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Validity of the M-3Y Force Equivalent *G*-Matrix Elements for Heavy Nuclei

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The matrix elements of the M-3Y force are adopted as equivalent *G*-matrix elements and the folded diagram method is used to calculate the spectra of ^{210}Pb, ^{206}Pb, ^{206}Hg, and ^{210}Po. Results show that the matrix elements of the M-3Y force as equivalent *G*-matrix elements are suitable for the microscopic calculations of the nuclear structure in the heavy nucleus region.

Key words: M-3Y force, folded diagram, nuclide ^{206}Pb, ^{210}Pb, ^{206}Hg, ^{210}Po, spectra.

1. INTRODUCTION

In recent years, it has been successful in searching for new rich-neutron nuclides in experiments. The ^{202}Pt [1], ^{208}Hg [2], and ^{185}Hf [3] have been discovered successively in China. There are few theoretical studies on these new nuclides; in particular, a work on the microscopic calculation has never been published. For the nuclei ^{202}Pt and ^{208}Hg, since they are not far away from the doubly closed shell nucleus, we suppose that it could be effective to calculate the structure of these nuclei in the simplified model [4,5] based on the Fermion "pairs."

The essential idea of the simplified model is the following: First, we define basic "pairs" as the lowest states of the eigenfunctions with certain angular momentum in the shell model calculation, for nuclei containing two nucleons (or holes) outside a doubly closed shell nucleus, and then calculate the spectra [6] of the system with four or more valence nucleons outside the doubly closed shell, with in the model space constructed in terms of the basic "pairs." Based on above starting point, at first we

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concentrate on the calculation for nuclei consisting of a pair valence nucleons (or holes) outside a doubly closed shell nucleus. For the s-d and f-p shell nuclei, microscopic calculations were carried out in terms of the M-3Y force, which is equivalent to the *G*-matrix elements [7—9]. It is shown that the M-3Y force equivalent *G*-matrix elements are completely suitable in these two regions. In order to establish the solid foundation for the study of ^{202}Pt and ^{208}Hg, in this paper we attempt to use the M-3Y force equivalent *G*-matrix elements to calculate the spectra of ^{210}Pb, ^{206}Pb, ^{206}Hg, and ^{210}Po, and examine if the M-3Y force equivalent *G*-matrix elements are suitable for the heavy mass nuclei. In Sections 2 and 3, we will briefly describe the calculation method. The results and discussions are given in Section 4.

2. METHOD OF CALCULATION

We will use the folded diagram method [10] to calculate the spectra of ^{210}Pb, ^{206}Pb, ^{206}Hg, and ^{210}Po. According to this method, the Schrödinger equation $H\psi_\lambda = E_\lambda\psi_\lambda$ with A nucleons can be reduced to an eigenequation of two nucleons (or holes) within a model space

$$PH_{\text{eff}}P\Psi_n=(E_n-E_0^c)P\Psi_n, \tag{2.1}$$

where H_{eff} is the effective Hamiltonian, $n = 1, 2, \ldots, d$, d is the dimension of the model space and P is a projection operator of the model space spanned by unperturbed shell-model states. For the cases of ^{210}Pb, ^{206}Pb, ^{206}Hg, and ^{210}Po, the ^{208}Pb is chosen to be the stable nuclear core, and the model space contains two valence nucleons (or holes) in the outside orbits. Since the isospin is no longer a good quantum number for heavy nuclei, the anti-symmetric basis for two nucleons can be written as

$$\Phi_J=\frac{1}{\sqrt{1+\delta_{ij}}}[a_{j_1}^+a_{j_2}^+]0\rangle=|j_1j_2\rangle_J. \tag{2.2}$$

The only differences of the basis among ^{210}Pb, ^{206}Pb, ^{206}Hg, and ^{210}Po are the different orbits selected. In this mode, Eq.(2.1) can then be deduced as the following equation:

$$\sum_j(E_j\delta_{ij}+V_{ij}^{\text{eff}})\chi_j^\lambda=(E_\lambda-E_0^c)\chi_i^\lambda, \tag{2.3}$$

$$i, j=1, 2, \cdots\cdots, d.$$

where $i, j = 1, 2,\ldots, d$, E_j is the zero-order energy, i.e., the sum of two single-particle energies of the valence nucleons, $E_j = E_{j_1} + E_{j_2}$, V_{ij}^{eff} is the effective interaction, χ_j^λ is the projection of the eigenfunction onto the model space corresponding to the eigenvalue E_λ, and E_0^c is the g.s. energy of ^{206}Pb.

