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动力学椭圆代数 $A_{q,p,\pi}(\widehat{gl_n})$ 的 Drinfeld 流

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摘要　从推广的 Yang-Baxter 关系"$RLL=LLR^*$"出发，利用高秩高斯分解，得到了动力学椭圆代数 $A_{q,p,\pi}(\widehat{gl_n})$ 及其对应的 Drinfeld 流. 其中 R, R^* 是 $A_{n-1}^{(1)}$ 面模型对应的谱参数有一关于代数中心平移的动力学 R 矩阵.

关键词　动力学椭圆代数 $A_{q,p,\pi}(\widehat{gl_n})$　高斯分解　Drinfeld 流

1　引言

量子场论和统计力学中，量子形式的基本泊松括号和 Yang-Baxter 关系在定义各种各样的量子代数中有十分重要的作用. 这样的量子代数和满足 Yang-Baxter 方程的 R 矩阵之间有密切的联系. Drinfeld 和 Jimbo[1,2] 发现了一类基本的量子代数 $U_q(g)$，其中 g 是任意维李代数. Faddeev, Reshetikhin 和 Takhtajan[3] 证明了在 g 是某些有限维李代数的情形下，代数 $U_q(g)$ 可以用 Yang-Baxter 关系来构造. 其中的 R 和谱参数无关. 后来 Reshetikhin 和 Semenov-Tian-Shansky[4] 构造了一类新的 q－破缺仿射代数. 他们采用的 R 是三角的 R 矩阵，并且其谱参数有一关于代数中心的平移. Ding 和 Frenkel[5] 证明了 Drinfeld 和 Jimbo 给出的代数与 Reshetikhin 和 Semenov-Tian-Shansky 给出的代数之间是一种同构关系.

Foda 等人[6] 在推广的 Yang-Baxter 关系"$RLL=LLR^*$"基础上提出量子代数 $A_{q,p}(\widehat{sl_2})$ 的一个椭圆扩张就是八顶角模型对应的代数. 这里的 R, R^* 是谱参数有一定平移的椭圆型八顶角 R 矩阵. 最近, Hou 等人[7] 把 Foda 等[6] 的工作拓宽到动力学的情况. 他们采用 $A_1^{(1)}$ 面型的动力学 R 矩阵，利用推广的 Yang-Baxter 关系，构造了椭圆代数 $A_{q,p,\pi}(\widehat{gl_2})$.

本文扩展 Hou 等人[7] 的工作到一般 n 的情况. 利用 Yang-Baxter 关系"$RLL=LLR^*$"构造出动力学椭圆代数 $A_{q,p,\pi}(\widehat{gl_n})$. 其中 R, R^* 是 $A_{n-1}^{(1)}$ 面模型的谱参

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数有一关于代数中心平移的动力学 R 矩阵. 然后对 L 算子进行高秩高斯分解，得到 $A_{q,p,\pi}(\widehat{gl_n})$ 对应的流代数.

2　动力学 $A_{n-1}^{(1)}$ 面模型

首先介绍指标. 设 ω 是虚部大于 0 的任意复数，r 是实数且 $r \geq n+2$，$x = e^{i\omega}$. $\epsilon_\mu(1 \leq \mu \leq n)$ 是 $\boldsymbol{R}^n$ 的一组正则基. 定义内积为：$<\epsilon_\mu, \epsilon_\nu> = \delta_{\mu\nu}$.

$A_{n-1}^{(1)}$ 面模型的权格子由 $\bar{\epsilon}_\mu$ 线性张成：

$$P = \sum_{\mu=1}^{n} \boldsymbol{Z}\bar{\epsilon}_\mu, \quad \bar{\epsilon}_\mu = \epsilon_\mu - \epsilon, \quad \epsilon = \frac{1}{n}\sum_{\mu=1}^{n}\epsilon_\mu.$$

定义椭圆函数

$$\theta\begin{bmatrix} a \\ b \end{bmatrix}(z,\tau) = \sum_{n \in Z}\exp\{i\pi[(m+a)^2\tau + 2(m+a)(z+b)]\},\ \mathrm{Im}(\tau) > 0,$$

$$\sigma_a = \sigma_{(a_1,a_2)} = \theta\begin{bmatrix} \frac{1}{2}+\frac{a_1}{2} \\ \frac{1}{2}+\frac{a_2}{2} \end{bmatrix}(z,\tau),\ \theta^{(k)}(z,\tau) = \theta\begin{bmatrix} -\frac{k}{2} \\ 0 \end{bmatrix}(z,2\tau).$$

