate states are treated to be on their mass shell, because their off-shell contribution can be attributed to the quark-level effects. We assume the on-shell contribution dominates in the final state interactions. The exchanged resonances are treated as virtual particles. Their propagators are taken as the Breit-Wigner form

$$\frac{i}{k^2 - m^2 + imΓ_{tot}}, \tag{9}$$

where $Γ_{tot}$ is the total decay width of the exchanged resonance. To the lowest order, the effective couplings of f_0 to PP and VV can be taken as the form

$$L_1 = g_{fPP}\phi^+\phi f, \tag{10}$$

$$L_1 = g_{fVV}A_\mu A^\mu f, \tag{11}$$

where ϕ is the pseudoscalar field, A_μ the vector field. Then the decay amplitudes of $f_0 \to$ PP and VV are

$$T_{fPP} = g_{fPP}, \tag{12}$$

$$T_{fVV} = g_{fVV}\epsilon_\mu\epsilon^\mu. \tag{13}$$

The coupling constants g_{fPP} and g_{fVV} can be extracted from the measured branching fractions of $f_0 \to$ PP and VV decays, respectively[16]. Because $f_0 \to$ VV decays have not been detected yet, we assume that their couplings are small. We do not consider the intermediate vector meson contributions of s-channel in this paper.

For the t-channel contribution, the concerned effective vertex is VPP, which can be related to the V decay amplitude. Explicitly the amplitude of V→ PP can be written as

$$T_{VPP} = g_{VPP}\epsilon \cdot (p_1 - p_2), \tag{14}$$

where p_1 and p_2 are the four-momentum of the two pseudoscalars, respectively. To extract g_{fPP} and g_{VPP} from experiments, one should square Eqs. (12) and (14) to get the decay widths

$$Γ(f \to PP) = \frac{1}{8\pi}|g_{fPP}|^2\frac{|\boldsymbol{p}|}{m_f^2},$$

$$Γ(V \to PP) = \frac{1}{3}\frac{1}{8\pi}|g_{VPP}|^2\left[m_V^2 - 2m_1^2 - 2m_2^2 + \frac{(m_1^2 - m_2^2)^2}{m_V^2}\right]\frac{|\boldsymbol{p}|}{m_V^2}, \tag{15}$$

where m_1 and m_2 are the masses of the two final particles PP, respectively, $|\boldsymbol{p}|$ is the momentum of one of the final particle P in the rest frame of V or f. From the above equations, one can see that only the magnitudes of the effective couplings $|g_{fPP}|$ and $|g_{VPP}|$ can be extracted from experiments. If there is any phase factor for the effective coupling, it would be dropped. Actually it is quite possible that there are imaginary phases for the effective couplings. As an example, let us see the effective coupling of $g_{K^*K\pi}$ shown in Fig.3, which is relevant to the process $K^* \to K_\pi$. On the quark level, the effective vertex can be depicted in Fig.4, which should be controlled by nonperturbative QCD. From this figure one can see that it is reasonable that a strong phase could appear in the effective coupling, which results from strong interactions. Therefore we can introduce a strong phase for each hadronic effective coupling. In the succeeding part of this paper, the symbol g will only be used to represent the magnitude of the relevant effective coupling. The total one should be $ge^{i\theta}$, where θ is the strong phase coming from Fig.4, For example, the effective couplings will be written in the form of $g_{fPP}e^{i\theta_{fPP}}$ and $g_{VPP}e^{i\theta_{VPP}}$.

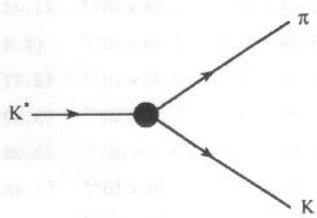


Fig.3. The effective coupling vertex on the hadronic level.

