Collective Motion in System with General Multipolar-Deformations *

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Abstract A general formalism is given for the collective motion in a system with general multipolar-deformations, which is treated as vibrations in body-fixed frame and rotation of whole system about the axes of Lab-system, as well as the coupling between vibrations and rotation. 18 various body-fixed frames are defined for octupole deformed system, which shows they can be put into 9 various classifies and the determinants of metric matrix in the body-fixed frames defined by the variables a_{30}, a_{31}, a_{32}, b_{31} and a_{30}, a_{31}, b_{31}, b_{32} are $9a_{32}^2$ and $9b_{32}^2$, which are the simplest.

Key words collective motion, body-fixed frame, multipolarities-deformations

1 Introduction

It is well known that, in spite of the fact that the atomic nuclei are not rigid bodies, the concept of an intrinsic or 'body-fixed' frame of reference is useful in the description of deformed nuclei. In the case of quadruple deformation the intrinsic frame of reference is linked to the principal axes of the nuclear surface. The quadruple shapes are fully described in terms of two independent intrinsic components of the corresponding quadruple tensor, which can be parameterized by means of the two Bohr's deformation parameters β and γ. The famous Bohr Hamiltonian had been derived with a simple quantized procedure[1]. When some other multipolarities are involved, one has additional deformation parameters, which are connected with the intrinsic components of the corresponding deformation tensors. The question arises of how to transform Bohr Hamiltonian in the laboratory system into body-fixed frame so as to derive the quantized Hamiltonian and study the collective motion of the system with multipole deformations. In recent years, the collective spectra of pure octupole deformed system have been paid to attention. The collective bands of octupole deformation have become one of the most heating frontier topic (Refs. [2,3] for several examples). The observation of the $\Delta I=4$

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staggering of superdeformation bands in some nuclei[4,5] has aroused great enthusiasm for study of hexadecapole deformation[6,7]. So, it is necessary to generalize the quadruple deformed theory to the general multipolarities deformed theories. In the following, we first give out a general formalism of the collective motion in system with general multipolar-deformations. Then, we apply the general formalism to octupole-deformed system and give out 18 various definitions of body-fixed frames so as to find the simplest body-fixed frame of octupole deformed system.

2 General Formalism of Collective Motion in System with General Multipolar-Deformations

For a system with general multipolar-deformations, its surface, in polar coordinates, can be expressed in terms of the nuclear radius as

$$R(\Omega) = R_0\left[1 + \sum_{lm}\alpha_{lm} Y_{lm}(\Omega)\right], \tag{1}$$

where R_0 is the radius of the nucleus in its spherical equilibrium shape and α_{lm} are the collective coordinates that describe deformations of the nuclear surface. We use for the Y_{lm} the spherical harmonics satisfying the Condon and Shortley convention, i. e., $Y_{l,-m} = (-)^m Y^*_{lm}$. As the radius has to be real, it follows that $\alpha_{l,-m} = (-)^m \alpha^*_{lm}$.

It is assumed that these variables change slowly with time, and therefore it is usual to express their kinetic energy as a quadratic function of the velocities as

$$T = \frac{1}{2}\sum_{lm} B_l \mid \dot{\alpha}_{lm} \mid^2, \tag{2}$$

where $\dot{\alpha}_{lm}$ is the time derivative of α_{lm}. It is well known that the coefficients α_{lm} written in lab-system and those written in the intrinsic system (that we will denote by β_{lm} to avoid confusions) are related by

$$\alpha_{lm} = \sum_{m'} D^l_{mm'}(\theta_i)\beta_{lm'}, \tag{3}$$

where $\theta_i \equiv (\theta_1, \theta_2, \theta_3)$ are three Euler angles. As usual the $D^l_{mm'}(\theta_i)$ functions, which are related to the matrix element of the rotation operator, are defined by

$$D^l_{mm'}(\theta_i) = \langle lm \mid e^{-i\theta_2 J_3} e^{-i\theta_1 J_2} e^{-i\theta_3 J_3} \mid lm' \rangle, \tag{4}$$

where J_1, J_2, J_3 are angular momentum along the coordinate axes in lab-system.

