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QCD中等时方程的赝标介子解

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摘　要

本文讨论在背景场量子色动力学框架中的等时方程赝标介子解。方程的积分核由微扰和非微扰贡献两部分组成.微扰部分是通常的单胶子交换图的贡献；非微扰部分的贡献来自于最低阶的夸克和胶子凝聚效应。当输入合理的夸克质量参数和耦合常数时，可以得到与实验相符合的赝标介子谱.

关键词　等时方程，真空凝聚，赝标解

1 引　言

量子色动力学理论（QCD）[1] 被认为是描述强相互作用的基本理论．它与弱电统一模型理论一起统称为粒子物理中的标准模型理论．由于 QCD 理论的渐近自由性质，微扰量子色动力学（PQCD）为大量的高动量迁移下的实验数据所证实．然而微扰 QCD 计算不适用于低能区域，更不能给出正确的强子谱．因为在低能区域，非微扰效应不可忽略，且逐渐成为主要的贡献．格点规范理论虽然是一种精确的非微扰方法，但是由于计算机能力的限制还不能较好地获得量子色动力学非微扰结构．目前在重介子谱的研究中取得成功的一种方案是唯象势模型，例如 Cornell 势[2]、Richardson 势[3]和 Martin 势[4]等．然而如何从 QCD 基本相互作用来导出这些唯象势模型以至于进一步对轻介子谱也获得较满意的结果仍然是粒子物理中的重要课题之一．

由 SVZ 提出的 QCD 求和规则方法[5]，通过在算符乘积展开中引进复合算符的真空凝聚值，将 QCD 从微扰区域扩展到了较低能量区域．他们假设 QCD 的物理真空很不同于微扰真空，复合算符的真空平均值不为零，例如夸克凝聚 $\langle\Omega|\psi\bar{\psi}|\Omega\rangle$ 和胶子凝聚 $\langle\Omega|G^2|\Omega\rangle$ 等．人们发现由这些真空凝聚产生的非微扰修正在较低能量区域内是很重要的．

背景场量子色动力学试图在场论框架中重现包括非微扰修正的 QCD 求和规则方法[6,7]．它假定物理真空充满了夸克和胶子背景场，量子夸克和胶子场是围绕背景场的一种量子涨落．因此，在背景场量子色动力学的拉氏函数中不仅有量子场之间的相互作用，而且有量子场与背景场之间的相互作用．背景场的真空平均值给出了 QCD 的非微扰效

应．这就给出一个比较清晰的物理图象和方法去计算强相互作用物理过程．

在先前的几篇文章中，我们已经在背景场量子色动力学的框架里证明了 Bethe-Salpeter 方程仍然成立[8]并在包含最低阶的夸克凝聚和胶子凝聚的积分核情况下求解了 B-S[9,10] 方程．本文试图在背景场 QCD 中讨论等时方程[11]的赝标介子解．等时方程是 Bethe-Salpeter 方程的一种约化形式，它是处于同一时间的两体相对论束缚态方程，其束缚态波函数的物理意义清楚．因此，用这一方程通过同时考虑微扰和非微扰效应的贡献来解出赝标介子谱将是有意义的工作．

2 等时方程相互作用核

背景场 QCD 有效拉氏量 $\mathscr{L}_{\rm eff}$[7] 可以写为：

$$\begin{aligned}\mathscr{L}_{\rm eff} = & \bar{\eta}(i\not{D}(A) - m)\eta + \frac{1}{2}\phi^a_\mu[g^{\mu\nu}D^2_{ac}(A) - \left(1 - \frac{1}{\alpha}\right)(D^\mu(A)D^\nu(A))_{ac} \\ & + 2gf_{abc}G^{\mu\nu}_b]\phi^c_\nu + g(\bar{\psi}\phi^a T^a\eta + \bar{\eta}\phi^a T^a\psi) - g^2 f^{adf} f_{abc} A^\mu_d \phi^\nu_f \phi^b_\mu \phi^c_\nu \\ & - gf^{abc}(\partial_\mu\phi^a_\nu)\phi^\mu_b\phi^\nu_c - \frac{1}{4}g^2 f^{abc} f_{adf}\phi^\mu_b\phi^\nu_c\phi^d_\mu\phi^f_\nu + g\bar{\eta}\phi^a T^a\eta,\end{aligned} \tag{2.1}$$

这里 $A^a_\mu(x)$ 和 $\psi(x)$ 代表胶子和夸克背景场，它们满足运动方程；$\phi^a_\mu(x)$ 和 $\eta(x)$ 表示围绕经典场解的量子涨落．

从(2.1)式可以看出，有效拉氏量 $\mathscr{L}_{\rm eff}$ 既包括了量子场之间的相互作用项，也包括了量子场与背景场之间的相互作用项．相应地，束缚态方程的积分核也应由微扰和非微扰两部分组成．为了简单起见，我们在微扰部分仅考虑单胶子交换（看图 1），在非微扰部分仅考虑夸克凝聚 $\langle\Omega|\psi\bar{\psi}|\Omega\rangle$ 和胶子凝聚 $\langle\Omega|G^2|\Omega\rangle$ 修正(看图 2)．

