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Microscopic Mechanism of Identical Multi-Quasiparticle Bands

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The intrinsic structure of identical bands is demonstrated by using the particle-number-conserving (PNC) treatment. The occurrence of almost identical moments of inertia is the result of competition among the shell effect (including shape variation), the pairing (anti-alignment) effect, and the blocking (antipairing) effect. The observed moments of inertia of identical multi-quasiparticle bands are reproduced quite well by the PNC calculation without free parameters.

Key words: identical multi-quasiparticle band, blocking effect, particle-number-conserving method.

1. INTRODUCTION

An unexpected and exciting discovery in high-spin nuclear physics is the finding of almost identical superdeformed (SD) bands in neighboring nuclei [1,2]. Several explanations [2-4] were put forward, which assume that the occurrence of identical bands is a specific property of the SD bands.

However, shortly afterwards, it was recognized that the identical bands also exist in normally deformed (ND) pairs of even- and odd-mass nuclei [5,6] and in pairs of even-mass nuclei at low spins [7,8], i.e., the occurrence of identical bands is not necessarily related to the phenomena of the superdeformation or excitation of high-spin states in nuclei. However, it has been well established that there exist strong pairing correlations in ND nuclei at low spins [9,10], which lead to the odd-even

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differences in various properties (mass, moment of inertia, etc.). Particularly, the pairing interaction is responsible for the observed reduction of the nuclear moments of inertia compared to that of a rigid rotor [10,11]. According to the conventional BCS approximation for treating the nuclear pairing correlation, the moments of inertia of a 1-quasiparticle band should be large than those of the ground state bands in adjacent even-even nuclei by 15-20%. Therefore, the occurrence of the identical bands in ND pairs of even- and odd-mass nuclei at low spins is a serious challenge to the mean field (BCS) approximation [5,6].

It is worthwhile to mention that many years ago Bohr and Mottelson [10] pointed out that there exist large fluctuations in the odd-even differences of moments of inertia. For example, while the band-head moment of inertia of the [642]5/2 band in ^{151}Dy ($\sim 160\hbar^2$MeV^{-1}) is over twice as large as those of the ground bands of ^{160}Dy and ^{162}Dy, the moment of inertia of the [402]5/2$^+$ band in ^{171}Lu is almost equal to that of the ground band of ^{170}Yb ($\sim 71\hbar^2$MeV^{-1}). In fact, there exists no distinct demarcation line between the "identical" and non-identical bands. Careful investigations show that various odd-even differences are intimately connected with the proper treatment of the blocking effects. However, just as emphasized by Rowe [12], although the blocking effects are straightforward, it is very difficult to treat them in the BCS formalism, because they introduce different quasiparticle bases for different blocked levels. To calculate the odd-even difference in the moments of inertia of the rare-earth nuclei, the particle-number-conserving (PNC) treatment [14] for the eigenvalue problem of the cranked shell model (CSM) was used in Ref. [13], in which the blocking effects were treated exactly. It has been shown that the odd-even difference in the moments of inertia is a pure quantum mechanical interference effect and the experimental large fluctuations in the odd-even difference of the moments of inertia, $\delta J / J = [J(A+1) - J_0(A)] / J_0(A)$ (A even), can be reproduced satisfactorily. The PNC calculation shows that $\delta J / J$ depends sensitively on the location relative to the Fermi surface of the blocked level ν_0 and the size of the Coriolis response ($|\langle\nu|j_x|\nu_0\rangle|$), i.e., (a) when ν_0 is a high j intruder orbit near the Fermi surface (e.g., neutron [642]5/2, [633]7/2, [624]9/2) $\delta J / J$ is especially large, (b) when ν_0 is a low j and high Ω orbit (e.g., proton [402]5/2, [404]7/2), or very far from the Fermi surface, $\delta J / J$ is very small, hence the identical band may appear, (c) when ν_0 is the other normal orbit, $\delta J / J \approx$ 10-20%.