采用以下缩写：

$$[v]_t = x^{\frac{v^2}{t}-v}\Theta_{x^{2t}}(x^{2v}) = \sigma_0\left(\frac{v}{t}, -\frac{1}{t\omega}\right)\times \text{const.}, t > 0,$$

$$\Theta_q(z) = (z;q)(qz^{-1};q)(q;q), (z;q_1,\cdots,q_m) = \prod_{i_1,\cdots,i_m=0}^{\infty}(1 - zq^{i_1}\cdots q^{i_m}).$$

定义 $A_{n-1}^{(1)}$ 面模型的动力学 R 矩阵[8]

$$R(v,\pi_{ii})_{ii}^{ii} = r_1(v) = x^{\frac{1-r}{r}v}\frac{g_1(v)}{g_1(-v)},\ i = 1,\cdots,n.$$

$$g_1(v) = \frac{\{x^{2+2v}\}\{x^{2r+2n+2v-2}\}}{\{x^{2r+2v}\}\{x^{2n+2v}\}},\ \{z\} = (z;x^{2r},x^{2n})_\infty,$$

$$\frac{R(v,\pi_{ij})_{ij}^{ij}}{R(v,\pi_{ii})_{ii}^{ii}} = \frac{[v]_r[\pi_{ij}-1]_r}{[v+1]_r[\pi_{ij}]_r},\ i < j,\quad j = 1,2,\cdots,n.$$

$$\frac{R(v,\pi_{ij})_{ij}^{ji}}{R(v,\pi_{ii})_{ii}^{ii}} = \frac{[v+\pi_{ij}]_r[1]_r}{[v+1]_r[\pi_{ij}]_r},\quad i < j.$$

$$\frac{R(v,\pi_{ij})_{ij}^{ij}}{R(v,\pi_{ii})_{ii}^{ii}} = \frac{[v]_r[\pi_{ij}+1]_r}{[v+1]_r[\pi_{ij}]_r},\quad i > j.$$

$$\frac{R(v,\pi_{ij})_{ij}^{ji}}{R(v,\pi_{ii})_{ii}^{ii}} = \frac{[v-\pi_{ij}]_r[1]_r}{[v+1]_r[-\pi_{ij}]_r},\quad i > j.$$

其中 π_{ij}，π_{ii} 是和面权有关的动力学变量. 并且 $\pi_\mu = \sqrt{r(r-1)}P_{\bar{\epsilon}_\mu}$，$\pi_{\mu\nu} = \pi_\mu - \pi_\nu$. $\pi_{\mu\nu}$ 作用到玻色 Fock 空间的真空态 $|l,k>$ 上为整数 $<\epsilon_\mu - \epsilon_\nu, rl-(r-1)k>$.

在上述 R 矩阵的定义中用到动力学变量的性质：$\pi_{ij}=-\pi_{ji}$. 所以 R 矩阵中实际上仅含有 n 个独立的动力学变量，而且矩阵元 $R^{ii}_{ii}(v,\pi_{ii})$ 不依赖动力学变量.

引入和 R 差一标量因子的 $R^{\pm}$ 矩阵

$$R^{\pm}(v,\pi)\equiv R^{\pm}(v,\pi,r)=\tau^{\pm}(v)R(v,\pi),\ \tau^{\pm}(v)=\tau(-v\pm\frac{1}{2}),$$

$$\tau(v)=x^{\frac{2(1-n)}{n}v}\frac{(x^{1+2v};x^{2n})(x^{2n-2v-1};x^{2n})}{(x^{2n+2v-1};x^{2n})(x^{1-2v};x^{2n})}.$$

动力学 $R^{\pm}(v,\pi)$ 矩阵满足动力学 Yang－Baxter 方程. 它除了幺正性和交叉幺正性[8]外，还具有以下的解析延拓性质：$R^{+}(v+r,\pi)=R^{-}(v,\pi)$.

