

Thermalization, Expansion, and Radial Flow in HIC

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The thermalization, expansion, and radial flow in the processes of intermediate energy heavy ion collisions are discussed via the BUU model. From the calculation of quadrupole momenta and density distributions, it is shown that the global equilibrium is reached for the collision system at lower energies, while at higher energies only the local equilibrium could be reached. Using the Skyrme interaction, the influences of radial flow on the equation of state are also studied, which shows that radial expansion flow is sensitive to the effective mass, but insensitive to the incompressibility.

Key words: heavy ion collision, thermalization, EOS, radial expansion.

1. INTRODUCTION

There is a threshold energy in heavy ion collisions, below which the quadrupole momentum is equal to zero ($Q_{zz} = 0$). It means that the momentum distribution of the collision system is isotropic, and the system is thermalized [1]. Above the threshold $Q_{zz} > 0$, which means that the momentum distribution is not isotropic, and the collision system is not thermalized. At lower energies, there is the global equilibrium. However, at higher energies it does not exist anymore. Therefore, it is inappropriate to use the statistical equilibrium theory for describing the processes of heavy ion collision. Is there a local equilibrium? Many more discussions about this are needed for medium energy heavy ion collisions, because the situation is quite different from lower energy heavy ion collisions which have rich experimental results, e.g., the measurement [2,3] of the angular distribution

Received December 27, 1995. Supported by the National Natural Science Foundation of China and LWTZ-1298, the Chinese Academy of Sciences.

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of evaporation particles. On the other hand, both the statistical model and hybrid model based on the assumption of statistical equilibrium can fit with the experimental data of IMFs distribution quite well [4]. Can we conclude from this fact that the hypothesis of statistical model is correct, i.e., in intermediate energies or higher energies heavy ion collisions the system is thermalized before the production of IMFs?

Experimental radial expansion has been studied recently. For example, by the measurement of the rapidity source of intermediate mass fragmentation in Au + Au (150 MeV/u) central collisions, it was [5] found that the average energy per particle as a function of Z suggested the existence of radial expansion which occupied about 1/4 of the available energy [6,7].

In this paper the thermalization and the relations of local equilibrium with the equation of state and incident energies are discussed, and the theoretical picture of radial expansion process is given; the effects of EOS on the radial expansion are also discussed.

2. MODEL

Our work is based on the BUU model with the single particle distribution satisfying the Boltzmann equation. Under the nonlocal approximation [4] of the interaction the BUU equation is as follows:

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m^*} \cdot \nabla_r - \nabla_r \cdot \Sigma^{\text{HF}} \cdot \nabla_p \right) f(\mathbf{r}, \mathbf{p}, t) = I_{\text{coll}}, \quad (1)$$

I_{coll} is the collision term of the Uehling-Uhlenbeck form,

$$I_{\text{coll}} = - \int \frac{d^3 p_2 d^3 p_1' d^3 p_2'}{(2\pi)^9} \sigma_{\text{NN}} v_{12} [f f_2 (1-f_1') (1-f_2') - f_1' f_2' (1-f) (1-f_2)] (2\pi)^3 \delta^3(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_1' - \mathbf{p}_2'), \quad (2)$$

where v_{12} is the relative velocity of the colliding nuclei, and σ_{NN} is the nucleon-nucleon collision cross section. The effective mass m^*/m due to nonlocality of interaction is defined as

$$m^*/m = \left(1 + \frac{m}{p} \cdot \nabla_p U \right)^{-1}. \quad (3)$$

which signifies the momentum-dependent mean field. The mean field for the nonlocal approximation is in Ref. [1].

3. RESULT AND DISCUSSION

3.1. Discussion of thermalization-local equilibrium

Usually, for a given collision system, the following criteria are applied to discuss whether the collision system is equilibrated: after a certain time evolution, the average energy for emission neutrons reaches a constant; there is no more change of the density in the overlapping region of the collision system; the nucleon-nucleon collision number becomes a constant; the quadrupole moment of momentum distribution oscillates around zero.