Before we address the microscopic mechanism of the identical band, we make a brief phenomenological analysis for the identical bands, particularly the identical multi-quasiparticle bands. It is well known that the identical bands exist in neighboring even-even nuclei [7-8] (nearly equal moments of inertia of 2-quasiparticle vacuum bands) and in neighboring even-even and odd-mass nuclei [5-6,13] (nearly equal moments of inertia of a 1 quasiparticle band and a quasiparticle vacuum band), one may easily imagine that the identical bands may appear in the neighboring odd-mass nuclei (nearly equal moments of inertia of two 1-quasiparticle bands). For example, the band-head moments of inertia: $2J \approx 70\hbar^2$MeV^{-1} for [404]7/2 bands in ^{171,173,175,177}Ta, $2J \approx 74\hbar^2$MeV^{-1} for [404]7/2 bands in ^{169,171,177}Lu, and $2J \approx 71\hbar^2$MeV^{-1} for [402]5/2 bands in ^{169,171}Lu. More careful observation shows that the 1-quasiparticle band in the odd-mass nucleus is almost identical with the 2-quasiparticle band in the neighboring even-even nucleus. Three typical examples are shown in Table 1 [the band-head moment of inertia is derived from the lowest two levels of the same signature in each band, and the level data are taken from [16] (even-even nuclei) and the nuclei data sheets (odd-A nuclei)]: These facts seem unexpected. From the BCS approximation, it is very difficult to account for the occurrence of these identical multi-quasiparticle bands in neighboring odd- and even-mass nuclei. Moreover, let J_0, J_μ, $J_{\mu\nu}$ denote the moments of inertia of the quasiparticle vacuum band $|0\rangle\rangle$, 1-quasiparticle band $\alpha_\mu^+|0\rangle\rangle$, and 2-quasiparticle band $\alpha_\mu^+\alpha_\nu^+|0\rangle\rangle$, respectively, then according to the BCS approximation and neglecting the residual quasiparticle interaction, the following additivity of the moments of inertia can be derived [15]

$$J_{\mu\nu} - J_0 = (J_\mu - J_0) + (J_\nu - J_0), \tag{1}$$

Table 1
Three identical pairs of 1- and 2-quasiparticle bands.

Rotational band	$2J$ ($\hbar^2$ MeV^{-1})
^{167}Er, g.b., n[633]7/2	112.4
^{168}Er, 1773.2keV, $K^\pi = 6^-$, nn[633]7/2 + [512]5/2	113.3
^{169}Yb, g.b., n[633]7/2	123.7
^{171}Yb, 95.272keV, n[633]7/2	122.1
^{170}Er, 1851.4keV, $K^\pi = 6^-$, nn[633]7/2 + [512]5/2	121.2
^{161}Ho, g.b., p[523]7/2	90.1
^{162}Er, 1985keV, $K^\pi = 7^-$, pp[523]7/2 + [404]7/2	89.9

or equivalently, the ratio

$$R = \frac{(J_\mu - J_0) + (J_\nu - J_0)}{J_{\mu\nu} - J_0} = 1 . \tag{2}$$

However, the experimental results show that in most cases the R ratios deviate from 1 significantly [15], which implies that the residual quasiparticle interaction due to blocking effects is rather significant. In this paper we will try to demonstrate the microscopic mechanism of identical multi-quasiparticle bands using the PNC treatment [13,14].

The CSM Hamiltonian of an axially symmetric nucleus is

$$H_{\mathrm{CSM}} = H_{\mathrm{SP}} - \omega J_x + H_{\mathrm{P}} = H_0 + H_{\mathrm{P}}, \tag{3}$$