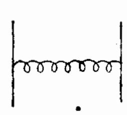

图 1 单胶子交换

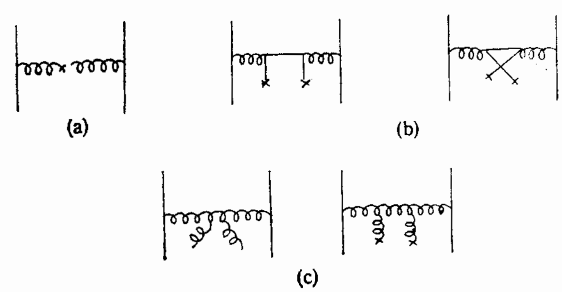

图 2 非微扰修正

(a) 胶子凝聚；(b) 夸克凝聚；(c) 胶子凝聚．

对胶子背景场 $A^a_\mu(x)$，选取固定点规范[12] $x^\mu A^a_\mu(x) = 0$，我们可以得到下列两个展开式[13]，

$$\langle\Omega|A^a_\mu(x)A^b_\nu(y)|\Omega\rangle = \frac{1}{384}x^\lambda y^\rho(g_{\lambda\rho}g_{\mu\nu} - g_{\lambda\nu}g_{\mu\rho})\langle\Omega|G^2|\Omega\rangle + \cdots, \tag{2.2}$$

$$\langle\Omega|\psi^i_\beta(x)\bar{\psi}^j_\alpha(0)|\Omega\rangle = \frac{1}{12}\left\{\delta_{ij} - \frac{1}{4}mx_\mu\gamma^\mu_{ij}\right\}\delta_{\alpha\beta}\langle\Omega|\psi\bar{\psi}|\Omega\rangle + \cdots, \tag{2.3}$$

这两个公式可用来计算 Wilson 算符乘积展开中的非微扰系数．文献[5,14]已给出了夸克和胶子凝聚的值，

$$\langle\Omega|u\bar{u}|\Omega\rangle=\langle\Omega|d\bar{d}|\Omega\rangle=1.2\langle\Omega|s\bar{s}|\Omega\rangle=(250\text{MeV})^3,$$
$$\langle\Omega|c\bar{c}|\Omega\rangle=\langle\Omega|b\bar{b}|\Omega\rangle=\langle\Omega|t\bar{t}|\Omega\rangle=0 \tag{2.4}$$
$$\left\langle\Omega\left|\frac{\alpha_s}{\pi}G^2\right|\Omega\right\rangle=(360\text{MeV})^4.$$

等时方程是 Bethe-Salpeter 方程的三维近似．等时方程的核是 Bethe-Salpeter 方程核的无穷叠代．在零级近似下，我们取等时方程的核等于 Bethe-Salpeter 方程的核，这时在质心系中的等时方程为[11]，

$$\begin{aligned}[P_0-H_1(\boldsymbol{p})+H_2(-\boldsymbol{p})]\phi(\boldsymbol{p})=&\int\frac{d^3\boldsymbol{p}'}{(2\pi)^3}\frac{dp_0}{2\pi}\frac{dp_0'}{2\pi}\\&\cdot[\{\beta\gamma_\mu[\phi(\boldsymbol{p}')\beta S_A(\boldsymbol{p}',p_0'-(1-s)P_0)-S_R(\boldsymbol{p}',p_0'+sP_0)\beta\phi(\boldsymbol{p}')]\\&\cdot\gamma^\mu\beta V(\boldsymbol{p}-\boldsymbol{p}',p_0-p_0')\}\beta S_f(p-(1-s)P_0)\\&-\{S_f(p+sP_0)\gamma_\mu[\phi(\boldsymbol{p}')\beta S_A(\boldsymbol{p}',p_0'-(1-s)P_0)\\&-S_R(\boldsymbol{p}',p_0'+sP_0)\beta\phi(\boldsymbol{p}')]\gamma^\mu V(\boldsymbol{p}-\boldsymbol{p}',p_0-p_0')\}],\end{aligned} \tag{2.5}$$

这里超前传播函数

$$\begin{aligned}\beta S_A(\boldsymbol{p},p_0'-(1-s)P_0)=&-\frac{\Lambda_2^+(-\boldsymbol{p}')}{p_0'-(1-s)P_0-\omega_2'-i\varepsilon}\\&-\frac{\Lambda_2^-(-\boldsymbol{p}')}{p_0'-(1-s)P_0+\omega_2'-i\varepsilon},\end{aligned}$$

推迟传播函数

$$S_R(\boldsymbol{p}',p_0'+sP_0)\beta=-\frac{\Lambda_1^+(\boldsymbol{p}')}{p_0'+sP_0-\omega_1'+i\varepsilon}-\frac{\Lambda_1^-(\boldsymbol{p}')}{p_0'+sP_0+\omega_1'+i\varepsilon},$$

$$s=\frac{m_1}{m_1+m_2},\ \Lambda_i^\pm(\boldsymbol{p}')=\frac{\omega_i'\pm H_i(\boldsymbol{p}')}{2\omega_i'},\ H_i(\boldsymbol{p}')=\boldsymbol{\alpha}\cdot\boldsymbol{p}'+\beta m_i,$$

$\omega_i'=\sqrt{\boldsymbol{p}'^2+m_i^2}$，$m_1$ 和 m_2 分别是组成介子的夸克和反夸克质量，P_0 是介子质量．