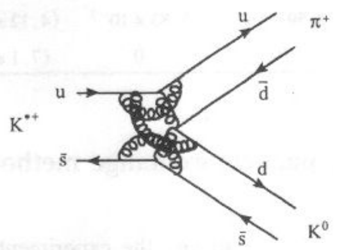


Fig.4. The effective coupling vertex on the quark level.

The decay amplitude of the s-channel final state interactions can be calculated from Fig.1

$$A_s^{FSI} = \frac{1}{2}\int\frac{d^3\boldsymbol{p}_1}{(2\pi)^3 2E_1}\int\frac{d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_D - p_1 - p_2)A(D \to P_1P_2)\frac{ig_1g_2e^{i(\theta_1+\theta_2)}}{k^2 - m^2 + imΓ_{tot}}, \tag{16}$$

where p_1 and p_2 represent the four-momenta of the pseudoscalar P_1 and P_2, the amplitude $A(D \to P_1P_2)$ is the direct decay amplitude. The effective coupling constants g_1 and g_2 should be g_{fPP} or g_{VPP} which can be obtained by comparing Eq.(15) with experimental data. By performing integrals, we obtain

$$A_s^{FSI} = \frac{1}{8\pi m_D}|\boldsymbol{p}_1| A(D \to P_1P_2)\frac{\mathrm{i}g_1g_2\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}}{k^2 - m^2 + \mathrm{i}m\Gamma_{tot}}. \tag{17}$$

The t-channel contribution via exchanging a vector meson (Fig.2(a)) is

$$A_{t,V}^{FSI} = \frac{1}{2}\int\frac{\mathrm{d}^3\boldsymbol{p}_1}{(2\pi)^3 2E_1}\int\frac{\mathrm{d}^3\boldsymbol{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_D - p_1 - p_2)\cdot A(D\to P_1P_2)g_1\epsilon_\lambda\cdot(p_1+p_3)\frac{\mathrm{i}\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}}{k^2 - m^2 + \mathrm{i}m\Gamma_{tot}}\cdot F(k^2)^2 g_2\epsilon_\lambda\cdot(p_2+p_4), \tag{18}$$

where $F(k^2) = (\Lambda^2 - m^2)/(\Lambda^2 - k^2)$ is the form factor which is introduced to compensate the off-shell effect of the exchanged particle at the vertices[17]. We choose the lightest resonance state as the exchanged particle that gives rise to the largest contribution to the decay amplitude.

We furthermore have

$$A_{t,V}^{FSI} = \int_{-1}^{1}\frac{\mathrm{d}(\cos\theta)}{16\pi m_D}|\boldsymbol{p}_1| A(D\to P_1P_2)\cdot g_1\frac{\mathrm{i}\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}}{k^2 - m^2 + \mathrm{i}m\Gamma_{tot}}F(k^2)^2 g_2 H, \tag{19}$$

where

$$H = -\Big[m_D^2 - \frac{1}{2}(m_1^2 \quad m_2^2 + m_3^2 + m_4^2) + (|\boldsymbol{p}_1||\boldsymbol{p}_4| + |\boldsymbol{p}_2||\boldsymbol{p}_3|)\cos\theta + E_1E_4 + E_2E_3\Big] - \frac{1}{m_V^2}(m_1^2 - m_3^2)(m_2^2 - m_4^2). \tag{20}$$

The t-channel contribution by exchanging a pseudoscalar meson (Fig.2(b)) is

$$A_{t,P}^{FSI} = \frac{1}{2}\int\frac{\mathrm{d}^3\boldsymbol{p}_1}{(2\pi)^3 2E_1}\int\frac{\mathrm{d}^3\boldsymbol{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_D - p_1 - p_2)\cdot\sum_{\lambda_1,\lambda_2}A(D\to V_1V_2)g_1\epsilon_{\lambda_1}\cdot(p_3 - k)\frac{\mathrm{i}\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}}{k^2 - m^2 + \mathrm{i}m\Gamma_{tot}}\cdot F(k^2)^2 g_2\epsilon_{\lambda_2}\cdot(k+p_4), \tag{21}$$