3 动力学椭圆代数 $A_{q,p,\pi}(\widehat{gl_n})$

动力学 L 算子具有以下唯一的分解形式：

$$L^{\pm}(v,\pi)=\begin{bmatrix}1&0&\cdots&0&0\\e^{\pm}_{2,1}(v,\pi_{21})&1&\cdots&0&0\\\cdots&\cdots&\cdots&1&0\\e^{\pm}_{n,1}(v,\pi_{n1})&\cdots&\cdots&e^{\pm}_{n,n-1}(v,\pi_{nn-1})&1\end{bmatrix}\times\begin{bmatrix}k^{\pm}_1(v,\pi_{11})&\cdots&0\\0&\cdots&0\\\cdots&\cdots&0\\0&\cdots&k^{\pm}_n(v,\pi_{nn})\end{bmatrix}\times$$

$$\begin{bmatrix}1&f^{\pm}_{1,2}(v,\pi_{12})&\cdots&f^{\pm}_{1,n}(v,\pi_{1n})\\0&1&\cdots&\cdots\\0&\cdots&\cdots&f^{\pm}_{n-1,n}(v,\pi_{n-1n})\\0&\cdots&\cdots&1\end{bmatrix}.\qquad(1)$$

其中 $e^{\pm}_{ij}(v,\pi_{ij}),f^{\pm}_{ij}(v,\pi_{ij}),k^{\pm}_{ij}(v,\pi_{ij})$ 是动力学椭圆代数 $A_{q,p,\pi}(\widehat{gl_n})$ 的生成元. 它们满足以下的对易关系：

$$R^{\pm}(v_1-v_2,\pi)L^{\pm}_1(v_1,\pi)L^{\pm}_2(v_2,\pi)=L^{\pm}_2(v_2,\pi)L^{\pm}_1(v_1,\pi)R^{*\pm}(v_1-v_2,\pi),\qquad(2)$$

$$R^{+}(v_1-v_2+\frac{c}{2},\pi)L^{+}_1(v_1,\pi)L^{-}_2(v_2,\pi)=L^{-}_2(v_2,\pi)L^{+}_1(v_1,\pi)R^{*+}(v_1-v_2-\frac{c}{2},\pi),\qquad(3)$$

$$R^{-}(v_1-v_2-\frac{c}{2},\pi)L^{-}_1(v_1,\pi)L^{+}_2(v_2,\pi)=L^{+}_2(v_2,\pi)L^{-}_1(v_1,\pi)R^{*-}(v_1-v_2+\frac{c}{2},\pi),\qquad(4)$$

$$L^{-}(v,\pi)=L^{+}(v-\frac{c}{2}+r,\pi).\qquad(5)$$

其中 $L^{+}_1(v,\pi)=L^{\pm}(v,\pi)\otimes id,L^{\pm}_2(v,\pi)=id\otimes L^{\pm}(v,\pi),R^{*\pm}(v,\pi)=R^{\pm}(v,-\pi,r-c)$. c 是代数的中心.

L 算子和动力学变量的交换关系

$$\pi_{\mu\nu}L^{(\pm)\mu'}_{\nu'}(v,\pi)=L^{(\pm)\mu'}_{\nu'}(v,\pi)(\pi_{\mu\nu}+(c-r)\langle\epsilon_\mu-\epsilon_\nu,\bar{\epsilon}_{\mu'}\rangle+r\langle\epsilon_\mu-\epsilon_\nu,\bar{\epsilon}_{\nu'}\rangle).\qquad(6)$$

动力学变量在 $R^{\pm}$ 中的周期是 r，而在 $R^{*\pm}$ 中是 $r-c$，于是可得：

$$R^{\pm}(v,\pi_{\mu\nu})L^{(\pm)\mu'}_{\nu'}(v,\pi)=L^{(\pm)\mu'}_{\nu'}(v,\pi)R^{\pm}(v,\pi_{\mu\nu}+c\langle\epsilon_\mu-\epsilon_\nu,\bar{\epsilon}_{\mu'}\rangle),$$

$$R^{*\pm}(v,\pi_{\mu\nu})L^{(\pm)\mu'}_{\nu'}(v,\pi)=L^{(\pm)\mu'}_{\nu'}(v,\pi)R^{*\pm}(v,\pi_{\mu\nu}+c\langle\epsilon_\mu-\epsilon_\nu,\bar{\epsilon}_{\nu'}\rangle).$$