利用公式(2.2)和(2.3)推算出图 1 和图 2 对势函数 $V(\boldsymbol{p}-\boldsymbol{p}',p_0-p_0')$的贡献[9]，并完成对 p_0 和 p_0' 的积分，则方程(2.5)成为

$$\begin{aligned}(P_0-H_1(\boldsymbol{p})+H_2(-\boldsymbol{p}))\Phi(\boldsymbol{p})=&\int\frac{d^3\boldsymbol{p}'}{(2\pi)^3}\{\beta\gamma_\mu\Phi(\boldsymbol{p}')[\Lambda_2^+(-\boldsymbol{p}')G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')\\&+\Lambda_2^-(-\boldsymbol{p})G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')]\gamma^\mu\Lambda_2^+(\boldsymbol{p})\beta+\beta\gamma_\mu[\Lambda_1^+(\boldsymbol{p}')G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')\\&+\Lambda_1^-(\boldsymbol{p}')G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')]\Phi(\boldsymbol{p}')\gamma^\mu\Lambda_2^-(\boldsymbol{p})\beta+\Lambda_1^+(\boldsymbol{p})\beta\gamma_\mu\Phi(\boldsymbol{p}')[\Lambda_2^+(-\boldsymbol{p}')\\&\cdot G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\Lambda_2^-(-\boldsymbol{p}')G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')]\gamma^\mu\beta\\&+\Lambda_1^-(\boldsymbol{p})\beta\gamma_\mu[\Lambda_1^+(\boldsymbol{p}')G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\Lambda_1^-(\boldsymbol{p}')G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')]\Phi(\boldsymbol{p}')\gamma^\mu\beta\}\end{aligned} \tag{2.6}$$

其中

$$\begin{aligned}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')=&\frac{4}{3}g^2\frac{1}{2|\boldsymbol{p}-\boldsymbol{p}'|}\frac{1}{|\boldsymbol{p}-\boldsymbol{p}'|+\omega_2-\omega_2'}\\&+\sum_f\frac{g^4\langle\Omega|\phi_f\bar{\phi}_f|\Omega\rangle}{18m_f|\boldsymbol{p}-\boldsymbol{p}'|^2}\left[\frac{1}{(|\boldsymbol{p}-\boldsymbol{p}'|+\omega_2-\omega_2')^2}\right.\\&\left.-\frac{1}{(|\boldsymbol{p}-\boldsymbol{p}'|+\omega_2-\omega_2')|\boldsymbol{p}-\boldsymbol{p}'|}\right]\end{aligned}$$

$$
-\sum_f \frac{g^4\langle\Omega|\phi_f\bar{\phi}_f|\Omega\rangle}{18m_f((\boldsymbol{p}-\boldsymbol{p}')^2+m_f^2)}\left[\frac{1}{(\sqrt{(\boldsymbol{p}-\boldsymbol{p}')^2+m_f^2}+\omega_2-\omega_2')^2}\right.
$$

$$
\left.+\frac{1}{(\sqrt{(\boldsymbol{p}-\boldsymbol{p}')^2+m_f^2}+\omega_2-\omega_2')\sqrt{(\boldsymbol{p}-\boldsymbol{p}')^2+m_f^2}}\right]+\frac{g^4\langle\Omega|G^2|\Omega\rangle}{16|\boldsymbol{p}-\boldsymbol{p}'|^3}
$$

$$
\cdot\left[\frac{1}{(|\boldsymbol{p}-\boldsymbol{p}'|+\omega_2-\omega_2')^3}+\frac{3}{2}\frac{1}{(\omega_2-\omega_2'+|\boldsymbol{p}-\boldsymbol{p}'|)^2|\boldsymbol{p}-\boldsymbol{p}'|}\right.
$$

$$
\left.+\frac{3}{2}\frac{1}{(\omega_2-\omega_2'+|\boldsymbol{p}-\boldsymbol{p}'|)|\boldsymbol{p}-\boldsymbol{p}'|^2}\right],
$$

$$
G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')=\frac{4}{3}g^2\frac{1}{2|\boldsymbol{p}-\boldsymbol{p}'|}\frac{1}{P_0+\omega_2'-\omega_1-|\boldsymbol{p}-\boldsymbol{p}'|}
$$

$$
+\sum_f\frac{g^4\langle\Omega|\phi_f\bar{\phi}_f|\Omega\rangle}{18m_f|\boldsymbol{p}-\boldsymbol{p}'|}\frac{P_0+\omega_2'-\omega_1-2|\boldsymbol{p}-\boldsymbol{p}'|}{(P_0+\omega_2'-\omega_1-|\boldsymbol{p}-\boldsymbol{p}'|)^2|\boldsymbol{p}-\boldsymbol{p}'|}
$$

$$
-\sum_f\frac{g^4\langle\Omega|\phi_f\bar{\phi}_f|\Omega\rangle}{18m_f((\boldsymbol{p}-\boldsymbol{p}')^2+m_f^2)}\frac{P_0+\omega_2'-\omega_1-2\sqrt{(\boldsymbol{p}-\boldsymbol{p}')^2+m_f^2}}{(P_0+\omega_2'-\omega_1-\sqrt{(\boldsymbol{p}-\boldsymbol{p}')^2+m_f^2})^2}
$$

$$
\cdot\frac{1}{\sqrt{(\boldsymbol{p}-\boldsymbol{p}')^2+m_f^2}}+\frac{g^4\langle\Omega|G^2|\Omega\rangle}{16|\boldsymbol{p}-\boldsymbol{p}'|^3}\left[\frac{3}{2}\frac{1}{(P_0+\omega_2'-\omega_1-|\boldsymbol{p}-\boldsymbol{p}'|)^2}\right.
$$

$$
\cdot\frac{1}{|\boldsymbol{p}-\boldsymbol{p}'|^2}+\frac{1}{(P_0+\omega_2'-\omega_1-|\boldsymbol{p}-\boldsymbol{p}'|)^3}
$$

$$
\left.-\frac{3}{2}\frac{1}{(P_0+\omega_2'-\omega_1-|\boldsymbol{p}-\boldsymbol{p}'|)^2|\boldsymbol{p}-\boldsymbol{p}'|}\right], \tag{2.7}
$$