and we obtain

$$A_{t,P}^{FSI} = \int_{-1}^{1}\frac{\mathrm{d}(\cos\theta)}{16\pi m_D}|\boldsymbol{p}_1|\frac{\mathrm{i}\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}}{k^2 - m^2 + \mathrm{i}m\Gamma_{tot}}\cdot X g_1 g_2 F(k^2)^2(-H_1 + H_2), \tag{22}$$

where

$$H_1 = 4\mathrm{i}m_{V_1}f_{V_1}(m_D + m_2)A_1\Big[\frac{1}{2}(m_D^2 - m_3^2 - m_4^2) - \frac{1}{m_1^2}(E_1E_3 - |\boldsymbol{p}_1||\boldsymbol{p}_3|\cos\theta)(E_1E_4 + |\boldsymbol{p}_1||\boldsymbol{p}_4|\cos\theta) - \frac{1}{m_2^2}(E_2E_4\ |\boldsymbol{p}_2||\boldsymbol{p}_4|\cos\theta)(E_2E_3 + |\boldsymbol{p}_2||\boldsymbol{p}_3|\cos\theta) + \frac{1}{2m_1^2m_2^2}(m_D^2 - m_1^2 - m_2^2)(E_1E_3 - |\boldsymbol{p}_1||\boldsymbol{p}_3|\cos\theta)(E_2E_4 - |\boldsymbol{p}_2||\boldsymbol{p}_4|\cos\theta\Big], \tag{23}$$

$$H_2 = \frac{8\mathrm{i}m_{V_1}f_{V_1}}{(m_D + m_2)}A_2\Big[E_2E_3 + |\boldsymbol{p}_2||\boldsymbol{p}_3|\cos\theta - \frac{1}{2m_1^2}(m_D^2 - m_1^2 - m_2^2)(E_1E_3 - |\boldsymbol{p}_1||\boldsymbol{p}_3|\cos\theta\Big]\cdot\Big[E_1E_4 + |\boldsymbol{p}_1||\boldsymbol{p}_4|\cos\theta - \frac{1}{2m_2^2}(m_D^2 - m_1^2 - m_2^2)(E_2E_4 - |\boldsymbol{p}_2||\boldsymbol{p}_4|\cos\theta)\Big], \tag{24}$$

and X represents the relevant direct decay amplitude of D decaying to the intermediate vector pair V_1 and V_2 divided by $\langle V_1|(V-A)_\mu|0\rangle\langle V_2|(V-A)^\mu|D\rangle$,

$$X \equiv \frac{A(D\to V_1V_2)}{\langle V_1|(V-A)_\mu|0\rangle\langle V_2|(V-A)^\mu|D\rangle}.$$

4 Numerical calculation and discussion

In order to calculate the FSI contribution of D decays to $\pi\pi$, $K\pi$ and KK one needs to analyze which channel can rescatter into the final states. The rescattering processes are $D\to\pi\pi\to\pi\pi$, $D\to KK\to\pi\pi$, $D\to\rho\rho\to\pi\pi$, $D\to K^*K^*\to\pi\pi$ for $D\to\pi\pi$ decays; $D\to K\pi\to K\pi$, $D\to K^*\rho\to K\pi$ for $D\to K\pi$ channels; $D\to\pi\pi\to KK$, $D\to KK\to KK$, $D\to\pi\eta\to KK$, $D\to\rho\rho\to KK$, $D\to K^*K^*\to KK$, $D\to\rho\phi\to KK$ for $D\to KK$ decays and pictorially shown in Fig.5, Fig.6 and Fig.7. These rescattering processes give the largest contributions, because the intermediate states have the largest couplings with the final states and the masses of exchanged mesons are small, giving the largest t-channel contributions. When we calculate the contribution of each diagram in Figs. 5—7 via Eqs. (19) and (22), we should, at first, consider all the possible isospin structure for each diagram and draw all the possible sub-diagrams on the quark level. Secondly, we write down the isospin factor for each sub-diagram. For example, the $u\bar{u}$ component in one final meson π^0 has an isospin factor

$\frac{1}{\sqrt{2}}$, and the $d\bar{d}$ component has $-\frac{1}{\sqrt{2}}$. For the intermediate state π^0, the factors $\frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$ should be dropped, otherwise, the isospin relation between different channels would be violated[11]. Third, we sum the contributions of all the possible sub-diagrams on the quark level to get the isospin factor for each diagram on the hadronic level.