In the folded diagram theory, V^{eff} is represented by the following infinite series:

$$V^{\text{eff}}=\hat{Q}-\hat{Q}'\int\hat{Q}+\hat{Q}'\int\hat{Q}\int\hat{Q}-\hat{Q}'\int\hat{Q}\int\hat{Q}\int\hat{Q}+\cdots\cdots, \tag{2.4}$$

Here Q-box is an infinite sum of all irreducible diagrams which must be linked by at least one passive (i.e., non-valence) line. The difference between $\hat{Q}'$ and $\hat{Q}$ is that $\hat{Q}'$ contains at least two

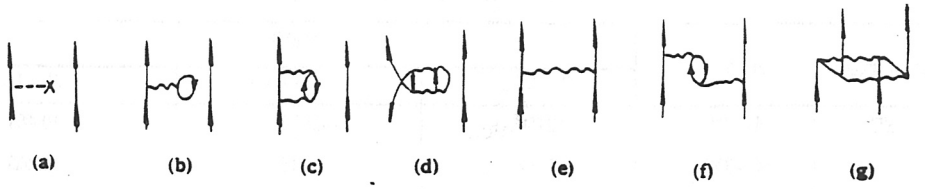

Fig. 1
The Goldstone diagrams in the first and second orders.

interaction vertexes. The symbol $\int$ is the energized folding operator. Equation (2.4) can also be expressed as follows

$$V^{\text{eff}} = F_0(\hat{Q}) + F_1(\hat{Q}) + F_2(\hat{Q}) + \cdots + F_m(\hat{Q}) + \cdots\cdots, \tag{2.5}$$

where $F_m(\hat{Q})$ represents the term from the m-th folded diagram. Because the convergence of the infinite series of folded diagrams is quite good [11], we only take terms up to the first folded diagram in our calculations. For the Q-box, we only take Goldstone diagrams up to the second order as Fig. 1. Graphs (a)-(d) in Fig. 1 are one-body diagrams, where graph (a) is the mean field, and the other three graphs express interactions between the core and the valence nucleons. In fact, the sum of contributions from all one-body diagrams determines the single-particle energies of ^{209}Pb, ^{207}Pb, ^{209}Bi, and ^{207}Tl. E_{ik} can be replaced by the experimental values of single-particle levels if all one-body diagrams are added to E_j on the left-hand side of Eq.(2.3). The one-body diagrams are no longer included in the first term of formula (2.4).

3. *G*-MATRIX ELEMENTS AND M-3Y FORCES

In the microscopic calculations of nuclear structure, in order to avoid the divergence of the integral of two-body matrix elements, which is caused by the strong repulsive core of the short range part of the realistic force obtained from free nucleon-nucleon scatterings, it is necessary to calculate G-matrix elements. For avoiding the complexity of G-matrix elements, in the two nuclear regions of *s-d* and *f-p* shells, some authors [5,6] used the equivalent G-matrix elements, which were obtained by calculating matrix elements with the M-3Y force proposed by G. Bertsch [7]. The M-3Y force equivalent G-matrix elements are in good agreement with those G-matrix elements obtained by using the Raid or Paris forces. We energized the M-3Y force equivalent G-matrix elements to heavy mass region and try to examine the validity of the M-3Y force in the region. The M-3Y force comprises the central, spin-orbit coupling, and tensor components:

$$V_{\text{NN}} = V_{\text{C}}(r_{12}) + V_{\text{LS}}(r_{12})\vec{L}\cdot\vec{S} + V_{\text{T}}(r_{12})S_{12}, \tag{3.1}$$

where

$$V_{\text{C}}(r_{12}) = \sum_i^3 V_{\text{C}i}\, Y(r_{12}/R_i),$$

$$V_{\text{LS}}(r_{12}) = \sum_i^3 V_{\text{LS}i}\, Y(r_{12}/R_i)\vec{L}\cdot\vec{S},$$

$$V_{\text{T}}(r_{12}) = \sum_i^3 V_{\text{T}i} r_{12}^2\, Y(r_{12}/R_i)S_{12},$$

Table 1
Parameters of the M-3Y force.

