

# Longitudinal Focusing Design of Space-Charge Dominated Beams\*

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**Abstract** Longitudinal focusing is of great importance in space-charge-dominated bunched beams such as heavy ion fusion. The bunches lengthen rapidly due to the strong space-charge forces during transportation. The electron ring at University of Maryland (UMD) is designed to operate with bunches having rectangular and parabolic profiles. Three induction cavities along the ring will be employed for longitudinal focusing. In this paper, we first briefly review the longitudinal dynamics of parabolic and rectangular bunches. The design of the induction cavities with their high repetition rate modulators is then presented. The preliminary test results of lower voltage and repetition rate version of two different modulators and the PSPICE simulation results show that modulators work well.

**Key words** longitudinal focusing, space-charge-dominated bunched beams, induction cavities, ear-field

## 1 Introduction

The longitudinal focusing of space-charge dominated beam is of great interest in recirculator systems for applications in high energy physics, heavy ion inertial fusion, spallation neutron source and so on. Longitudinal expansion due to the strong space-charge force will result in rapid change in the beam profile. An initially rectangular profile of a drifting beam does not remain rectangular, thus, so-called "ear-field" must be applied to the edges of the beam to focus the expanded beam back to the rectangular shape. A linear force should also be applied to focus a parabolic beam. Three induction modules will be employed for longitudinal focusing in order to maintain a proper beam profile in the UMD electron ring<sup>[1]</sup>. The focusing voltages for beams with rectangular and parabolic profiles are given in section 2. The design of the induction cavity and its pulse modulators, as well as the test result are presented in this paper. More details on the development of high voltage, 5 MHz-repetition-rate modulator will be described elsewhere later.

## 2 Longitudinal expansion of bunched beams with rectangular and parabolic line charge density distributions

The particles in the leading edge of bunched beams will be accelerated, and particles in the trailing edge will be decelerated due to the space-charge force. The line charge density and the velocity distributions along the bunches will change with drifting distance  $S$ .

For an initial rectangular beam with a uniform velocity, the line charge density and velocity in the beam frame can be obtained from the solution of one-dimension cold-fluid equations<sup>[2-4]</sup> in the simple wave region as

$$\frac{\lambda}{\lambda_0} = \begin{cases} 1, & |t| < t_1 \\ \left[ \frac{2}{3} + \frac{S_{\text{comp}}}{3S} \left( 1 - \frac{2|t|}{\tau_0} \right) \right]^2, & t_1 \leq |t| \leq t_2 \\ 0, & |t| > t_2 \end{cases} \quad (1)$$

$$\frac{v}{C_s} = \begin{cases} -\frac{2}{3} \frac{|t|}{t} \left[ 1 - \frac{S_{\text{comp}}}{S} \left( 1 - \frac{2|t|}{\tau_0} \right) \right], & t_1 \leq |t| \leq t_2 \\ 0, & t_2 < |t| < t_1 \end{cases} \quad (2)$$

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$$t_1 = \left( \frac{1}{2} - \frac{S}{2S_{\text{cusp}}} \right) \tau_0, \quad t_2 = \left( \frac{1}{2} + \frac{S}{2S_{\text{cusp}}} \right) \tau_0,$$

$$S_{\text{cusp}} = v_0^2 \tau_0 / 2C_*, \quad C_* = \sqrt{\frac{egI_i}{4\pi\epsilon_0 mv_0 \gamma^5}}.$$

Here,  $\lambda$  is the line charge density,  $v$  the velocity in the beam frame,  $\epsilon_0$  the permittivity of free space,  $e/m$  the ratio of charge to mass of the particles,  $\gamma$  the Lorentz factor,  $g$  a geometry factor in the order of unity.  $I_i$  the initial beam current,  $v_0$  the beam velocity,  $\tau_0$  the pulse width of beam, and  $C_*$  the rarefaction wave velocity.

According to Eq.(1), a 10keV, 0.1A, 50ns rectangular beam in the UMD ring will reach the "cusp" point after about three turns<sup>[4]</sup>. In order to maintain a good beam profile during transportation, an external force must be applied to the beam before the "cusp" point.