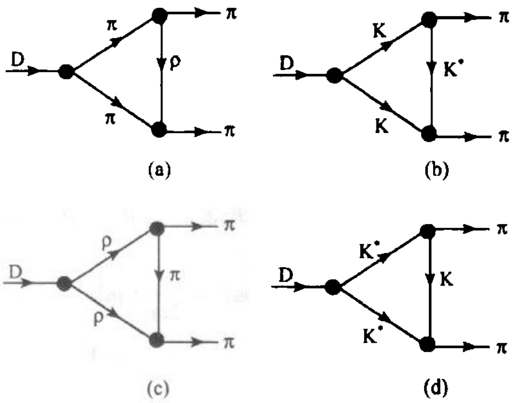
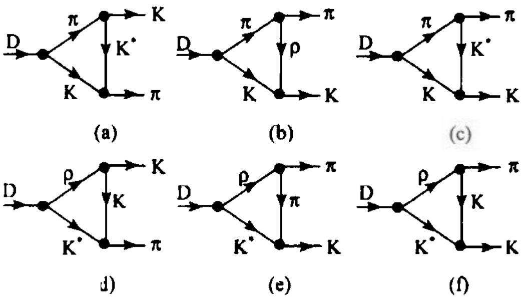


Fig.5. Intermediate states in rescattering process for $D\to\pi\pi$ decays.



Intermediate states in rescattering process for $D\to K\pi$ decays.

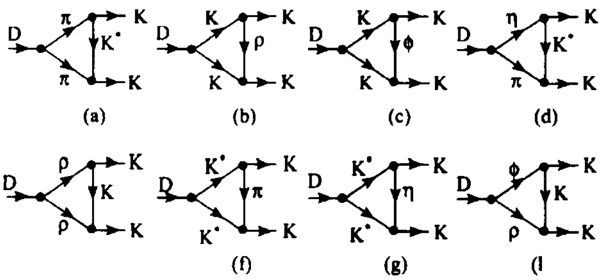


Fig.7. Intermediate states in rescattering process for $D\to KK$ decays.

The FSI contributions of the Cabibbo suppressed decays $D^+\to\pi^+\pi^0$ and $D^0\to\pi^0\pi^0$ depend on the couplings and phases $g_{K^*K\pi}e^{i\theta_{K^*K\pi}}$ and $g_{\rho\pi\pi}e^{i\theta_{\rho\pi\pi}}$ respectively, while $D^0\to\pi^+\pi^-$ depends on both of them. In the Cabibbo favored decays $D^+\to\bar{K}^0\pi^+$, $D^0\to K^-\pi^+$ $D^0\to\bar{K}^0\pi^0$ and the doubly Cabibbo suppressed decays $D^0\to K^+\pi^-$, $D^+\to K^+\pi^0$, $D^+\to K^0\pi^+$ and $D^0\to K^0\pi^0$, the branching ratios, including both the direct decay and the rescattering effect, depend not only on $g_{K^*K\pi}e^{i\theta_{K^*K\pi}}$ and $g_{\rho\pi\pi}e^{i\theta_{\rho\pi\pi}}$ but also on $g_{\rho KK}e^{i\theta_{\rho KK}}$. For the Cabibbo suppressed decay modes $D^+\to K^+\bar{K}^0$, $D^0\to K^+K^-$ and $D^0\to K^0\bar{K}^0$, the FSI effects come from $g_{K^*K\pi}e^{i\theta_{K^*K\pi}}$, $g_{\rho KK}e^{i\theta_{\rho KK}}$, $g_{\phi KK}e^{i\theta_{\phi KK}}$ and $g_{K^*K\eta}e^{i\theta_{K^*K\eta}}$.