Channel	Name	V_i(MeV)		
		$R_1=0.25$	$R_2=0.40$	$R_3=1.141$
SE	M-3Y1	12454	-3835	-10.463
	M-3Y2	12454	-3835	-10.463
TE	M-3Y1	21227	-6622	-10.463
	M-3Y2	21227	-6622	-10.463
SO	M-3Y1	5018	1810	0.0
	M-3Y2	-29580	-3464	31.389
TO	M-3Y1	0.0	0.0	3.448
	M-3Y2	12052	-1990	3.448
TNE	M-3Y1	0.0	-1259.6	-28.41*
	M-3Y2	0.0	-1260	-28.41*
TNO	M-3Y1	0.0	283.0	13.62*
	M-3Y2	0.0	263.0	13.80*
LSE	M-3Y1	0.0	-813.0	0.0
	M-3Y2	-4382	-352.0	0.0
LSO	M-3Y1	-3733	-427.3	0.0
	M-3Y2	-2918	-488.0	0.0

* $R_3=0.7$fm.

Table 2
Single-particle states and energies.

Nuclide	nlj	E(MeV)	Nuclide	nlj	E(MeV)
^{209}Pb	$1g_{9/2}$	-3.94	^{209}Bi	$0h_{9/2}$	-3.77
	$0i_{11/2}$	-3.15		$1f_{7/2}$	-2.87
	$0j_{15/2}$	-2.53		$0i_{13/2}$	-2.16
	$2d_{5/2}$	-2.36		$2p_{3/2}$	-0.95
	$3s_{1/2}$	-1.91			
^{207}Pb	$0i_{13/2}$	9.01	^{207}Tl	$1d_{5/2}$	9.70
	$2p_{3/2}$	8.27		$0h_{11/2}$	9.37
	$1f_{5/2}$	7.95		$1d_{3/2}$	8.38
	$2p_{1/2}$	7.38		$2s_{1/2}$	8.08

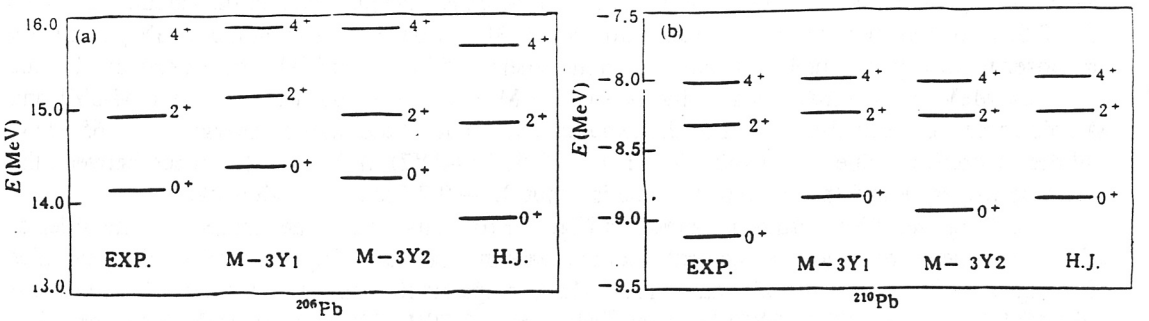

Fig. 2

Nuclear spectra. (a) ^{206}Pb; (b) ^{210}Pb.

In each figure, the first column represents experimental levels; the second and the third columns are the levels calculated in terms of the M-3Y1 and M-3Y2 forces, respectively; the fourth column is the levels calculated by T.T.S. Kuo in terms of the Hamada-Johnston force [3].

in which, $Y(X) = e^{-x}/x$, and S_{12} is the tensor force operator. In the calculations of the matrix elements the Moshinsky transformation [12] is used.

4. RESULTS AND DISCUSSIONS

For nuclei ^{210}Pb, ^{206}Pb, ^{206}Hg, and ^{210}Po, the harmonic oscillator parameter $\hbar\omega = 6.448$ MeV is used. In order to calculate M-3Y force matrix elements, two sets of parameters shown in Table 1 are chosen. They are labeled as M-3Y1 [7] and M-3Y2 [12].