For a parabolic bunch, the line charge density and velocity distributions can be expressed as follows<sup>[5]</sup> if  $z_m - z_i \ll z_i$ :

$$\lambda(z, S) = \frac{I_i z_i}{\beta c z_m} \left( 1 - \frac{z^2}{z_m^2} \right), \quad (3)$$

$$v \approx E_i \frac{4\alpha g S}{I_0 \beta^4 m v_0 c} t, \quad (4)$$

where

$$\alpha = \frac{4gI_i}{\beta_0^3 I_0 \gamma^5}, \quad z_m \approx \frac{gI_i}{I_0 \beta^3 z_i} \frac{C}{N_g} + z_i.$$

Here,  $z_i$  is the initial bunch length,  $E_i$  the beam energy,  $C$  the circumference of the UMD ring,  $N_g$  the number of the induction gaps along the ring, and  $I_0 = 1.7 \times 10^4$  A the characteristic current for electrons.

### 3 Design of longitudinal focusing in the UMD ring

#### 3.1 Voltages for longitudinal focusing in the UMD ring

Because the equations of motion are time reversible<sup>[3]</sup>, we could restore the beam profile by tilting the beam velocity distribution in the beam frame properly in a very short time. This can be realized by applying a voltage pulse from a small induction gap, during which the transition time of the particles could be negligible.

From Eq.(2), we can derive the voltage for restoring the rectangular bunch as

$$V = \begin{cases} \frac{4t}{3e} |mv_0 C_* \left[ 1 - \frac{S_{\text{cusp}}}{S} \left( 1 - \frac{2|t|}{\tau_0} \right) \right]|, & t_1 < |t| < t_2 \\ 0, & t_2 < |t| < t_1 \end{cases} \quad (5)$$

In the UMD ring design, three induction modules will be employed for longitudinal focusing for good beam profiles. In this case, the expansion distance  $S$  is 1.92m. For an initial rectangular beam of 10keV, 0.1A, 50ns, the voltage waveform for restoring the beam after the expansion distance  $S$  is plotted in Fig.1.

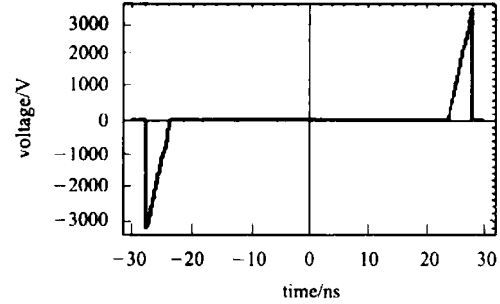


Fig.1. "Ear-field" for focusing rectangular bunch.

This is the so-called "ear-field". The particles in the leading edge will be decelerated by the negative "ear", while the particles in the trailing edge will be accelerated by the positive "ear". The beam profile will be restored after another distance of  $S$ . This process repeats periodically, so that the longitudinal beam envelope will oscillate along the ring. For a matched space-charge dominated beam bunch with a parabolic profile, we get the voltage for restoring the beam profile from the equation of the longitudinal beam envelope<sup>[6]</sup>,

$$V(t) = \frac{2cgI_i C}{I_0 N_g \gamma^2 (z_m/z_i)^2} \frac{mc^2}{q} t. \quad (6)$$

To focus the parabolic bunch, the voltage waveform is shown in Fig.2. Due to the short revolution time of the UMD ring, the repetition rate for both voltage pulses

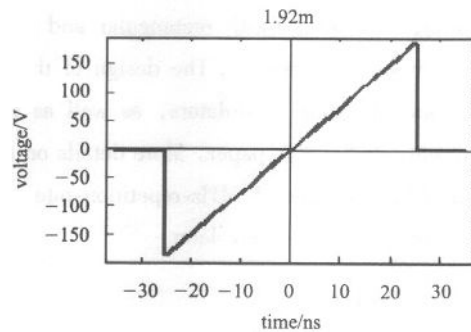


Fig.2. Voltage for focusing parabolic bunch.

should be 5.08MHz. This is very tough for the pulse switch technology, especially for the fast rise/fall time “ear-field”

### 3.2 Longitudinal matching

Longitudinal matching is also important in longitudinal focusing. In order to match the beams between the injection and the ring, the distance between the electron gun and the first focusing cavity should be 1.92m.