In the numerical calculation, we use the input parameters: 1) the decay constants $f_\pi=0.133$GeV, $f_K=0.162$GeV, $f_\rho=0.2$GeV, $f_{K^*}=0.221$GeV, $f_\phi=0.233$GeV; 2) the form factors $F^{D\pi}(0)=0.692$, $F^{DK}(0)=0.762$, $A_1^{DK^*}(0)=0.880$, $A_2^{DK^*}(0)=1.147$, $A_1^{D\rho}(0)=0.775$, $A_2^{D\rho}(0)=0.923$[1]. Except for the decay constants, the values of the form factors have not been known exactly yet. We therefore have to take them from model-dependent calculations. The parameter Λ in the off-shellness compensating function $F(k^2)$ introduced in Eq. (18) takes the value of 0.513GeV, which is in the mass ranges of the final state mesons. In order to get the branching ratios which include both the direct decays and the rescattering effects, we use Eq. (15) and the central values of the measured decay width of $K^*\to K\pi$, $\rho\to\pi\pi$ and $\phi\to KK$[16] to obtain $g_{K^*K\pi}=4.59$, $g_{\rho\pi\pi}=6.0$ and $g_{\phi KK}=5.77$. We take $g_{\rho KK}=\sqrt{\lambda}\,g_{\rho\pi\pi}$ with the $s\bar{s}$ suppression parameter $\lambda=0.28$[18]. Since there is no data for $K^*\to K\eta$ (K^* is not heavy enough to decay into $K\eta$), we estimate the value of the strong coupling $g_{K^*K\eta}\approx3.5$ by comparing $g_{\rho\pi\pi}$ with $g_{K^*K\pi}$, and considering *SU*(3) flavor symmetry with 20%—30% violation. When the nonfactorizable parameter χ is not taken into account, we can not reproduce the experimental data for all the $D\to PP$ decays simultaneously. So we need to keep it as a phenomenological parameter. By taking $\chi=-8.6\times10^{-2}$, the experimental data of all the detected $D\to PP$ decays can be well accommodated within the experimental errors.

The strong phases of the effective hadronic couplings $\theta_{K^*K\pi}$, $\theta_{\rho\pi\pi}$, $\theta_{\rho KK}$, $\theta_{\phi KK}$, and $\theta_{K^*K\eta}$ can not be known from direct experimental measurements or from nonperturbative calculations, because there is no any such kind of computations yet. The only information is that the values of these phases should not differ too much, according to *SU*(3) flavor symmetry. We fit the experimental data to get the values for these phase parameters, and find that it is possible to reproduce the experimental data of these D decays with small

$SU(3)$ flavor symmetry violation. To show this situation, in which the experimental data are accommodated, Table 2 gives the numerical results of the branching ratios at $\theta_{K^*K\pi} = 53.9°$, $\theta_{\rho\pi\pi} = 57.3°$, $\theta_{\rho KK} = 71.8°$, $\theta_{\phi KK} = 58.7$ and $\theta_{K^*K\eta} = 65°$, with a small $SU(3)$ symmetry breaking effects. Column 'Factorization' is for the branching ratio predicted in the naive factorization approach, where the nonfactorizable correction is small. We find that the data of D→PP cannot be accommodated without including the contribution of the nonfactorizable effect. Column 'Factorization + FSI' is for the branching ratio of the naive factorization including the final state interaction. The contributions of final state rescattering effects are large, which can improve the predictions of naive factorization to be consistent with the experimental data. The strong phases introduced for the effective hadronic couplings $g_{K^*K\pi}$, $g_{\rho\pi\pi}$, $g_{\rho KK}$, $g_{\phi KK}$ and $g_{K^*K\eta}$ are important for explaining the experimental data, otherwise, it is quite difficult to get the correct results for these decay modes at the same time by varying other input parameters.