$$
G_{22}^{-+}=G_{22}^{++}(\omega_2'\to-\omega_2'),\quad G_{21}^{-+}=G_{21}^{++}(\omega_2'\to-\omega_2'),
$$

$$
G_{11}^{--}=G_{22}^{++}(\omega_2\to\omega_1,\omega_2'\to\omega_1'),\quad G_{11}^{-+}=G_{22}^{++}(\omega_2\to\omega_1,\omega_2'\to-\omega_1'),
$$

$$
G_{12}^{--}=G_{12}^{++}(\omega_2'\to\omega_1',\omega_1\to\omega_2),\quad G_{12}^{+-}=G_{12}^{--}(\omega_1'\to-\omega_1'),
$$

$$
\omega_i=\sqrt{\boldsymbol{p}^2+m_i^2}\quad i=1,2.
$$

(2.7) 式正是在背景场 QCD 中应求的等时方程．这是一个协变的三维方程．在输入耦合常数、夸克质量和真空凝聚值这些参数以后，此方程对系统质量 P_0 构成本征值问题．但从(2.7)式看出，当 $\boldsymbol{p}=\boldsymbol{p}'$ 时，$G_{22}^{++},\cdots,G_{12}^{+-}$ 存在红外发散，在数值积分中这是无法回避的问题．所以在实际的计算中，我们用一个阻尼因子放在每一个发散项分母上，即作替换 $|\boldsymbol{p}-\boldsymbol{p}'|\to\sqrt{(\boldsymbol{p}-\boldsymbol{p}')^2+\mu^2}$，对于 μ^2，我们应取得尽可能小，以至于方程求解结果对它不敏感．

3 方程组和数值解

对于赝标介子，等时波函数的一般形式是

$$
\Phi(\boldsymbol{p})=\gamma_5\Phi_1(\boldsymbol{p})+\gamma_4\gamma_5\Phi_2(\boldsymbol{p})+\frac{\gamma_4\boldsymbol{\gamma}\cdot\boldsymbol{p}}{m_1+m_2}\Phi_3(\boldsymbol{p})+\frac{\boldsymbol{\gamma}\cdot\boldsymbol{p}}{m_1+m_2}\Phi_4(\boldsymbol{p}), \tag{3.1}
$$

这里 $\Phi_i(\boldsymbol{p})$ $(i=1,2,3,4)$ 是 $\boldsymbol{p}$ 的偶函数．

将(3.1)式代入(2.6)式，我们得到波函数四个分量满足的联立方程，

$$P_0\Phi_1(\boldsymbol{p})-(m_1+m_2)\Phi_2(\boldsymbol{p})-\frac{2\boldsymbol{p}^2}{m_1+m_2}\Phi_3(\boldsymbol{p})$$

$$=\int\frac{d^3\boldsymbol{p}'}{(2\pi)^3}\Big\{\Phi_1(\boldsymbol{p}')\Big[\Big(1-\frac{m_2^2}{2\omega_2\omega_2'}\Big)G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\Big(1+\frac{m_2^2}{2\omega_2\omega_2'}\Big)G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\Big(1-\frac{m_1m_2}{2\omega_1'\omega_2}\Big)G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\Big(1+\frac{m_1m_2}{2\omega_1'\omega_2}\Big)G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\Big(1+\frac{m_1m_2}{2\omega_1'\omega_2}\Big)G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\Big(1-\frac{m_1m_2}{2\omega_1'\omega_2}\Big)G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\Big(1+\frac{m_1^2}{2\omega_1'\omega_1}\Big)G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\Big(1-\frac{m_1^2}{2\omega_1'\omega_1}\Big)G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]$$

$$+\Phi_2(\boldsymbol{p}')\Big[\Big(\frac{m_2}{2\omega_2}-\frac{m_2}{\omega_2'}\Big)G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\Big(\frac{m_2}{2\omega_2}+\frac{m_2}{\omega_2'}\Big)G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\Big(\frac{m_1}{\omega_1'}-\frac{m_2}{2\omega_2}\Big)G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\Big(-\frac{m_1}{\omega_1'}-\frac{m_2}{2\omega_2}\Big)G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\Big(-\frac{m_2}{\omega_2'}-\frac{m_1}{2\omega_1}\Big)G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\Big(\frac{m_2}{\omega_2'}-\frac{m_1}{2\omega_1}\Big)G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\Big(\frac{m_1}{\omega_1'}+\frac{m_1}{2\omega_1}\Big)G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\Big(\frac{m_1}{2\omega_1}-\frac{m_1}{\omega_1'}\Big)G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big] \qquad (3.2a)$$

$$+\Phi_3(\boldsymbol{p}')\Big[-\frac{\boldsymbol{p}'^2}{(m_1+m_2)\omega_2'}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\frac{\boldsymbol{p}'^2}{(m_1+m_2)\omega_2'}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{\boldsymbol{p}'^2}{\omega_1'(m_1+m_2)}G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')-\frac{\boldsymbol{p}'^2}{\omega_1'(m_1+m_2)}G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')$$

$$-\frac{\boldsymbol{p}'^2}{\omega_2'(m_1+m_2)}G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\frac{\boldsymbol{p}'^2}{\omega_2'(m_1+m_2)}G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{\boldsymbol{p}'^2}{\omega_1'(m_1+m_2)}G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')-\frac{\boldsymbol{p}'^2}{\omega_1'(m_1+m_2)}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]$$