Drinfeld 全流 $E_i(v,\pi)$, $F_i(v,\pi)$ 可定义为

$$E_i(v) \equiv E_i(v,\pi_{i+1i}) = e^+_{i+1i}(v,\pi_{i+1i}) - e^-_{i+1i}(v+\frac{c}{2},\pi_{i+1i}),$$

$$F_i(v) \equiv F_i(v,\pi_{ii+1}) = f^+_{ii+1}(v+\frac{c}{2},\pi_{ii+1}) - f^-_{ii+1}(v,\pi_{ii+1}).$$

为方便起见，采用以下标记：

$$f^\pm_i(v) = f^\pm_{ii+1}(v,\pi_{ii+1}),\ e^\pm_i(v) = e^\pm_{i+1i}(v,\ \pi_{i+1i}),\ K^\pm_i(v) = k^\pm_i(v,\pi_{ii}),$$

$$R^\pm(v,\pi)^{kl}_{ij} \equiv R^\pm(v,\pi_{ij})^{kl}_{ij},\ R^{*\pm}(v,\pi)^{kl}_{ij} \equiv R^{*\pm}(v,\pi_{ij})^{kl}_{ij}.$$

4　动力学椭圆代数 $A_{q,p,\pi}(\widehat{gl_n})$ 的 Drinfeld 流

因为矩阵元 $R^\pm(v,\pi)^{ii}_{ii}$ 与 $R^{*\pm}(v,\pi)^{ii}_{ii}$ 和动力学参数无关，所以它们和 L 算子对易. 在 $n=2$ 情况下，全流 $E(v)$, $F(v)$, $K^\pm_i(v)(i=1,2)$ 满足以下对易关系

$$R^\pm(v_1-v_2)^{11}_{11}K^\pm_i(v_1)K^\pm_i(v_2) = K^\pm_i(v_2)K^\pm_i(v_1)R^{*\pm}(v_1-v_2)^{11}_{11}, \tag{7}$$

$$R^\pm(v_1-v_2)^{12}_{12}K^\pm_1(v_1)K^\pm_2(v_2) = K^\pm_2(v_2)K^\pm_1(v_1)R^{*\pm}(v_1-v_2)^{12}_{12}, \tag{8}$$

$$R^+\left(v_1-v_2+\frac{c}{2}\right)^{11}_{11}K^+_i(v_1)K^-_i(v_2) = K^-_i(v_2)K^+_i(v_1)R^{*+}\left(v_1-v_2-\frac{c}{2}\right)^{11}_{11}, \tag{9}$$

$$R^+\left(v_1-v_2+\frac{c}{2}\right)^{12}_{12}K^+_1(v_1)K^-_2(v_2) = K^-_2(v_2)K^+_1(v_1)R^{*+}\left(v_1-v_2-\frac{c}{2}\right)^{12}_{12}, \tag{10}$$

$$R^-\left(v_1-v_2-\frac{c}{2}\right)^{12}_{12}K^-_1(v_1)K^+_2(v_2) = K^+_2(v_2)K^-_1(v_1)R^{*-}\left(v_1-v_2-\frac{c}{2}\right)^{12}_{12}, \tag{11}$$

$$K^+_1(v_1)E(v_2)K^+_1(v_1)^{-1} = \frac{R^+(v_1-v_2)^{11}_{11}}{R^+(v_1-v_2)^{12}_{12}}E(v_2), \tag{12}$$

$$K^+_2(v_1)E(v_1)K^+_2(v_2)^{-1} = E(v_1)\frac{R^+(v_1-v_2)^{11}_{11}}{R^+(v_1-v_2)^{12}_{12}}, \tag{13}$$

$$K^-_1(v_1)E(v_2)K^-_1(v_1)^{-1} = \frac{R^+\left(v_1-v_2-\frac{c}{2}\right)^{11}_{11}}{R^+\left(v_1-v_2-\frac{c}{2}\right)^{12}_{12}}E(v_2), \tag{14}$$

$$K^-_2(v_2)E(v_1)K^-_2(v_2)^{-1} = E(v_1)\frac{R^+\left(v_1-v_2+\frac{c}{2}\right)^{11}_{11}}{R^+\left(v_1-v_2+\frac{c}{2}\right)^{12}_{12}}, \tag{15}$$