Table 2. The branching ratios of D→PP.

Decay mode	Factorization	Factorization + FSI	Experiment
$D^+ \to \pi^+\pi^0$	2.71×10^{-3}	1.8×10^{-3}	$(2.5 \pm 0.7) \times 10^{-3}$
$D^0 \to \pi^+\pi^-$	2.65×10^{-3}	1.49×10^{-3}	$(1.43 \pm 0.07) \times 10^{-3}$
$D^0 \to \pi^0\pi^0$	1.39×10^{-5}	1.06×10^{-3}	$(8.4 \pm 2.2) \times 10^{-4}$
$D^+ \to K^0\pi^+$	9.98×10^{-2}	2.95×10^{-2}	$(2.77 \pm 0.18) \times 10^{-2}$
$D^0 \to K^-\pi^+$	5.14×10^{-2}	3.72×10^{-2}	$(3.80 \pm 0.09) \times 10^{-2}$
$D^0 \to \bar{K}^0\pi^0$	4.1×10^{-4}	2.09×10^{-2}	$(2.28 \pm 0.22) \times 10^{-2}$
$D^0 \to K^+\pi^-$	2.01×10^{-4}	1.41×10^{-4}	$(1.48 \pm 0.21) \times 10^{-4}$
$D^+ \to K^+\pi^0$	2.56×10^{-4}	2.96×10^{-4}	-
$D^+ \to K^0\pi^+$	5.39×10^{-6}	7.56×10^{-4}	-
$D^0 \to K^0\pi^0$	1.06×10^{-6}	2.84×10^{-4}	-
$D^+ \to K^+\bar{K}^0$	9.76×10^{-3}	6.4×10^{-3}	$(5.8 \pm 0.6) \times 10^{-3}$
$D^0 \to K^+K^-$	3.83×10^{-3}	4.0×10^{-3}	$(4.12 \pm 0.14) \times 10^{-3}$
$D^0 \to K^0K^0$	0	5.73×10^{-4}	$(7.1 \pm 1.9) \times 10^{-4}$

As we have mentioned earlier, the branching ratios have not been detected in experiments for doubly Cabibbo-suppressed decay modes $D^+ \to K^+\pi^0$, $D^+ \to K^0\pi^+$ and $D^0 \to K^0\pi^0$, except $D^0 \to K^+\pi^-$. In our method, they are all predicted to be at order of $O(10^{-4})$.

To conclude this section, we shall give some comments. There are some free parameters, such as the D decay form factors which have not been well determined in experiments yet. They need to be measured from leptonic and semileptonic decays of D mesons, which are quite possible in the CLEO-c program in the near future. The other input parameters that may cause uncertainties are the shape of the off-shell compensating function $F(k^2)$ and the nonfactorizable parameter χ, which are needed to be studied by some nonperturbative methods based on QCD in the future. Certainly to completely understand final state interactions, more experimental data and more theoretical works are needed.

5 Summary

We have studied two-body nonleptonic charmed meson decays into two pseu-doscalar mesons. The total decay amplitude includes both direct weak decays and final state rescattering effects. The direct weak decays are calculated in the factorization approach, and the final state interaction effects are studied in the one-particle-exchange method. The prediction of the naive factorization is far away from the experimental data. After including the contribution of final state interactions, as well as the nonfactorizable corrections, the theoretical predictions can accommodate the experimental data within experimental errors, where the strong phases of the effective couplings are quite necessary to reproduce experimental data. The branching ratios are predicted for the three doubly Cabibbo-suppressed decay modes.

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D→PP 衰变过程的末态相互作用*

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摘要 利用单粒子交换方法,研究了D介子衰变到两个赝标粒子中的末态相互作用. 通过实验数据中抽取的强相角来分析末态相互作用的效应.

关键词 因子化 粲介子衰变 味道对称性

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