where H_{SP} is the single-particle (e.g., Nilsson) Hamiltonian, $-\omega J_x$ is the Coriolis interaction, H_{P} is the pairing interaction, $H_0 = H_{\mathrm{SP}} - \omega J_x$ is the cranked Nilsson (CN) Hamiltonian, which is a one-body operator. $H_0 = \Sigma_i h_0(i)$, $h_0 = h_{\mathrm{SP}} - \omega j_x$ is for a single particle. Let $h_0|\mu\rangle = \varepsilon_\mu|\mu\rangle$, where $|\mu\rangle$ is the cranked Nilsson single-particle state with nondegenerate eigenvalue ε_μ. The good quantum numbers characterizing $|\mu\rangle$ are the parity π_μ and signature $r_\mu = e^{-i\pi\alpha_\mu} = \pm i$, ($\alpha_\mu = \mp 1/2$). For a n-particle system, the eigenstate $|i\rangle$ of H_0 $H_0|i\rangle = E_i|i\rangle$, may be described by the occupation of n particles $|i\rangle = |\mu_1\mu_2\ldots\mu_n\rangle$, $E_i = \sum_{k=1}^{n} \varepsilon_{\mu_k}$. $|i\rangle$ is called a cranked many-particle configuration (CMPC), characterized by E_i, parity π, and signature $\alpha = \sum_{k=1}^{n} \alpha_{\mu_k}$. The eigenstate $|\psi\rangle$ of H_{CSM}, $H_{\mathrm{CSM}}|\psi\rangle = E|\psi\rangle$, can be expressed in the CMPC space as

$$|\psi\rangle = \sum_i C_i |i\rangle . \tag{4}$$

For the yrast and low-lying eigenstates of H_{CSM}, the accurate solution of $|\psi\rangle$ can be obtained by diagonalizing H_{CSM} in a sufficiently large CMPC space, $(E_i - E_0) < E_c$, E_0 is the energy of the lowest CMPC state, $|i_0\rangle$, and E_c is sufficiently large. As in Refs. [13,14], E_c is chosen to be $0.85\ \hbar\omega_0$ in the calculation, and in this case, over twenty Nilsson orbits near the Fermi surface are involved (see Table 3). Calculations show that all the main CMPC (weight $> 10^{-3}$) have been included in our calculation, so the calculated low-lying eigenstates are accurate enough. In the calculation the Nilsson parameters ε_2, ε_4, κ, μ, and $\hbar\omega_0$ are all taken from the Lund systematics [17], and the paring interaction strength G is determined by the observed odd-even mass differences [18], so there is no free parameter in our calculation. The angular momentum alignment of the state $|\psi\rangle$ is $\langle\psi|J_x|\psi\rangle$. According to the CSM, the kinematic moment of inertia of $|\psi\rangle$ is

$$J = \frac{1}{\omega}\langle\psi|J_x|\psi\rangle = \frac{1}{\omega}\sum_i |C_i|^2\langle i|J_x|i\rangle + \frac{2}{\omega}\sum_{i<j} C_i^* C_j\langle i|J_x|j\rangle . \tag{5}$$

Table 2
Band-head moments of inertia of the yrast and 2-quasiparticle bands of ^{168}Er, and the 1-quasiparticle band of ^{167}Er.

Rotational band	$2J$ ($\hbar^2$ MeV^{-1})		
	Cal.		Exp.
	$G = 0$	$G \neq 0$	
^{168}Er g.b.	118.1	70.7	75.0
^{168}Er [633]7/2 + [512]5/2	127.9	113.8	113.3
^{167}Er [633]7/2	129.8	112.3	112.4
^{167}Er [512]5/2	139.7	84.0	84.6

Table 3
The calculated moments of inertia (in units of $\hbar^2$MeV^{-1}) of the ^{168}Er ground band, 1-quasiparticle bands ^{167}Er[512] 5/2 and ^{167}Er[633]7/2, and 2-quasiparticle band ^{168}Er, $K^\pi = 6^-$, [633]7/2 + [512]5/2.