$$+\Phi_4(\boldsymbol{p}')\Big[-\frac{\boldsymbol{p}'^2m_2}{2(m_1+m_2)\omega_2\omega_2'}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{\boldsymbol{p}'^2m_2}{2(m_1+m_2)\omega_2\omega_2'}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_2\boldsymbol{p}'^2}{2(m_1+m_2)\omega_1'\omega_2}G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$-\frac{m_2\boldsymbol{p}'^2}{2(m_1+m_2)\omega_1'\omega_2}G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_1\boldsymbol{p}'^2}{2(m_1+m_2)\omega_1\omega_2'}G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')$$

$$-\frac{m_1\boldsymbol{p}'^2}{2(m_1+m_2)\omega_1\omega_2'}G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')-\frac{m_1\boldsymbol{p}'^2}{2(m_1+m_2)\omega_1\omega_1'}G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{m_1\boldsymbol{p}'^2}{2(m_1+m_2)\omega_1\omega_1'}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]\Big\}$$

$$(-m_1-m_2)\Phi_1(\boldsymbol{p})+P_0\Phi_2(\boldsymbol{p})$$

$$=\int\frac{d^3\boldsymbol{p}'}{(2\pi)^3}\Big\{\Phi_1(\boldsymbol{p}')\Big[\Big(-\frac{m_2}{\omega_2}+\frac{m_2}{2\omega_2'}\Big)G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\left(-\frac{m_2}{\omega_2}-\frac{m_2}{2\omega_2'}\right)G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')+\left(\frac{m_2}{\omega_2}-\frac{m_1}{2\omega_1'}\right)G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\left(\frac{m_2}{\omega_2}+\frac{m_1}{2\omega_1'}\right)G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')+\left(\frac{m_1}{\omega_1}+\frac{m_2}{2\omega_2'}\right)G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\left(\frac{m_1}{\omega_1}-\frac{m_2}{2\omega_2}\right)G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')+\left(-\frac{m_1}{\omega_1}-\frac{m_1}{2\omega_1'}\right)G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\left(\frac{m_1}{2\omega_1'}-\frac{m_1}{\omega_1}\right)G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]+\Phi_2(\boldsymbol{p}')\Big[\frac{2m_2^2+\boldsymbol{p}\cdot\boldsymbol{p}'-\omega_2\omega_2'}{2\omega_2\omega_2'}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{-\omega_2\omega_2'-\boldsymbol{p}\cdot\boldsymbol{p}'-2m_2^2}{2\omega_2\omega_2'}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')+\frac{2m_1m_2-\omega_1'\omega_2-\boldsymbol{p}\cdot\boldsymbol{p}'}{2\omega_1'\omega_2}$$

$$\cdot G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\frac{\boldsymbol{p}\cdot\boldsymbol{p}'-\omega_1'\omega_2-2m_1m_2}{2\omega_1'\omega_2}G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{\boldsymbol{p}\cdot\boldsymbol{p}'-\omega_1\omega_2'-2m_1m_2}{2\omega_1\omega_2'}G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\frac{2m_1m_2-\omega_1\omega_2'-\boldsymbol{p}\cdot\boldsymbol{p}'}{2\omega_1\omega_2'}$$

$$\cdot G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')+\frac{-\omega_1\omega_1'-\boldsymbol{p}\cdot\boldsymbol{p}'-2m_1^2}{2\omega_1\omega_1'}G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{2m_1^2+\boldsymbol{p}\cdot\boldsymbol{p}'-\omega_1\omega_1'}{2\omega_1\omega_1'}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]+\Phi_3(\boldsymbol{p}')\Big[\frac{2m_2\boldsymbol{p}'^2-m_2\boldsymbol{p}\cdot\boldsymbol{p}'}{2(m_1+m_2)\omega_2\omega_2'}$$

$$\cdot G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_2\boldsymbol{p}\cdot\boldsymbol{p}'-2m_2\boldsymbol{p}'^2}{2\omega_2(m_1+m_2)\omega_2'}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}') \tag{3.2b}$$

$$+\frac{2m_2\boldsymbol{p}'^2+m_1\boldsymbol{p}\cdot\boldsymbol{p}'}{2(m_1+m_2)\omega_1'\omega_2}G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\frac{-2m_2\boldsymbol{p}'^2-m_1\boldsymbol{p}\cdot\boldsymbol{p}'}{2(m_1+m_2)\omega_1'\omega_2}$$

$$\cdot G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')+\frac{-2m_1\boldsymbol{p}'^2-m_2\boldsymbol{p}\cdot\boldsymbol{p}'}{2(m_1+m_2)\omega_1\omega_2'}G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{m_2\boldsymbol{p}\cdot\boldsymbol{p}'+2m_1\boldsymbol{p}'^2}{2\omega_1\omega_2'(m_1+m_2)}G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_1\boldsymbol{p}\cdot\boldsymbol{p}'-2m_1\boldsymbol{p}'^2}{2\omega_1\omega_1'(m_1+m_2)}G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{2m_1\boldsymbol{p}'^2-m_1\boldsymbol{p}\cdot\boldsymbol{p}'}{2\omega_1\omega_1'(m_1+m_2)}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]$$