$$K^+_1(v_1)^{-1}F(v_2)K^+_1(v_1) = F(v_2)\frac{R^{*+}\left(v_1-v_2-\frac{c}{2}\right)^{11}_{11}}{R^{*+}\left(v_1-v_2-\frac{c}{2}\right)^{12}_{12}}, \tag{16}$$

$$K_2^+(v_2)^{-1}F(v_1)K_2^+(v_2)=\frac{R^{**}\left(v_1-v_2+\frac{c}{2}\right)_{11}^{11}}{R^{**}\left(v_1-v_2+\frac{c}{2}\right)_{12}^{12}}F(v_1),\tag{17}$$

$$K_1^-(v_1)^{-1}F(v_2)K_1^-(v_1)=F(v_2)\frac{R^{**}(v_1-v_2)_{11}^{11}}{R^{**}(v_1-v_2)_{12}^{12}},\tag{18}$$

$$K_2^-(v_2)^{-1}F(v_1)K_2^+(v_2)=\frac{R^{**}(v_1-v_2)_{11}^{11}}{R^{**}(v_1-v_2)_{12}^{12}}F(v_1),\tag{19}$$

$$E(v_1)\frac{R^{\pm}(v_1-v_2)_{11}^{11}}{R^{\pm}(v_1-v_2)_{12}^{12}}E(v_2)=E(v_2)\frac{R^{\pm}(v_2-v_1)_{11}^{11}}{R^{\pm}(v_2-v_1)_{12}^{12}}E(v_1),\tag{20}$$

$$F(v_1)\frac{R^{*\pm}(v_2-v_1)_{11}^{11}}{R^{*\pm}(v_2-v_1)_{12}^{12}}F(v_2)=F(v_2)\frac{R^{*\pm}(v_1-v_2)_{11}^{11}}{R^{*\pm}(v_1-v_2)_{12}^{12}}F(v_1),\tag{21}$$

$$[E(v_1),F(v_2)]=\{x-x^{-1}\}^{-1}\left\{\{\delta\left(v_2-v_1-\frac{c}{2}\right)K_2^-\left(v_1+\frac{c}{2}\right)\frac{[\pi]_{r-c}[1]_{r-c}}{\theta'\ [\pi-1]_{r-c}}\right.$$
$$\left.K_1^-\left(v_1+\frac{c}{2}\right)^{-1}-\delta\left(v_2-v_1+\frac{c}{2}\right)K_2^+(v_1)\frac{[\pi]_{r-c}[1]_{r-c}}{\theta'_{r-c}[\pi-1]_{r-c}}K_1^+(v_1)^{-1}\}\right\}.\tag{22}$$

其中：$K_i^-(v)=K_i^+\left(v-\frac{c}{2}+r\right)$，$\theta'_t=(x-x^{-1})\frac{\partial}{\partial v}[v]_t|_{v=0}$.

在 $n=3$ 的情况下，利用 Ding 和 Frenkel[5] 提供的方法，我们只需给出 $K_1^{\pm}(v)$，$f_1^{\pm}(v)$，$e_1^{\pm}(v)$和 $K_3^{\pm}(v)$，$f_2^{\pm}(v)$，$e_2^{\pm}(v)$之间的关系，其它的可通过直接观察或利用 $n=2$ 的结果得到．采用指标法，同时注意动力学变量和 L 算子间的对易关系(6)，对方程(2)—(4)中的 L 算子高斯分解，经过复杂的计算可得