In order to evaluate the kinetic energy, the time derivative of α_{lm} must be written down explicitly:

$$\dot{\alpha}_{lm} = \sum_{m'}\left[D^l_{mm'}(\theta_i)\dot{\beta}_{lm'} + \dot{D}^l_{mm'}(\theta_i)\beta_{lm'}\right]. \tag{5}$$

The time derivative of the $D^l_{mm'}(\theta_i)$ functions can be expressed as [8]

$$\dot{D}^l_{mm'}(\theta_i) = -i\sum_k D^l_{mk}(\theta_i)\langle lk \mid \boldsymbol{\omega}\cdot\boldsymbol{J} \mid lm' \rangle, \tag{6}$$

where

$$\begin{aligned} \omega_1 &= \dot{\theta}_1 \sin\theta_3 - \dot{\theta}_2 \sin\theta_1 \cos\theta_3, \\ \omega_2 &= \dot{\theta}_1 \cos\theta_3 + \dot{\theta}_2 \sin\theta_1 \sin\theta_3, \\ \omega_3 &= \dot{\theta}_3 + \dot{\theta}_2 \cos\theta_1, \end{aligned} \tag{7}$$

are the angular velocities around the axes in the body-fixed frame. Now the kinetic energy can be expressed as

$$T = T_{\text{vib}} + T_{\text{rot}} + T_{\text{coupling}}, \tag{8}$$

where the vibrational energy is represented

$$= \frac{1}{2} \sum_{lm} B_l \mid \dot{\beta}_{lm} \mid^2 \tag{9}$$

while the rotational part is

$$= \frac{1}{2} \sum_{i,j} \omega_i \omega_j \mathcal{I}_{ij} . \tag{10}$$

with the inertia tensor defined as

$$\mathcal{I}_{ij} = \frac{1}{2} \sum_{lmm'} B_l \langle lm' \mid \{J_i, J_j\} \mid lm \rangle \beta_{lm} \beta^*_{lm'} . \tag{11}$$

The coupling between the internal and rotational degrees of freedom maintains a complicated structure, i.e.,

$$T_{\text{coupling}} = \sum_i \omega_i \kappa_i , \tag{12}$$

where the κ_i are given by

$$\kappa_i = -\mathrm{Im} \sum_{lmm'} B_l \langle lm' \mid J_i \mid lm \rangle \dot{\beta}_{lm} \beta^*_{lm'} . \tag{13}$$

With above formalism, the collective motion in system with general multipolar-deformations can be expressed into three parts: vibrations in body-fixed frame and rotation of whole system about the axes of lab-system, as well as the coupling between vibrations and rotation. Here, the internal variables β_{lm} are plural. For simplicity, a set of real variables a_{lm} and b_{lm} is used to cover for all the variables β_{lm} with the relations $\beta_{l0} = a_{l0}$, $\beta_{lm} = \frac{a_{lm} - \mathrm{i} b_{lm}}{\sqrt{2}}$. When we represent potential energy as $V = V(a_{lm}, b_{lm})$, the collective Hamiltonian of the multipolarly deformed system has been transformed into body-fixed frame. Formulae (8)—(13) have given a general formalism of describing collective motion of multipolarly deformed system. It can be used to describe not only pure single multipolarly deformed system, but also the multiple multipolarly deformed system which may involve both quadruple and octupole deformations and so on at the same time.

However, the a_{lm} and b_{lm} are not independent one other. Three of them have already been replaced by the three Euler angles (θ_i). Then, how to choose the deformation parameters is the key to define the most proper body-fixed frame. To a pure quadruple deformed system, when a_{20} and a_{22} are chosen to define the body-fixed frame of quadruple deformation, the T_{coupling} disappears. Bohr Hamiltonian can be obtained conveniently with a simple quantized procedure. To a pure octupole deformed system, there are 35 different ways to define the body-fixed frame. Which choice of parameter makes the definition of body-fixed frame is the most convenient? In order to answer this question; let's analyze the body-fixed frame in each choice of parameters.