Rotational band	$G = 0$			$G \neq 0$			Reduction of moment of inertia			J / J (^{168}Er g.b.)	
	$2J_p$	$2J_n$	$2J$	$2J_p$	$2J_n$	$2J$	$\frac{J_p(G \neq 0)}{J_p(G=0)}$	$\frac{J_n(G \neq 0)}{J_n(G=0)}$	$\frac{J(G \neq 0)}{J(G=0)}$	$G = 0$	$G \neq 0$
^{168}Er g.b	40.73	77.35	118.1	25.46	45.20	70.66	62.5%	58.4%	59.8%	1	1
^{167}Er [512]5/2	40.73	98.93	139.7	25.46	58.54	84.0	62.5%	59.2%	60.1%	1.18	1.19
^{167}Er [633]7/2	40.73	89.07	129.8	25.46	86.86	112.3	62.5%	97.5%	86.5%	1.10	1.59
^{168}Er $K^\pi = 6^-$	40.73	87.19	127.9	25.46	88.34	113.8	62.5%	101.3%	89.0%	1.08	1.61

For $G = 0$ and $G \neq 0$ (the pairing interaction strength is determined by the observed odd-even mass difference [18], $G_n = 238.8$ keV, and $G_p = 282.6$ keV, see Ref. [14]). J_p and J_n are the contributions from protons and neutrons, respectively.

Because J_x is a one-body operator, $\langle i|J_x|j\rangle$ is nonzero only when $|i\rangle$ and $|j\rangle$ differ by one particle occupation. Suppose that after a certain permutation of the creation operators, $|i\rangle$ and $|j\rangle$ can be reduced into the form $|i\rangle = (-)^{M_{i\mu}}|\mu\cdots\rangle$, $|j\rangle = (-)^{N_{j\nu}}|\nu\cdots\rangle$, where the ellipses stand for the same particle occupation and $(-)^{M_{i\mu}} = \pm 1$, $(-)^{N_{j\nu}} = \pm 1$ according to the even or odd permutation. Thus, J can be expressed in terms of the single-particle picture as

$$J = \frac{1}{\omega}\langle\psi|J_x|\psi\rangle = \sum_{\mu} J_{\mu\mu} + \sum_{\mu<\nu} J_{\mu\nu}, \tag{6}$$

$$\sum_{\mu} J_{\mu\mu} = \frac{1}{\omega}\sum_{\mu}\langle\mu|j_x|\mu\rangle\sum_{i}|C_i|^2 P_{i\mu} = \frac{1}{\omega}\sum_{\mu}\langle\mu|j_x|\mu\rangle n_\mu, \tag{7}$$

$$J_{\mu\nu} = \frac{2}{\omega}\langle\mu|j_x|\nu\rangle\sum_{i<j}(-)^{M_{i\mu}+N_{j\nu}} C_i^* C_j, \ (\mu \neq \nu) \tag{8}$$

where $n_\mu = \Sigma_i|C_i|^2 P_{i\mu}$ is the particle occupation probability of the CN orbit $|\mu\rangle$ in the state $|\psi\rangle$ and $P_{i\mu} = 1$, if $|\mu\rangle$ is occupied in $|i\rangle$, and $P_{i\mu} = 0$, otherwise. If the pairing interaction is missing ($G = 0$), only one CMPC appears in $|\psi\rangle$ and all the interference terms $J_{\mu\nu}$ vanish. Then, for the lowest CMPC $|i_0\rangle$,

$$J - \frac{1}{\omega}\langle i_0|J_x|i_0\rangle = \frac{1}{\omega}\sum_{\mu\,(\text{occ. in }|i_0\rangle)}\langle\mu|j_x|\mu\rangle, \tag{9}$$

which, in general, is near the rigid-body value, but shows strong shell effects.

Table 4
The contributions to the moments of inertia (in units of $\hbar^2$MeV^{-1}) from neutron $N = 5$, 6 shells.