The single-particle levels are chosen as the energy levels of ^{209}Pb, ^{207}Pb, ^{207}Ti, and ^{209}Bi, respectively, as shown in Table 2. The model space consists all configuration in Table 2. The calculation of folded diagram actually is attributed to the calculation of the derivative of the Q-box [10]. Because the M-3Y matrix elements are independent of energy, the energy dependence of the Q-box only appears in the energy denominator. Besides, in our calculations all probable core excitations up to $2\hbar\omega$ are taken into account. Results of calculations for ^{210}Pb and ^{206}Pb are shown in Fig. 2. From Fig. 2, one can see that our theoretical values of low-lying levels from both sets of the

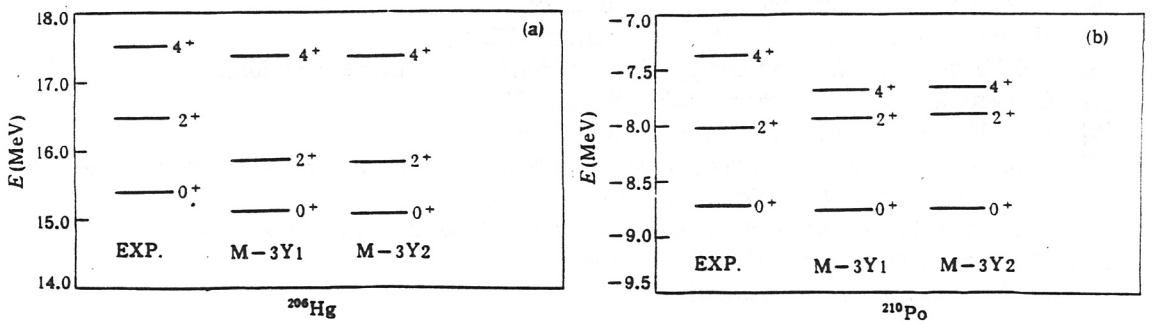

Fig. 3

Nuclear spectra. (a) ^{206}Hg; (b) ^{210}Po.

In each figure, the first column represents experimental levels, the second and the third columns are the levels calculated in terms of the M-3Y1 and M-3Y2 forces, respectively.

M-3Y force parameters are in good agreement with the experimental data and the calculated results of T.T.S. Kuo *et al.* with the Hamada-Johnston force [3], but our results from the M-3Y parameters are closer to the experimental data. For the binding energy of the g.s. of ^{210}Pb, the experimental value is -1.24 MeV and the theoretical value is -0.965 MeV and -1.065 MeV from the M-3Y1 and M-3Y2 force, respectively. For ^{206}Pb, the experimental value of the binding energy is -0.65 MeV, and the theoretical value is -0.442(M-3Y1) or -0.527(M-3Y2) MeV. The difference between the experimental value and the theoretical value is about 0.1—0.2 MeV for both nuclei.

For ^{206}Hg and ^{210}Po, results are shown in Fig. 3. From this figure, one can see that the order of calculating levels are in good agreement with the experimental data. Experimental binding energies of ^{206}Hg and ^{210}Po are -0.65 and -1.17 MeV, respectively, and the theoretical values are -0.94(M-3Y1) or -0.99(M-3Y2) MeV for ^{206}Hg, and -1.20(M-3Y1) or -1.21(M-3Y2) for ^{210}Po, respectively. Theoretical results are in good agreement with the experimental data for the two sets of the M-3Y parameters. However, for the spacings of levels of ^{206}Hg and ^{210}Po, there are some differences between the theoretical results and the experimental data. The theoretical spacing between 4^+ and 2^+ of ^{206}Hg is larger than the spacing between 2^+ and 0^+ and the theoretical spacing between 4^+ and 2^+ of ^{210}Po is smaller than the spacing between 2^+ and 0^+, but spacings between 0^+, 2^+, and 4^+ are nearly equal intervals for both nuclei. The differences between the theoretical results and the experimental data probably come from the coulomb interactions among protons and the higher order terms of the effective interactions.

From above analysis, we come to the conclusions: First, it is feasible to use the M-3Y force equivalent *G*-matrix elements to calculate the low-lying levels of nuclei in the heavy-mass nuclear region instead of the *G*-matrix elements from the realistic force obtained from the free nucleon-nucleon scattering. Second, for nuclei consisting of valence protons (or holes), the coulomb interactions and higher order terms of effective interactions would probably have some influence on the results.

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