$$R^{\pm}(v_1-v_2,\pi)_{13}^{13}K_1^{\pm}(v_1)K_3^{\pm}(v_2)=K_3^{\pm}(v_2)K_1^{\pm}(v_1)R^{*\pm}(v_1-v_2,\pi)_{13}^{13},\tag{23}$$
$$R^{*\pm}(v_1-v_2,\pi+c)_{13}^{13}f_1^{\pm}(v_1)K_3^{\pm}(v_2)=K_3^{\pm}(v_2)f_1^{\pm}(v_1)R^{*\pm}(v_1-v_2,\pi)_{23}^{13},\tag{24}$$
$$R^{\pm}(v_1-v_2,\pi)_{23}^{23}e_1^{\pm}(v_1)K_3^{\pm}(v_2)=K_3^{\pm}(v_2)e_1^{\pm}(v_1)R^{\pm}(v_1-v_2,\pi-c)_{13}^{13},\tag{25}$$
$$f_2^{\pm}(v_2)K_1^{\pm}(v_1)R^{*\pm}(v_1-v_2,\pi)_{13}^{13}=K_1^{\pm}(v_1)f_2^{\pm}(v_2)R^{*\pm}(v_1-v_2,\pi)_{12}^{12},\tag{26}$$
$$R^{\pm}(v_1-v_2,\pi)_{12}^{12}e_2^{\pm}(v_2)K_1^{\pm}(v_1)=R^{\pm}(v_1-v_2,\pi)_{13}^{13}K_1^{\pm}(v_1)e_2^{\pm}(v_2),\tag{27}$$
$$f_2^{\pm}(v_2)e_1^{\pm}(v_1)=e_1^{\pm}(v_1)f_2^{\pm}(v_2),\tag{28}$$
$$e_2^{\pm}(v_2)f_1^{\pm}(v_1)=f_1^{\pm}(v_1)e_2^{\pm}(v_2),\tag{29}$$

$$R^{\pm}\left(v_1-v_2\pm\frac{c}{2},\pi\right)_{13}^{13}K_1^{\pm}(v_1)K_3^{\mp}(v_2)=K_3^{\mp}(v_2)K_1^{\pm}(v_1)R^{*\pm}\left(v_1-v_2\mp\frac{c}{2},\pi\right)_{13}^{13},\tag{30}$$

$$R^{*\mp}\left(v_1-v_2\pm\frac{c}{2},\pi+c\right)_{13}^{13}f_1^{\mp}(v_1)K_3^{\pm}(v_2)=K_3^{\pm}(v_2)f_1^{\mp}(v_1)R^{*\mp}\left(v_1-v_2\pm\frac{c}{2},\pi\right)_{23}^{23},\tag{31}$$

$$R^{\mp}\left(v_1-v_2\mp\frac{c}{2},\pi\right)_{23}^{23}e_1^{\mp}(v_1)K_3^{\pm}(v_2)=K_3^{\pm}(v_2)e_1^{\mp}(v_1)R^{\mp}\left(v_1-v_2\mp\frac{c}{2},\pi-c\right)_{13}^{13},\tag{32}$$

$$f_2^{\pm}(v_2)K_1^{\mp}(v_1)R^{*\mp}\left(v_1-v_2\pm\frac{c}{2},\pi\right)_{13}^{13}=K_1^{\mp}(v_1)f_2^{\pm}(v_2)R^{*\mp}\left(v_1-v_2\pm\frac{c}{2},\pi\right)_{12}^{12},\tag{33}$$

$$R^{\mp}(v_1 - v_2 \mp \frac{c}{2}, \pi)_{12}^{12} e_2^{\pm}(v_2) K_1^{\mp}(v_1) = R^{\mp}(v_1 - v_2 \mp \frac{c}{2}, \pi)_{13}^{13} K_1^{\mp}(v_1) e_2^{\pm}(v_2), \tag{34}$$

$$f_2^{\pm}(v_2) e_1^{\mp}(v_1) = e_1^{\mp}(v_1) f_2^{\pm}(v_2), \tag{35}$$

$$e_2^{\pm}(v_2) f_1^{\mp}(v_1) = f_1^{\mp}(v_1) e_2^{\pm}(v_2), \tag{36}$$

$$E_2(v_2)E_1(v_1) \quad \frac{R^{+}(v_1 - v_2, \pi)_{23}^{23} R^{+}(v_1 - v_2, \pi + 2c)_{12}^{12}}{R^{+}(v_1 - v_2, \pi + c)_{13}^{13} R^{+}(v_1 - v_2, \pi)_{22}^{22}} E_1(v_1)E_2(v_2), \tag{37}$$

$$F_1(v_1)F_2(v_2) = \frac{R^{*+}(v_1 - v_2, \pi + c)_{23}^{23} R^{*+}(v_1 - v_2, \pi)_{12}^{12}}{R^{*+}(v_1 - v_2, \pi + 2c)_{13}^{13} R^{*+}(v_1 - v_2, \pi)_{22}^{22}} F_2(v_2)F_1(v_1), \tag{38}$$