3 Body-Fixed Frame of Octupole Deformed System

Considering the a_{30} is a symmetrical parameter of octupole deformation, it should be involved in body-fixed frame of octupole deformation. Then, there are 20 various ways for us to choose three parameters from the rest six parameters (a_{31}, a_{32}, a_{33}, b_{31}, b_{32}, b_{33}). In the following, we discuss every choice of parameters and definition of body-fixed frame.

(1) The a_{30}, a_{31}, a_{32}, b_{31} body-fixed frame

The kinetic energy of octupole deformed system

$$T_3 = \frac{1}{2} B_3 (\dot{a}_{30}^2 + \dot{a}_{31}^2 + \dot{a}_{32}^2 + \dot{b}_{31}^2) + B_3 \sum_i \omega_i \kappa_i + \frac{1}{2} B_3 \sum_{i,j} \omega_i \omega_j \mathcal{I}_{ij} , \tag{14}$$

here

$$\kappa_1 = \sqrt{6}(a_{30}\dot{b}_{31} - \dot{a}_{30} b_{31}) + \sqrt{\frac{5}{2}}(a_{32}\dot{b}_{31} - \dot{a}_{32} b_{31})$$

$$\kappa_2 = \sqrt{6}(a_{31}\dot{a}_{30} - \dot{a}_{31}a_{30}) + \sqrt{\frac{5}{2}}(a_{32}\dot{a}_{31} - \dot{a}_{32}a_{31}),$$
$$\kappa_3 = a_{31}\dot{b}_{31} - \dot{a}_{31}b_{31},$$
$$\mathcal{J}_{11} = 6a_{30}^2 + \frac{5}{2}a_{31}^2 + 2\sqrt{15}a_{30}a_{32} + 4a_{32}^2 + \frac{17}{2}b_{31}^2,$$
$$\mathcal{J}_{22} = 6a_{30}^2 + \frac{17}{2}a_{31}^2 - 2\sqrt{15}a_{30}a_{32} + 4a_{32}^2 + \frac{5}{2}b_{31}^2,$$
$$\mathcal{J}_{33} = a_{31}^2 + 4a_{32}^2 + b_{31}^2,$$
$$\mathcal{J}_{12} = -6a_{31}b_{31},$$
$$\mathcal{J}_{13} = \sqrt{6}a_{30}a_{31} + 3\sqrt{\frac{5}{2}}\, a_{32}a_{31},$$
$$\mathcal{J}_{23} = \sqrt{6}a_{30}b_{31} - 3\sqrt{\frac{5}{2}}\, a_{32}b_{31}.$$

As for the kinetic energy can be written as

$$T_3 = \frac{1}{2}B_3\sum_{i,j}G_{ij}(q)\dot{q}_i\dot{q}_j, \tag{16}$$

where $\dot{q}_i = (\dot{a}_{30}, \dot{a}_{31}, \dot{a}_{32}, \dot{b}_{31}, \omega_1, \omega_2, \omega_3)$, the metric matrix G_{ij} have been obtained as follows

$$G = \begin{pmatrix} I & K \\ \tilde{K} & J \end{pmatrix},$$

here, I is identity matrix of 4×4, $J = (\mathcal{J}_{ij})$, K equals to

$$K \begin{bmatrix} -\sqrt{6}b_{31} & \sqrt{6}a_{31} & 0 \\ 0 & -\sqrt{6}a_{30} + \sqrt{\frac{5}{2}}a_{32} & -b_{31} \\ -\sqrt{\frac{5}{2}}b_{31} & -\sqrt{\frac{5}{2}}a_{31} & 0 \\ \sqrt{6}a_{30} + \sqrt{\frac{5}{2}}a_{32} & 0 & a_{31} \end{bmatrix}$$

The determinant of metric matrix is calculated as

$$g = \det G = 9a_{32}^6. \tag{19}$$

With the same procedure as above, the kinetic energy of the collective motion, metric matrix and its determinant can also be obtained in the rest of body-fixed frames of octupole deformed system. All of determinants are listed in the following.