$$+\Phi_4(\boldsymbol{p}')\Big[\frac{\omega_2\boldsymbol{p}'^2-\omega_2'\boldsymbol{p}\cdot\boldsymbol{p}'}{2\omega_2\omega_2'(m_1+m_2)}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')-\frac{\omega_2'\boldsymbol{p}\cdot\boldsymbol{p}'+\omega_2\boldsymbol{p}'^2}{2\omega_2\omega_2'(m_1+m_2)}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{\omega_1'\boldsymbol{p}'\cdot\boldsymbol{p}+\boldsymbol{p}'^2\omega_2}{2\omega_1'\omega_2(m_1+m_2)}G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\frac{\omega_1'\omega_2\boldsymbol{p}'\cdot\boldsymbol{p}-\boldsymbol{p}'^2\boldsymbol{p}^2}{2\boldsymbol{p}^2\omega_1'\omega_2}G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{\omega_1\omega_2'\boldsymbol{p}\cdot\boldsymbol{p}'-\boldsymbol{p}^2\boldsymbol{p}'^2}{2\boldsymbol{p}^2\omega_1\omega_2'}G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\frac{\omega_1\omega_1'\boldsymbol{p}\cdot\boldsymbol{p}'+\boldsymbol{p}^2\boldsymbol{p}'^2}{2\omega_1\omega_1'\boldsymbol{p}^2}G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{\omega_1\omega_1'\boldsymbol{p}\cdot\boldsymbol{p}'-\boldsymbol{p}^2\boldsymbol{p}'^2}{2\omega_1\omega_1'\boldsymbol{p}^2}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]\Big\}$$

$$-(m_1+m_2)\Phi_1(\boldsymbol{p})+P_0\Phi_3(\boldsymbol{p})+(m_2-m_1)\Phi_4(\boldsymbol{p})$$

$$=\int\frac{d^3\boldsymbol{p}'}{(2\pi)^3}\Big\{\Phi_1(\boldsymbol{p}')\Big[-\frac{m_1+m_2}{\omega_2}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')-\frac{m_1+m_2}{\omega_2}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{m_1+m_2}{\omega_2}G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_1+m_2}{\omega_2}G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{m_1+m_2}{\omega_1}G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_1+m_2}{\omega_1}G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$-\frac{m_1+m_2}{\omega_1}G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')-\frac{m_1+m_2}{\omega_1}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]$$

$$+\Phi_2(\boldsymbol{p}')\Big[\frac{(m_1+m_2)m_2(2\boldsymbol{p}^2-\boldsymbol{p}\cdot\boldsymbol{p}')}{2\omega_2\omega_2'\boldsymbol{p}^2}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{(m_1+m_2)m_2(\boldsymbol{p}\cdot\boldsymbol{p}'-2\boldsymbol{p}^2)}{2\omega_2\omega_2'\boldsymbol{p}^2}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{(m_1+m_2)(m_2\boldsymbol{p}\cdot\boldsymbol{p}'+2m_1\boldsymbol{p}^2)}{2\omega_1'\omega_2\boldsymbol{p}^2}G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$-\frac{(m_1+m_2)(2m_1\boldsymbol{p}^2+m_2\boldsymbol{p}\cdot\boldsymbol{p}')}{2\omega_1'\omega_2\boldsymbol{p}^2}G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')$$

$$-\frac{(m_1+m_2)(2m_2\boldsymbol{p}^2+m_1\boldsymbol{p}\cdot\boldsymbol{p}')}{2\omega_1\omega_2'\boldsymbol{p}^2}G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{(m_1+m_2)(m_1\boldsymbol{p}\cdot\boldsymbol{p}'+2m_2\boldsymbol{p}^2)}{2\omega_1\omega_2'\boldsymbol{p}^2}G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}') \tag{3.2c}$$

$$+\frac{(m_1+m_2)m_1(\boldsymbol{p}\cdot\boldsymbol{p}'-2\boldsymbol{p}^2)}{2\omega_1\omega_1'\boldsymbol{p}^2}G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{(m_1+m_2)m_1(2\boldsymbol{p}^2-\boldsymbol{p}\cdot\boldsymbol{p}')}{2\omega_1\omega_1'\boldsymbol{p}^2}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]$$

$$+\Phi_3(\boldsymbol{p}')\Big[\frac{2\boldsymbol{p}'^2\boldsymbol{p}^2+m_2^2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_2'\omega_2}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')-\frac{2\boldsymbol{p}^2\boldsymbol{p}'^2+m_2^2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_2\omega_2'}$$

$$\cdot G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')+\frac{2\boldsymbol{p}'^2\boldsymbol{p}^2-m_1m_2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_1'\omega_2}G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{m_1m_2\boldsymbol{p}\cdot\boldsymbol{p}'-2\boldsymbol{p}^2\boldsymbol{p}'^2}{2\boldsymbol{p}^2\omega_1'\omega_2}G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_1m_2\boldsymbol{p}\cdot\boldsymbol{p}'-2\boldsymbol{p}^2\boldsymbol{p}'^2}{2\boldsymbol{p}^2\omega_1\omega_2'}$$

$$\cdot G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\frac{2\boldsymbol{p}'^2\boldsymbol{p}^2-m_1m_2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_1\omega_2'}G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{\omega_1'\boldsymbol{p}\cdot\boldsymbol{p}'-\omega_2\boldsymbol{p}'^2}{2\omega_1'\omega_2(m_1+m_2)}G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')+\frac{\omega_1\boldsymbol{p}'^2-\omega_2'\boldsymbol{p}\cdot\boldsymbol{p}'}{2\omega_1\omega_2'(m_1+m_2)}G_{21}^{+-}(\boldsymbol{p}-\boldsymbol{p}')$$

$$+\frac{-\omega_1\boldsymbol{p}'^2-\omega_2'\boldsymbol{p}\cdot\boldsymbol{p}'}{2\omega_1\omega_2'(m_1+m_2)}G_{21}^{--}(\boldsymbol{p}-\boldsymbol{p}')+\frac{\omega_1\boldsymbol{p}'^2+\omega_1'\boldsymbol{p}\cdot\boldsymbol{p}'}{2\omega_1\omega_1'(m_1+m_2)}$$