Rotational band	$N = 5$ shell					$N = 6$ shell					All shells		
	$G = 0$ diag.	$G \neq 0$			f	$G = 0$ diag.	$G \neq 0$			f	$2J_n$		f
		Diag.	Off-diag.	Total			Diag.	Off-diag.	Total		$G = 0$	$G \neq 0$	
^{168}Er g.b.	34.25	35.80	−11.84	23.96	70%	43.10	52.06	−30.08	21.20	49%	77.35	45.20	58.4%
^{167}Er [512]5/2	32.27	33.90	−3.06	30.84	95.6%	66.66	63.71	−36.07	27.63	41.5%	38.93	58.54	59.2%
^{167}Er [633]7/2	34.19	35.22	−10.19	25.03	73.2%	54.87	55.75	+6.04	61.79	112.6%	89.07	86.86	97.5%
^{168}Er $K^\pi = 6^-$	32.31	33.16	−42.26	28.94	89.6%	54.87	55.28	+4.08	59.36	108.2%	87.19	88.34	101.3%

The contributions from $N \leq 4$ shells are zero for $G = 0$. For $G \neq 0$, the contributions from these shells are still very small and not included in this table. Diag. $= 2\Sigma_\mu J_{\mu\mu}$, off-diag. $= 2\Sigma_{\mu<\nu} J_{\mu\nu}$, and $f = J_n(G\neq 0)/J_n\,(G = 0)$ stands for the reduction of moments of inertia of each major shell due to the pairing correlation.

As illustrative examples, the band-head moments of inertia of the ground band and the $K^\pi = 6^-$nn[633]7/2 + [512]5/2 band in ^{168}Er, and the ^{167}Er[633]7/2 and [512]5/2 bands are calculated and the results are shown in Tables 3-5. The comparison of the calculated band-head moments of inertia of these bands with experiments is given in Table 2. Considering no free parameter as being involved in the calculation, the calculated results are quite satisfactory. Particularly, the experimental phenomena that the moment of inertia of the 2-quasiparticle band ^{168}Er[633]7/2 + [512]5/2 is almost equal to that of the 1-quasiparticle band ^{167}Er[633]7/2, but much larger than that of the 1-quasiparticle band ^{167}Er[512]5/2, are reproduced quite well in the PNC calculation. Now let us discuss the underlying physics of such large variation in the moments of inertia.

(1) For $G = 0$, the calculated moments of inertia are, in general, near the rigid-body value ($\sim 140\hbar^2$MeV^{-1}) as expected, but show significant shell effects. The contribution of a closed shell configuration to the moment of inertia is zero. For the rare-earth nuclei, no contribution comes from the closed major shells $N = 0$, 1, 2, 3, and the main contributions are from the neutron $N = 5$, 6 shells and proton $N = 4$, 5 shells. It is well known that because of the strong spin-orbit coupling, the high-j intruder orbits are relatively far away from the normal orbits in the same N major shell, and j is approximately a good quantum number. These orbits may be approximately simulated by a single-j shell model [19,20]. In the single j model, the contribution to the moments of inertia from the particles occupying the upper orbits is negative ($d\langle j_x\rangle / d\omega > 0$). This is the reason why the J_n values of the ^{167}Er[633]7/2 and ^{168}Er[633]7/2 + [512]5/2 bands are larger than that of ^{168}Er (g.b.) by about 10% (see Table 3). For the ^{167}Er[512]5/2 band, no neutron occupies the [633]7/2 orbit (for $G = 0$), hence the J_n value of ^{167}Er[512]5/2 is larger than that of ^{168}Er (g.b.) by about 20% (see Table 3).

(2) When the pairing interaction is taken into account, many of the CMPC are mixed into the low-lying excited eigenstates of H_{CSM}. Due to the resulting destructive interference effects the off-diagonal part of $J_{\mu\nu}$ ($\mu \neq \nu$), is generally negative (see Tables 4 and 5), so the calculated moment of inertia is strongly reduced. Physically, considering the anti-alignment effect of the pairing interaction, this can be easily understood. In particular, when μ and ν are the high-j intruder orbits near the Fermi surface, $|J_{\mu\nu}|$ is especially large and in fact, the reduction of the moments of inertia due to the paring interaction mainly comes from these orbits. On the other hand, when the pairing interaction is taken into account, the diagonal part, $\Sigma J_{\mu\mu}$, changes only a little, because the particle occupation changes very little. From the above discussions, we can understand why the moments of inertia of the ground bands of even-even nuclei are reduced due to the pairing interaction by a factor of about 1/2.