(2) The a_{30}, a_{31}, b_{31}, b_{32} body-fixed frame

The determinant of metric matrix

$$g = \det G = 9b_{32}^6. \tag{20}$$

(3) The a_{30}, a_{31}, a_{32}, b_{32} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{9}{4}\, a_{31}^2(a_{32}^2 + b_{32}^2)^2. \tag{21}$$

(4) The a_{30}, a_{32}, b_{31}, b_{32} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{9}{4}\, b_{31}^2(a_{32}^2 + b_{32}^2)^2. \tag{22}$$

(5) The a_{30}, a_{32}, a_{33}, b_{33} body-fixed frame

The determinant of metric matrix

$$g = \det G = (12a_{30}^2 - 5a_{32}^2)^2 a_{32}^2 . \tag{23}$$

(6) The a_{30}, a_{33}, b_{32}, b_{33} body-fixed frame

The determinant of metric matrix

$$g = \det G = (12a_{30}^2 - 5a_{32}^2)^2 b_{32}^2 . \tag{24}$$

(7) The a_{30}, a_{32}, a_{33}, b_{32} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{9}{4}(-12a_{30}^2 + 5a_{32}^2 + 5b_{32}^2)^2 a_{33}^2 . \tag{25}$$

(8) The a_{30}, a_{32}, b_{32}, b_{33} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{9}{4}(-12a_{30}^2 + 5a_{32}^2 + 5b_{32}^2)^2 b_{33}^2 . \tag{26}$$

(9) The a_{30}, a_{31}, a_{33}, b_{33} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{1}{4}(-5a_{31}^2 + 3a_{33}^2 + 3b_{33}^2)^2 a_{31}^2 . \tag{27}$$

(10) The a_{30}, a_{33}, b_{31}, b_{33} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{1}{4}(-5b_{31}^2 + 3a_{33}^2 + 3b_{33}^2)^2 b_{31}^2 . \tag{28}$$

(11) The a_{30}, a_{31}, a_{33}, b_{31} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{9}{4}(5a_{31}^2 - 3a_{33}^2 + 5b_{31}^2)^2 a_{33}^2 . \tag{29}$$

(12) The a_{30}, a_{31}, b_{31}, b_{33} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{9}{4}(5a_{31}^2 - 3b_{33}^2 + 5b_{31}^2)^2 b_{33}^2 . \tag{30}$$

(13) The a_{30}, a_{33}, b_{31}, b_{32} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{15}{4}(6a_{30}a_{33}b_{31} + 3a_{33}^2 b_{32} + b_{31}^2 b_{32} - 2b_{32}^3)^2 \tag{31}$$

(14) The a_{30}, a_{31}, b_{32}, b_{33} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{15}{4}(6a_{30}a_{31}b_{33} - 3b_{33}^2 b_{32} - a_{31}^2 b_{32} + 2b_{32}^3)^2 \tag{32}$$

(15) The a_{30}, a_{31}, a_{33}, b_{32} body-fixed frame

The determinant of metric matrix

$$g = \det G = 9a_{30}^2(15a_{31}^2 a_{33}^2 - 6\sqrt{15}a_{31}a_{33}^3 + 9a_{33}^4 + 4\sqrt{15}a_{31}a_{33}b_{32}^2 - 12a_{33}^2 b_{32}^2 + 4a_{32}^4). \tag{33}$$

(16) The a_{30}, b_{31}, b_{32}, b_{33} body-fixed frame

The determinant of metric matrix

$$g = \det G = 9a_{30}^2(15b_{31}^2 b_{33}^2 + 6\sqrt{15}b_{31}b_{33}^3 + 9b_{33}^4 - 4\sqrt{15}b_{31}b_{32}^2 b_{33} - 12b_{32}^2 b_{33}^2 + 4b_{32}^4). \tag{34}$$