Often we use the last one to judge the thermalization of a collision system, i.e., if $Q_{xx} = \sum (2p_z^2 - p_x^2 - p_y^2)$ equals zero, the collision system is thermalized; otherwise it is not. For

central collisions of a symmetric system $^{40}\text{Ca} + ^{40}\text{Ca}$ it has been known from the quadrupole momentum and density distribution that the collision system cannot reach a global equilibrium when the incident energy is over 50 MeV/u [10]. This is just the phase transition energy when the reaction mechanism is changing from incomplete fusion to the fragmentation process. Could the collision system reach

local equilibrium over this energy? Here, we define $R_E = \frac{\sum (p_x^2 + p_y^2)}{2p_z^2}$. Similar to the criterion of the quadrupole moment, if $R_E = 1$, the collision system is equilibrated; if $R_E < 1$, the equilibrium is violated.

As the global equilibrium is broken, we study the thermalization in the sphere of radius $R = R_0 = 6$ fm in the C.M. system.

(1) The relations between local equilibrium and incident energy. For the $^{93}\text{Nb} + ^{93}\text{Nb}$ central collision, $E_{\text{lab}} = 70$ MeV/u, it has been shown that [1] the global equilibrium is broken. In the sphere $R = R_0 = 6$ fm, our calculation shows that R_E is very small at $t = 0$, but with the increase of time step, R_E changes gradually from $R_E < 1$ to $R_E > 1$ and finally it oscillates around 1. This can be explained in the following way: at the beginning the nucleon-nucleon collision is weak; at $t = 30$ fm/c it becomes violent; in the course of emission of light particles, it arrives at an equilibrium. Similar results are also obtained for $R_0 = 2, 3$ fm. Since the compound system is thermalized in the sphere of $R_0 = 6$ fm, the system in the smaller sphere inside this region should be also thermalized. The time steps of thermalization are 110, 100, and 105 fm/c for $R_0 = 2, 3$, and 5 fm, respectively, which are consistent on the time scale. As we know, the fluctuation of the mean field can cause the fluctuation of density. At higher energies the density fluctuation is quite strong and the distribution of density is not uniform. Nucleons will transport from a higher density region to lower ones. At the same time, higher energies can produce squeeze-out, and the distribution of squeeze-out fragments is not uniform, so the fragments or particles with high energies will be emitted and this will also violate the global equilibrium of the collision system. However, after particle emission it is possible to reach equilibrium in a small region of the system and that is the local equilibrium. For different energies, the obtained thermalization times are 105, 85, 80, 80 fm/c for $E_{\text{lab}} = 50, 70, 90, 110$ MeV/u, respectively. It shows that with the increase of incident energies, the thermalization time decreases gradually. There is a larger difference between the thermalization times at 50 and 70 MeV/u. It turns out that the phase transition region is different from others.

(2) The relations of thermalization with EOS. From above we know that the collision system can reach local equilibrium at higher incident energies. Here we use the Skyrme interaction potential which characterizes the intrinsic properties of nuclear matter by using effective mass and incompressibility. The following four groups of parameters are used in our calculation: the different incompressibility coefficients $K = 260$ MeV, 214 MeV and approximate effective masses $m^*/m = 0.611, 0.79$ for SKA, SKM, respectively; and $K = 359$ MeV, 251 MeV, $m^*/m = 0.614, 0.614$ for SGOI, SGOII. For the same collision system Nb+Nb at $E_{\text{lab}} = 70$ MeV/u the thermalization times are studied in central collisions. The thermalization times are 60 fm/c, 60 fm/c, 55 fm/c, and 50 fm/c for SKA, SKM, SGOI, and SGOII, respectively, at a given $R = 6$ fm. There is little difference among them. From this we know that the thermalization is insensitive to the incompressibility and effective mass, i.e., EOS.

3.2. Expansion and radial flow

The collective flow predicted by hydrodynamics appears in two ways: one is the side-slash of spectator in the reaction plane, the other is the bounce-off of participant perpendicular to beam direction, i.e., isotropic radial expansion flow [8]. Experimental evidence also suggested the existence of radial expansion. In this paper we discuss radial expansion in theory and also give its picture. To consider the expansion process of heavy ion collisions, we study the changes of the particle number with time in the sphere of radius R .

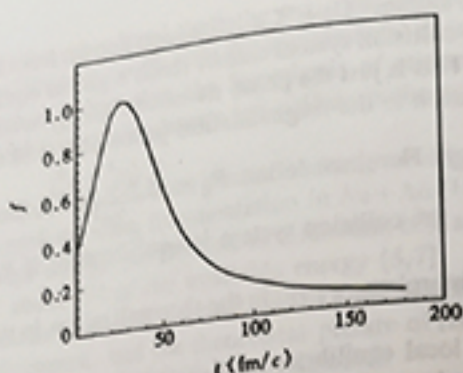


Fig. 1
The evolution of f with the time at $R = 8$ fm for
 $^{93}\text{Nb} + ^{93}\text{Nb}$ central collision, $E_{\text{lab}} = 90$ MeV/u.