$$[F_1(v_1), E_2(v_2)] = 0. \tag{39}$$

在一般 n 的情况下，仅需给出 $K_1^{\pm}(v)$, $f_1^{\pm}(v)$, $e_1^{\pm}(v)$和 $K_n^{\pm}(v)$, $f_n^{\pm}(v)$, $e_n^{\pm}(v)$之间的对易关系. 采用类似的方法，得到：

$$R^{\pm}(v_1 - v_2, \pi)_{1n}^{1n} K_1^{\pm}(v_1) K_n^{\pm}(v_2) = K_n^{\pm}(v_2) K_1^{\pm}(v_1) R^{*+}(v_1 - v_2, \pi)_{1n}^{1n}. \tag{40}$$

$$R^{\pm}(v_1 - v_2, \pi)_{1n-1}^{1n-1} e_{n-1}^{\pm}(v_1) K_1^{\pm}(v_1) = R^{\pm}(v_1 - v_2, \pi)_{1n}^{1n} K_1^{\pm}(v_1) e_{n-1}^{\pm}(v_2). \tag{41}$$

$$R^{*\pm}(v_1 - v_2, \pi + c)_{1n}^{1n} f_1^{\pm}(v_1) K_n^{\pm}(v_2) = K_n^{\pm}(v_2) f_1^{\pm}(v_1) R^{*\pm}(v_1 - v_2, \pi)_{2n}^{2n}. \tag{42}$$

$$f_{n-1}^{\pm}(v_2) K_1^{\pm}(v_1) R^{*\pm}(v_1 - v_2, \pi)_{1n}^{1n} = K_1^{\pm}(v_1) f_2^{\pm}(v_2) R^{*\pm}(v_1 - v_2, \pi)_{1n-1}^{1n-1}. \tag{43}$$

$$R^{\pm}(v_1 - v_2, \pi)_{2n}^{2n} e_1^{\pm}(v_1) K_n^{\pm}(v_2) = K_n^{\pm}(v_2) e_1^{\pm}(v_1) R^{\pm}(v_1 - v_2, \pi - c)_{1n}^{1n}. \tag{44}$$

$$f_{n-1}^{\pm}(v_2) e_1^{\pm}(v_1) = e_1^{\pm}(v_1) f_{n-1}^{\pm}(v_2), \tag{45}$$

$$e_{n-1}^{\pm}(v_2) f_1^{\pm}(v_1) = f_1^{\pm}(v_1) e_{n-1}^{\pm}(v_2). \tag{46}$$

$$R^{\pm}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{1n}^{1n} K_1^{\pm}(v_1) K_n^{\mp}(v_2) = K_n^{\mp}(v_2) K_1^{\pm}(v_1) R^{*+}(v_1 - v_2 \mp \frac{c}{2}, \pi)_{1n}^{1n}. \tag{47}$$

$$R^{*\pm}(v_1 - v_2 \mp \frac{c}{2}, \pi + c)_{1n}^{1n} f_1^{\pm}(v_1) K_n^{\mp}(v_2) = K_n^{\mp}(v_2) f_1^{\pm}(v_1) R^{*\pm}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{2n}^{2n}. \tag{48}$$

$$R^{\pm}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{2n}^{2n} e_1^{\pm}(v_1) K_n^{\mp}(v_2) = K_n^{\mp}(v_2) e_1^{\pm}(v_1) R^{\pm}(v_1 - v_2 \pm \frac{c}{2}, \pi - c)_{1n}^{1n}. \tag{49}$$

$$f_{n-1}^{\pm}(v_2) K_1^{\mp}(v_1) R^{*\mp}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{1n}^{1n} = K_1^{\mp}(v_1) f_{n-1}^{\pm}(v_2) R^{*\mp}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{1n-1}^{1n-1}. \tag{50}$$

$$R^{\pm}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{1n-1}^{1n-1} e_{n-1}^{\mp}(v_2) K_1^{\pm}(v_1) = R^{+}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{1n}^{1n} K_n^{\pm}(v_1) e_{n-1}^{\mp}(v_2). \tag{51}$$

$$f_{n-1}^{\pm}(v_2) e_1^{\mp}(v_1) = e_1^{\mp}(v_1) f_{n-1}^{\pm}(v_2), \tag{52}$$