Table 5

The main off-diagonal parts of $J_{\mu\nu}$ [see Eq.(8)] for the ^{168}Er g.b., ^{167}Er[512]5/2, ^{167}Er[633]7/2, and ^{168}Er, $K^\pi = 6^-$, [633]7/2 + [512]5/2 bands.

Nilsson orbit		$2J_{\mu\nu}$ ($\hbar^2$ MeV^{-1})							
		^{168}Er g.b. signature α		^{167}Er [512]5/2 signature α		^{167}Er [633]7/2 signature α		^{168}Er $K^\pi = 6^-$ signature α	
μ	ν	1/2	−1/2	1/2	−1/2	1/2	−1/2	1/2	−1/2
[514]7/2	[505]9/2	−0.16	−0.16	−0.21	−0.21	−0.14	−0.14		
[530]1/2	[521]3/2	−0.25	−0.25	−0.33	−0.33	−0.21	−0.21	−0.19	−0.19
[530]1/2	[521]1/2	−0.31	−0.24	−0.34	−0.25	−0.31	−0.24	−0.34	−0.26
[532]3/2	[523]5/2	−0.35	−0.35	−0.38	−0.38	−0.34	−0.34	−0.14	−0.14
[532]3/2	[521]1/2	−0.29	−0.35	−0.28	−0.35	−0.49	−0.36	−0.28	−0.35
[521]3/2	[523]5/2	−0.08	−0.08	−0.11	−0.11				
[521]3/2	[512]5/2	−1.91	−1.91	1.41[a]	1.97[a]	−1.84	−1.84	0.43[a]	0.58[a]
[523]5/2	[514]7/2	−1.55	−1.55	−1.42	−1.42	−1.47	−1.47	−1.39	−1.40
[521]1/2	[510]1/2	−0.38	−0.32	−0.15	−0.13	−0.17	−0.14	−0.16	−0.13
[521]1/2	[512]3/2	−0.29	−0.34	−0.09	−0.10	−0.13	−0.15	−0.09	−0.11
[512]5/2	[514]7/2	−0.13	−0.13	0.08[a]					0.09[a]
[512]5/2	[503]7/2	−0.18	−0.18						
[514]7/2	[505]9/2	−0.06	−0.06						
[660]1/2	[651]3/2	−0.28	−0.55	−0.35	−0.69	−0.28	−0.53	−0.14	−0.27
[651]3/2	[642]5/2	−1.48	−1.46	−1.85	−1.83	−1.15	−1.26	−0.78	−0.81
[642]5/2	[633]7/2	−6.54	−6.54	−13.78	−13.78	3.87[a]	3.12[a]	2.43[a]	1.85[a]
[633]7/2	[624]9/2	−6.63	−6.63	−1.85	−1.85	0.82[a]	1.47[a]	1.06[a]	0.75[a]
[624]9/2	[615]11/2	−0.20	−0.20						
[651]1/2	[640]1/2	−0.05	−0.15						
[651]1/2	[642]3/2		−0.11						
$N = 4$ shell									
$N = 5$ shell		−11.84		−3.06		−10.19		−4.23	
$N = 6$ shell		−30.86		−36.07		+6.04		+4.08	
All shells		−42.71		−39.13		−4.16		−0.15	

Very small contributions $|J_{\mu\nu}| \leq 0.01$ are omitted in this table. The positive $J_{\mu\nu}$ (due to blocking effects) is indicated by [a].

(3) For the 1-quasiparticle band in odd-A nuclei and the pair-broken (2-quasiparticle) bands in even-even nuclei, we must consider the influence of the blocking effects, which are especially important for the low-lying excited and low-spin states. Physically, the blocking effect of an unpaired particle will weaken the effective pairing interaction. So it is an antipairing effect and will decrease the reduction of the moment of inertia due to pairing, or equivalently it will promote the spin alignment $\langle J_x \rangle$ along the rotating x-axis. Therefore, the moments of inertia associated with multi-quasiparticle excitation states, in general, will be larger than those of the ground bands (the quasiparticle vacuum) in neighboring even-even nuclei. Mathematically, the blocking effects lead to a sign change of $J_{\mu\nu}$; if the orbit μ or ν is blocked (see Table 5) $J_{\mu\nu}$ will become positive when μ or ν is [633]7/2 for the ^{167}Er[633]7/2 band and when μ or ν is [512]5/2 for the ^{167}Er[512]5/2 band. Our calculation shows that the blocking of the high-j intruder orbits near the Fermi surface leads to a strong alignment of the angular momentum because these orbits have very strong Coriolis responses. From this we can