We define

$$f = \frac{N_{\text{freeze-out}}}{N_{\text{max}}} \quad (4)$$

where $N_{\text{freeze-out}}$, N_{max} are the nucleon numbers in the sphere corresponding to the thermalization and the maximal compression times, respectively. In the $^{93}\text{Nb} + ^{93}\text{Nb}$ central collisions with $E_{\text{lab}} = 90$ MeV/u, $R_0 = 8$ fm, for hard potential, the evolution of f with time shows that at $t = 30$ fm/c the collision system reaches a maximal compression, then expands rapidly with the evolution of time and reaches thermalization at $t = 80$ fm/c (Fig. 1). We also calculated the relation of f with the beam energies at $R = 6$ fm. For $E_{\text{lab}} = 50, 70, 90$ MeV/u, the smaller f corresponds to larger beam energies and stronger expansion (Fig. 2).

The momentum p of a nucleon has two components: a flow component p_{flow} and a random thermal component p_{th} . It can be written as

$$p_i = p_{\text{flow}} + p_{\text{th}} \quad (5)$$

The flow profile $\beta(r)$ is as follows:

$$\beta(r) = \frac{1}{N_R} \sum_{i=1}^{N_R} \frac{p_i}{E_i} \cdot \frac{R_i}{R_i} \quad (6)$$

where N_R is the particle number in the sphere of radius R . As shown from preceding discussion, the collision system is thermalized in the sphere of radius R , so in the above equation the thermal component is equal to zero, and Fig. 3 gives the relation of $\beta(r)$ with R . The velocity decreases with the increase of R . This shows that the velocity at the center is larger than the one at the edge of the sphere. It gives the radial expansion.

The calculation shows [9] that the radial expansion flow is insensitive to incompressibility, which is consistent with the results of M.A. Lisa [10]. However, it is sensitive to effective mass. The larger the effective mass, the smaller the radial expansion energy. For smaller effective mass, the potential is mainly repulsive and this leads to the enhancement of the radial expansion and the expansion energy. In comparison with the experiment, the effective mass which equals 0.8 gives reasonable fitting and the expansion energy also increases with the increase of incident energy.

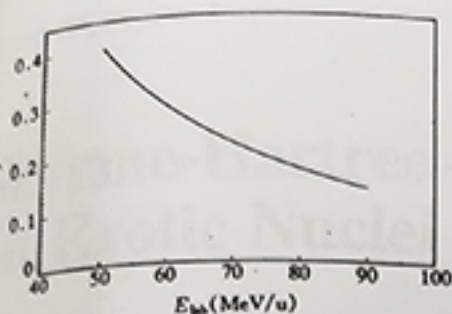


Fig. 2

The relation of f with the incident energy at $R = 6$ fm for $^{93}\text{Nb} + ^{93}\text{Nb}$ central collision, $E_{\text{lab}} = 90$ MeV/u.

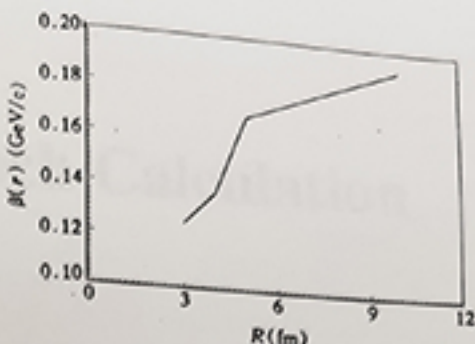


Fig. 3

For $^{93}\text{Nb} + ^{93}\text{Nb}$ central collision, $E_{\text{lab}} = 90$ MeV/u, the velocity distribution as a function of the radius R .

4. CONCLUSION

In intermediate energy heavy ion collisions, there is a threshold energy. Below this energy, the collision system can reach a global equilibrium. Over this energy, the global equilibrium is violated, and the system leads to local equilibrium. The thermalization is insensitive to the effective mass and incompressibility. The picture of the radial expansion process is also given. The radial expansion is sensitive to the effective mass, but insensitive to incompressibility.

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