$$e_{n-1}^{\pm}(v_2) f_1^{\mp}(v_1) = f_1^{\mp}(v_1) e_{n-1}^{\pm}(v_2). \tag{53}$$

$$\frac{R^{*\pm}(v_1 - v_2, \pi)_{1n-1}^{1n-1}}{R^{*\pm}(v_1 - v_2, \pi)_{1n}^{1n}} f_{n-1}^{\pm}(v_2) f_1^{\pm}(v_1) = f_1^{\pm}(v_2) f_{(n-1)}^{\pm}(v_2) \frac{R^{*\pm}(v_1 - v_2, \pi)_{2n-1}^{2n-1}}{R^{*\pm}(v_1 - v_2, \pi)_{2n}^{2n}}. \tag{54}$$

$$\frac{R^{\pm}(v_1 - v_2, \pi)_{2n-1}^{2n-1}}{R^{\pm}(v_1 - v_2, \pi)_{2n}^{2n}} e_{n-1}^{\pm}(v_2) e_1^{\pm}(v_1) = e_1^{\pm}(v_2) e_{n-1}^{\pm}(v_2) \frac{R^{\pm}(v_1 - v_2, \pi)_{1n-1}^{1n-1}}{R^{\pm}(v_1 - v_2, \pi)_{1n}^{1n}}. \tag{55}$$

$$\frac{R^{*\pm}(v_1 - v_2 \mp \frac{c}{2}, \pi)_{1n-1}^{1n-1}}{R^{*\pm}(v_1 - v_2 \mp \frac{c}{2}, \pi)_{1n}^{1n}} f_{n-1}^{\mp}(v_2) f_1^{\pm}(v_1) = f_1^{\pm}(v_1) f_{(n-1)}^{\mp}(v_2) \frac{R^{*\pm}(v_1 - v_2 \mp \frac{c}{2}, \pi)_{2n-1}^{2n-1}}{R^{*\pm}(v_1 - v_2 \mp \frac{c}{2}, \pi)_{2n}^{2n}}. \tag{56}$$

$$\frac{R^{\pm}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{2n-1}^{2n-1}}{R^{\pm}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{2n}^{2n}} e_{n-1}^{\mp}(v_2) e_1^{\pm}(v_1) = e_1^{\pm}(v_1) e_{(n-1)}^{\mp}(v_2) \frac{R^{\pm}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{1n-1}^{1n-1}}{R^{\pm}(v_1 - v_2 \pm \frac{c}{2}, \pi)_{1n}^{1n}}. \tag{57}$$

5 讨论

本文在动力学 Yang - Baxter 关系"$RLL = LLR^*$"的基础上,利用高秩高斯分解,得到了动力学椭圆代数 $A_{q,p,\pi}(\widehat{gl}_n)$的 Drinfeld 流,我们所采用的 R, R^* 是 $A_{n-1}^{(1)}$面模型对应的谱参数有一个关于代数中心平移的动力学 R 矩阵. 本文的思路和方法可以直接运用到其它面模型.

一般来讲,可以利用 $A_{q,p,\pi}(\widehat{gl}_n)$的量子行列式得到动力学椭圆代数 $A_{q,p,\pi}(\widehat{sl}_n)$. 但是直到现在人们还没有弄清楚 $A_{q,p,\pi}(\widehat{gl}_2)$的量子行列式. 因此研究 $A_{q,p,\pi}(\widehat{sl}_n)$是一件非常有意义的工作. 此外,$A_{q,p,\pi}(\widehat{gl}_n)$的 Drinfeld 流还可以生成 q - 破缺 W_n 代数的屏蔽流. 我们将在以后的文章中加以阐明.

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Drinfeld Currents of Dynamical Elliptic Algebra $A_{q,p,\pi}(\widehat{gl_n})$

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Abstract From the generalized Yang-Baxter relations $RLL = LLR^*$, where R and R^* are the dynamical R-matrix of $A_{n-1}^{(1)}$ type face model with the elliptic module shifted by the center of the algebra, using the Ding – Frenkel correspondence, we obtain the Drinfeld currents of algebra $A_{q,p,\pi}(\widehat{gl_n})$.

Key words dynamical elliptic algebra $A_{q,p,\pi}(\widehat{gl_n})$, Gauss decomposition, Drinfeld currents

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