$$\cdot G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\frac{\omega_1'\boldsymbol{p}\cdot\boldsymbol{p}'-\omega_1\boldsymbol{p}'^2}{2\omega_1\omega_1'(m_1+m_2)}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]\Big\}$$

$$(m_2-m_1)\Phi_3(\boldsymbol{p})+P_0\Phi_4(\boldsymbol{p})$$

$$=\int\frac{\mathrm{d}^3\boldsymbol{p}'}{(2\pi)^3}\Big\{\Phi_1(\boldsymbol{p}')\Big[-\frac{m_1+m_2}{2\omega_2\omega_2'}m_2G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_1+m_2}{2\omega_2\omega_2'}m_2G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')$$

$$
\begin{aligned}
&-\frac{m_1+m_2}{2\omega_1'\omega_2}m_1G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_1+m_2}{2\omega_1'\omega_2}m_1G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')\\
&-\frac{m_1+m_2}{2\omega_1\omega_2'}m_2G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_1+m_2}{2\omega_1\omega_2'}m_2G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')\\
&-\frac{m_1+m_2}{2\omega_1\omega_1'}m_1G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_1+m_2}{2\omega_1\omega_1'}m_1G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]\\
&+\Phi_2(\boldsymbol{p}')\Big[\frac{(m_1+m_2)(\omega_2'\boldsymbol{p}^2-\omega_2\boldsymbol{p}\cdot\boldsymbol{p}')}{2\boldsymbol{p}^2\omega_2\omega_2'}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')\\
&+\frac{(m_1+m_2)(\omega_2'\boldsymbol{p}^2+\omega_2\boldsymbol{p}\cdot\boldsymbol{p}')}{2\boldsymbol{p}^2\omega_2\omega_2'}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')\\
&+\frac{(-m_1-m_2)(\boldsymbol{p}^2\omega_1'+\boldsymbol{p}\cdot\boldsymbol{p}'\omega_2)}{2\boldsymbol{p}^2\omega_1'\omega_2}G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')\\
&+\frac{(m_1+m_2)(\omega_2\boldsymbol{p}\cdot\boldsymbol{p}'-\omega_1'\boldsymbol{p}^2)}{2\boldsymbol{p}^2\omega_1'\omega_2}G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')\\
&+\frac{(-m_1-m_2)(\omega_1\boldsymbol{p}\cdot\boldsymbol{p}'-\omega_2'\boldsymbol{p}^2)}{2\boldsymbol{p}^2\omega_1\omega_2'}G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')\\
&+\frac{(m_1+m_2)(\omega_1\boldsymbol{p}\cdot\boldsymbol{p}'+\omega_2'\boldsymbol{p}^2)}{2\boldsymbol{p}^2\omega_1\omega_2'}G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}') \qquad (3.2\mathrm{d})\\
&+\frac{(-m_1-m_2)(\omega_1\boldsymbol{p}\cdot\boldsymbol{p}'+\omega_1'\boldsymbol{p}^2)}{2\boldsymbol{p}^2\omega_1\omega_1'}G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')\\
&+\frac{(m_1+m_2)(\omega_1\boldsymbol{p}\cdot\boldsymbol{p}'-\omega_1'\boldsymbol{p}^2)}{2\boldsymbol{p}^2\omega_1\omega_1'}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]\\
&+\Phi_3(\boldsymbol{p}')\Big[\frac{m_2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_2'}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')-\frac{m_2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_2'}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')\\
&+\frac{m_1\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_1'}G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')-\frac{m_1\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_1'}G_{12}^{--}(\boldsymbol{p}-\boldsymbol{p}')\\
&+\frac{m_2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_2'}G_{21}^{++}(\boldsymbol{p}-\boldsymbol{p}')-\frac{m_2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_2'}G_{21}^{-+}(\boldsymbol{p}-\boldsymbol{p}')\\
&+\frac{m_1\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_1'}G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')-\frac{m_1\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_1'}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]\\
&+\Phi_4(\boldsymbol{p}')\Big[\frac{\omega_2\omega_2'\boldsymbol{p}\cdot\boldsymbol{p}'-\boldsymbol{p}'^2\boldsymbol{p}^2}{2\boldsymbol{p}^2\omega_2'\omega_2}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')\\
&+\frac{\boldsymbol{p}\cdot\boldsymbol{p}'\omega_2\omega_2'+\boldsymbol{p}^2\boldsymbol{p}'^2}{2\boldsymbol{p}^2\omega_2'\omega_2}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')+\frac{\omega_1'\omega_2\boldsymbol{p}\cdot\boldsymbol{p}'+\boldsymbol{p}^2\boldsymbol{p}'^2}{2\boldsymbol{p}^2\omega_1'\omega_2}G_{12}^{+-}(\boldsymbol{p}-\boldsymbol{p}')\\
&-\frac{2\boldsymbol{p}^2\boldsymbol{p}'^2+m_1^2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_1\omega_1'}G_{11}^{+-}(\boldsymbol{p}-\boldsymbol{p}')+\frac{2\boldsymbol{p}^2\boldsymbol{p}'^2+m_1^2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_1\omega_1'}G_{11}^{--}(\boldsymbol{p}-\boldsymbol{p}')\Big]\\
&+\Phi_4(\boldsymbol{p}')\Big[\frac{m_2\boldsymbol{p}'\cdot\boldsymbol{p}}{2\boldsymbol{p}^2\omega_2}G_{22}^{++}(\boldsymbol{p}-\boldsymbol{p}')+\frac{m_2\boldsymbol{p}\cdot\boldsymbol{p}'}{2\boldsymbol{p}^2\omega_2}G_{22}^{-+}(\boldsymbol{p}-\boldsymbol{p}')
\end{aligned}
$$