(17) The a_{30}, a_{32}, a_{33}, b_{31} body-fixed frame

The determinant of metric matrix

$g = \det G =$

$$\frac{3}{4}(48a_{30}^2a_{32}^4 - 16\sqrt{15}a_{30}a_{32}^5 + 20a_{32}^6 - 144a_{30}^2a_{32}^2a_{33}^2 + 48\sqrt{15}a_{30}a_{32}^3a_{33}^2 - 60a_{32}^4a_{33}^2 + 108a_{30}^2a_{33}^4 - 36\sqrt{15}a_{30}a_{32}a_{33}^4 + 45a_{32}^2a_{33}^4 + 8\sqrt{15}a_{30}a_{32}^3b_{31}^2 - 20a_{32}^4b_{31}^2 - 12\sqrt{15}a_{30}a_{32}a_{32}^2b_{31}^2 + 30a_{32}^2a_{33}^2b_{31}^2 + 5a_{32}^2b_{31}^4) \quad . \tag{35}$$

(18) The a_{30}, a_{31}, a_{32}, b_{33} body-fixed frame

The determinant of metric matrix

$$g = \det G = \frac{3}{4}(48a_{30}^2a_{32}^4 + 16\sqrt{15}a_{30}a_{32}^5 + 20a_{32}^6 - 144a_{30}^2a_{32}^2b_{33}^2 - 48\sqrt{15}a_{30}a_{32}^3b_{33}^2 - 60a_{32}^4b_{33}^2 + 108a_{30}^2b_{33}^4 + 36\sqrt{15}a_{30}a_{32}b_{33}^4 + 45a_{32}^2b_{33}^4 - 8\sqrt{15}a_{30}a_{32}^3a_{31}^2 - 20a_{32}^4a_{31}^2 + 12\sqrt{15}a_{30}a_{32}a_{31}^2b_{33}^2 + 30a_{31}^2a_{32}^2b_{33}^2 + 5a_{32}^2a_{31}^4) \quad . \tag{36}$$

For the body-fixed frame defined by the variables a_{30}, a_{31}, a_{32}, a_{33} or a_{30}, a_{33}, b_{31}, b_{33}, the determinant of metric matrix in body-fixed frame is equal to naught. So these two definitions of the body-fixed frame are not proper. In Refs. [9—11], the body-fixed frame was defined by setting $b_{31} = b_{32} = b_{33} = 0$ and keeping a_{30}, a_{31}, a_{32} and a_{33} is not correct.

4 Conclusions

From the above discussions, we can see, with the general formalism expressed in formulae (8)—(13), the body-fixed frames of octupole deformed system can be defined conveniently. 18 various body-fixed frames of octupole deformed system can be parted into 9 various classes. The determinants of metric matrices in body-fixed frame defined by the variables a_{30}, a_{31}, a_{32}, b_{31} and a_{30}, a_{31}, b_{31}, b_{32} are the simplest, so this definition of the two intrinsic frames maybe the most convenient, where the inverse of metric matrix can be calculated easily, the quantized Hamiltonian can be derived out.

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多极形变系统的集体运动*

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摘要　给出了一个描写多极形变系统集体运动的普遍公式,在那里,动能被拆分成三部分:体坐标框架下的振动,体坐标系围绕实验室系的转动以及振动与转动的耦合.定义了18种不同的八极形变内禀系,按其度规矩阵的行列式分为9个不同的类.其中以参数$a_{30},a_{31},a_{32},b_{31}$和$a_{30},a_{31},b_{31},b_{32}$定义的八极形变内禀系,度规矩阵的行列式(分别为$9a_{32}^2$和$9b_{32}^2$)是最简单的.

关键词　集体运动　体坐标框架　多极形变

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