## 4 Design and development of compact induction module

The principle of induction cavities has been described in many places<sup>[7,8]</sup>. Here we just describe some special requirements on the induction cavity design for the UMD ring.

### 4.1 Special requirements on the cavity design

#### 4.1.1 High frequency response

Due to the high rep-rate and the fast rise/fall time of the ear field, most magnetic materials cannot be used. A special Nickel-Zinc ferrite toroid of material CMD5005, which can run in a frequency range up to more than 100MHz with low core loss and high permeability, has been chosen for the cavity design. The initial permeability of the ferrite is 1600 and the maximum flux density is 3200 Gauss. The dimensions of the ferrite toroids are 6” out-diameter, 2.1” inner-diameter and 1.3” thickness.

#### 4.1.2 High vacuum and bakable

The ring must be capable of vacuum pressure as low as  $10^{-10}$  Torr and must be bakable. To meet these requirements, a glass-metal seal has been designed for the induction gap, so the cavity is outside the vacuum. This not only avoids the possible problems which could occur if

the ferrite be put in vacuum, but also makes the special mechanical design possible which makes a cavity bakable by splitting the ferrite toroid and its housing (see Fig.3). The C-type toroids can be taken out before the system is baked.

#### 4.1.3 The transverse focusing due to the gap voltages

The induction gap voltage plays a role in transverse focusing<sup>[6]</sup>. The focusing length depends on the gap voltage<sup>[9]</sup>. In the case of the “ear-field”, the focus length varies from infinite to 25 m with the “ear” from 0V to  $V_{peak}$ . The minimum focus length is about  $1500m^{[10]}$  for parabolic bunch. So the effects on transverse focusing in both cases are negligible.

### 4.2 Modulator design for driving the cavity

A circuit that generates the “ear-field” has been designed for the rectangular bunch. A triangular pulses have been generated to focus the parabolic bunch. PSPICE simulations and tests of the circuits have been carried out. Fig.4 is the obtained “ear-field” waveform. The results show that the circuits work well. More details about developments of the high voltage, 5 MHz-rep-rate modulators will be described elsewhere.

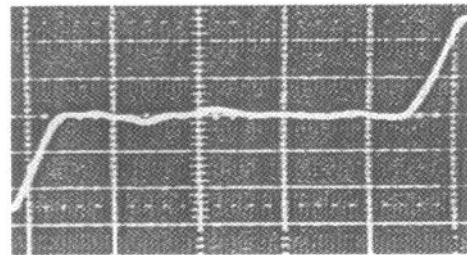


Fig.4. The experimental waveform of the 5MHz rep-rate “ear-field” pulses.  
Vert: 1.5kV/div; Hori: 10ns/div.

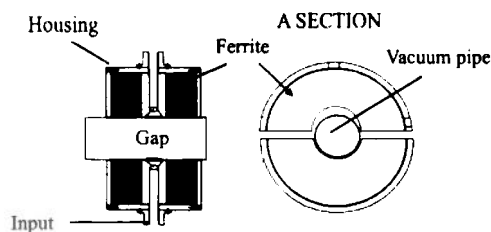


Fig.3. Mechanical design of the compact cavity.

## 5 Conclusions

The line charge density and velocity distributions of both rectangular and parabolic bunches after expansion have been given. The voltages for focusing the rectangular and parabolic bunches have also been derived individually. A compact, special designed bakable induction module for longitudinal focusing has been developed.

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## 空间电荷主导电子束的纵向聚焦设计

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**摘要** 在空间电荷主导电子束研究中,纵向聚焦非常重要,在输运过程中由于强空间电荷力的影响,束团将很快变长并崩溃,马里兰大学的电子环设计用于方形束团和抛物线形束团的研究,环中需要用三个感应腔对束团进行纵向聚焦.文中首先介绍了方形束团和抛物线形束团的纵向束流动力学,推导了两种束团纵向聚焦所需要的 5MHz 重复率高压脉冲波形,并给出了实现纵向聚焦的感应腔设计及其特殊要求.

**关键词** 纵向聚焦 空间电荷主导电子束 感应腔 耳朵场

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