understand why the moment of inertia of the ^{167}Er[633]7/2 band is much larger than that of ^{168}Er (g.b.) by about 50%. However, if the blocked level is a normal orbit, such as the neutron [512]5/2, which has a moderate Coriolis response, the increase of the moment of inertia due to the blocking is moderate. From this we can understand why the moment of inertia of the ^{167}Er[512]5/2 band is larger than that of ^{168}Er (g.b.) by only about 13%.

(4) However, it should be emphasized that the blocking effects in the multi-quasiparticle bands are by no means additive. In a pair-broken (2-quasiparticle) band, e.g., in the ^{168}Er[633]7/2 + [512]5/2 band, the presence of the odd neutron [512]5/2 will attenuate the blocking effect on the pairing correlation of the odd neutron [633]7/2, i.e., the blocking effect of the [633]7/2 neutron in the ^{168}Er[633]7/2 + [512]5/2 band cannot be displayed so sufficiently as in the ^{167}Er[633]7/2 band. The non-additivity of the moments of inertia originates from the cancellation of the blocking (antipairing) effects in a pair-broken band, which manifests clearly in the variation of the off-diagonal part of $J_{\mu\nu_0}$ (with ν_0 being the blocked level). For example, for μ = [642]5/2, ν_0 = [633]7/2, $J_{\mu\nu_0} = -13.08, 6.99, 4.28\hbar^2\text{MeV}^{-1}$ in ^{168}Er (g.b.), ^{167}Er[633]7/2, ^{168}Er[633]7/2 + [512]5/2, respectively. For μ = [521]3/2, ν_0 = [512]5/2, $J_{\mu\nu_0} = -3.81, 3.38, 1.01\hbar^2\text{MeV}^{-1}$ in ^{168}Er (g.b.), ^{167}Er[512]5/2, ^{168}Er[633]7/2 + [512]5/2, respectively (see Table 5). From the above discussions we can understand why the moment of inertia of the 2-quasiparticle band ^{168}Er[633]7/2 + [512]5/2 is almost equal to that of the 1-quasiparticle band ^{167}Er[633]7/2. In addition, considering [633]7/2 as being a high-j intruder orbit, but [512]5/2 being a normal orbit, we can understand why the moment of inertia of the ^{168}Er[633]7/2 + [512]5/2 band is still much larger than that of the ^{167}Er[512]5/2 band.

(5) Moreover, it is interesting to note that from Eq.(2) the experimental R ratio $R_{\text{exp}} = 1.23$, for the ^{168}Er (g.b.), ^{167}Er[633]7/2, ^{167}Er[512]5/2, and ^{168}Er[633]7/2 + [512]5/2 bands, is reproduced rather well by the PNC calculation ($R_{\text{cal}} = 1.27$). Of course, in our calculation, the deformation chosen for the multi-quasiparticle bands are the same as the quasiparticle vacuum band. The deformation change due to the blocking effects should be taken into account in a more sophisticated calculation.

CONCLUSION

In summary, the identical 1-quasiparticle and 2-quasiparticle bands in neighboring odd- and even-mass nuclei are recognized. The occurrence of almost identical moments of inertia is the result of the competition among the shell effect (including deformation change), the pairing (anti-alignment) effect, and the blocking (antipairing) effects. The observed moments of inertia of the identical multi-quasiparticle bands are reproduced quite well by the PNC calculation, in which the blocking effects are taken into account exactly and no free parameter is involved. This approach may be, in principle, applied to investigate the identical SD bands and it will be addressed in a subsequent paper.

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