$$- \frac{m_2 \boldsymbol{p} \cdot \boldsymbol{p}'}{2\boldsymbol{p}^2 \omega_2} G_{12}^{+-}(\boldsymbol{p} - \boldsymbol{p}') - \frac{m_2 \boldsymbol{p} \cdot \boldsymbol{p}'}{2\boldsymbol{p}^2 \omega_2} G_{12}^{--}(\boldsymbol{p} - \boldsymbol{p}')$$

$$+ \frac{m_1 \boldsymbol{p} \cdot \boldsymbol{p}'}{2\omega_1 \boldsymbol{p}^2} G_{21}^{++}(\boldsymbol{p} - \boldsymbol{p}') + \frac{m_1 \boldsymbol{p} \cdot \boldsymbol{p}'}{2\boldsymbol{p}^2 \omega_1} G_{21}^{-+}(\boldsymbol{p} - \boldsymbol{p}')$$

$$- \frac{m_1 \boldsymbol{p}' \cdot \boldsymbol{p}}{2\boldsymbol{p}^2 \omega_1} G_{11}^{+-}(\boldsymbol{p} - \boldsymbol{p}') - \frac{m_1 \boldsymbol{p}' \cdot \boldsymbol{p}}{2\boldsymbol{p}^2 \omega_1} G_{11}^{--}(\boldsymbol{p} - \boldsymbol{p}') \Big] \Big\}$$

方程(3.2)对 m_1 和 m_2 有交换对称性，这意味着同位旋对称性，即介子和反介子有相同质量.

最后，采用参数 $\alpha_s = 0.35$，$m_u = 360\text{MeV}$，$m_d = 360\text{MeV}$，$m_s = 535\text{MeV}$，$m_c = 1676\text{MeV}$，$m_b = 5140\text{MeV}$，$\mu = 0.08\text{GeV}$，求出方程(3.2)的数值解，结果放在表 1 中.

表 1 赝标介子谱计算值与实验值的比较(MeV)

	π	K	D	D_s	η_c	B	B_s	η_b
计算值	139	496	1860	2100	2960	5270	5380	9410
实验值	139.5679 ±0.0007	493.646 ±0.009	1869.3 ±0.5	1968.8 ±0.7	2978.8 ±1.7	5278.6 ±2.0	/	/

从上表可以看出，由等时方程所获得的数值结果与实验数据是基本一致的.

4 结 论

至此，我们在背景场量子色动力学框架中利用等时方程求解了基态赝标介子谱. 众所周知，如果在梯形近似下只计及单胶子交换的微扰 QCD 相互作用势是不能得到与实验相符合的强子谱(参见文献[16])，这是因为在低能大距离区域内 QCD 物理真空所产生的真空凝聚效应将起主导作用，不可忽略. 然而目前尚未有一套计及全部非微扰物理效应的有效方法去求解强子谱. 本文尝试考虑最低级真空凝聚效应对相互作用势的贡献来考察对赝标介子谱的影响，特别是求解中计及了从轻夸克到重夸克范围内的介子谱获得了比较满意的结果，这是十分令人鼓舞的. 这表明只要我们在相互作用势中包含了非微扰物理效应的贡献就有可能将检验量子色动力学理论的研究推广到较低能量区域以至于推进对强子结构动力学理论的深入理解.

我们曾利用上述相互作用积分核近似下求解了 Bethe-Salpeter 方程以及 B-S 方程的变形形式 Salpeter 方程和等时方程. 这些方程的解是相互自洽的.这不仅表明求解结果的可靠性而且从不同的角度论证了上述结论.

然而，如果认真地分析上述近似下的相互作用积分核就不难发现，通常微扰单胶子交换图(见图 1)的贡献给出的是库仑势，夸克凝聚和胶子凝聚(量纲为 4) 图(见图 2)的贡献给出的相互作用势不仅有线性势，还有立方势. 由此给出的总贡献是[9,10,17]

$$-\frac{\alpha}{r} + pr - qr^3,$$

特别注意的是由胶子凝聚贡献的立方势与线性势的符号相反，在大距离下失去了禁闭势的性质. 实际上在 $r < 0.8\text{fm}$ 以内，立方势的影响很小，因而对赝标介子谱的影响也很

小. 这样就可以理解为什么我们能在此近似下获得合理的结果. 如果要想寻找大于 1fm 的物理,人们必须考虑高维算符凝聚的物理效应,它们将给出高幂次的势行为 $r^{2n+1}(n \geq 1)$. 我们将在另一篇文章中详细讨论(见文献[18]).

感谢何祚庥、庆承瑞两位教授将等时方程介绍给我们,我们也感谢他们的热情帮助和有益的讨论.

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Pseudo-Scalar Meson Solutions of the Equal-Time Equation IN QCD

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Abstract

We present the pseudo-scalar solutions of the equal-time equation for the quark-antiquark bound states in QCD in the background fields. The kernel includes both of perturbative and nonpertubative interactions. The perturbative part is usual one gluon exchange, nonperturbative part comes from the contributions of the lowest order quark and gluon condensates. With the reasonable parameters, we obtain the mass spectrum in good agreement with the data.

Key words Equal-time equation, Vacuum condensate, Pseudo-scalar solution.