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Monte Carlo Investigation of the Fractal Structure of Multiparticle Production in High Energy Collisions

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Fluctuations of the produced particles in the final states of high energy collisions may be the breakthrough in the investigation of multiparticle dynamics. How to eliminate the statistical noise of the system is one of the most important problems. In this paper, the influence of the statistical noise on fractal structure is analyzed and a method for eliminating the statistical noise is proposed by using the Monte Carlo simulation based on a simple dynamical model.

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1. INTRODUCTION

Since the 1970's, in the investigation on the complicated multiparticle production in high energy collisions, the emphasizes have usually been placed on the average features of the studied system, such as the parametric representation of multiplicity distributions, the Koba-Nielson-Olesen (KNO) scaling and its violation, the single-particle rapidity distribution and the correlations between the rapidity of two particles and the forward-backward multiplicity correlation. Many phenomenological models have been suggested to interpret these average features [1]. In recent years, a large number of events with high dense "spikes" in rapidity space have been obtained from the experiments of multiparticle production at high energies. For instance, several events with local fluctuation of rapidity density up to $dn/d\eta \approx 300$ were obtained in 1983 by JACEE collaboration [2]. NA22 collaboration [3] in 1987 found many events with density up to $dn/d\eta = 100$. Another collaboration, the EMU01, also saw some events with $dn/d\eta = 140$ in 1988 [4]. Such large fluctuation of the local

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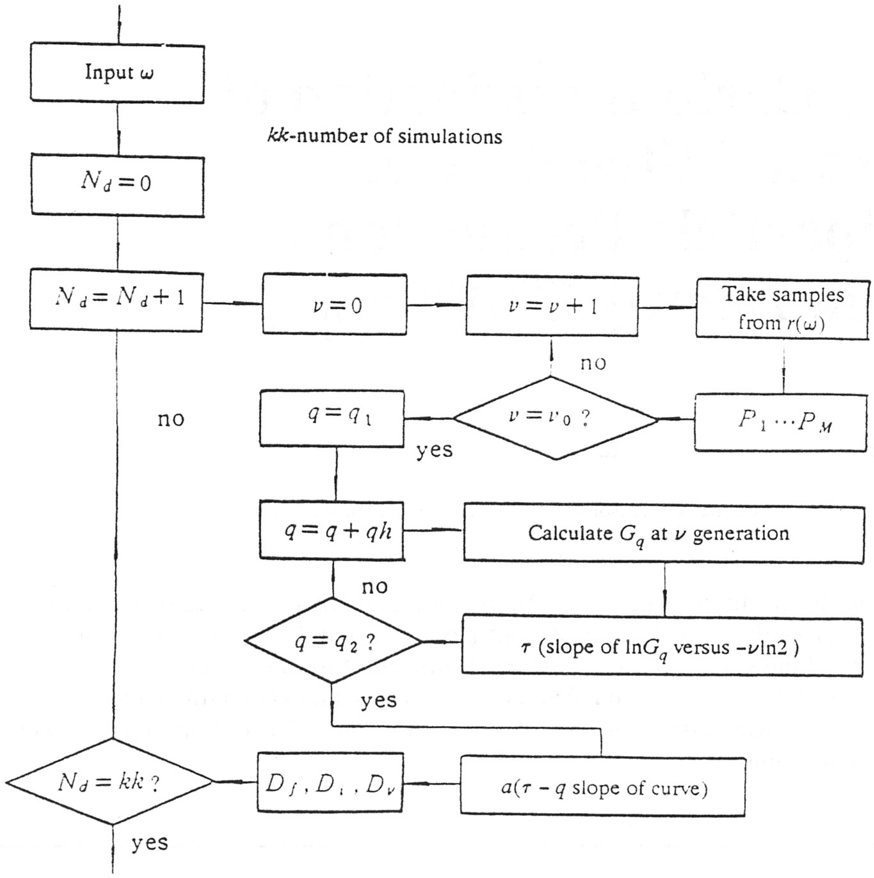

Fig. 1
Flow chart of MC simulation for a pure dynamical system.

density forced people to think that there may exist not only statistical noises but also dynamical fluctuations in the system.

Bialas and Peschanski [5] suggested a method which can erase the statistical fluctuation of a system and might be used in the investigation of the dynamical fluctuation. For a system with dynamical fluctuation, the factorial moments have anomalous scales

$$\langle F_i \rangle \propto \delta^{-\phi_i} \quad i \geq 2, \tag{1}$$

where δ is the size of rapidity sub-interval and ϕ_i denotes the ith intermittent index. Such anomalous scale means that the system has fractal property. The fractal dimensions (i.e., generalized dimensions) are

$$d_i = 1 - \frac{\phi_i}{i-1}. \tag{2}$$

This gives rise to extensive interests in studying the fractal structure of multiparticle production. The factorial moments (F-moments) have the advantage of being able to erase statistical noise and recover the dynamical fractalities. But by means of F-moments one can only obtain the dimension

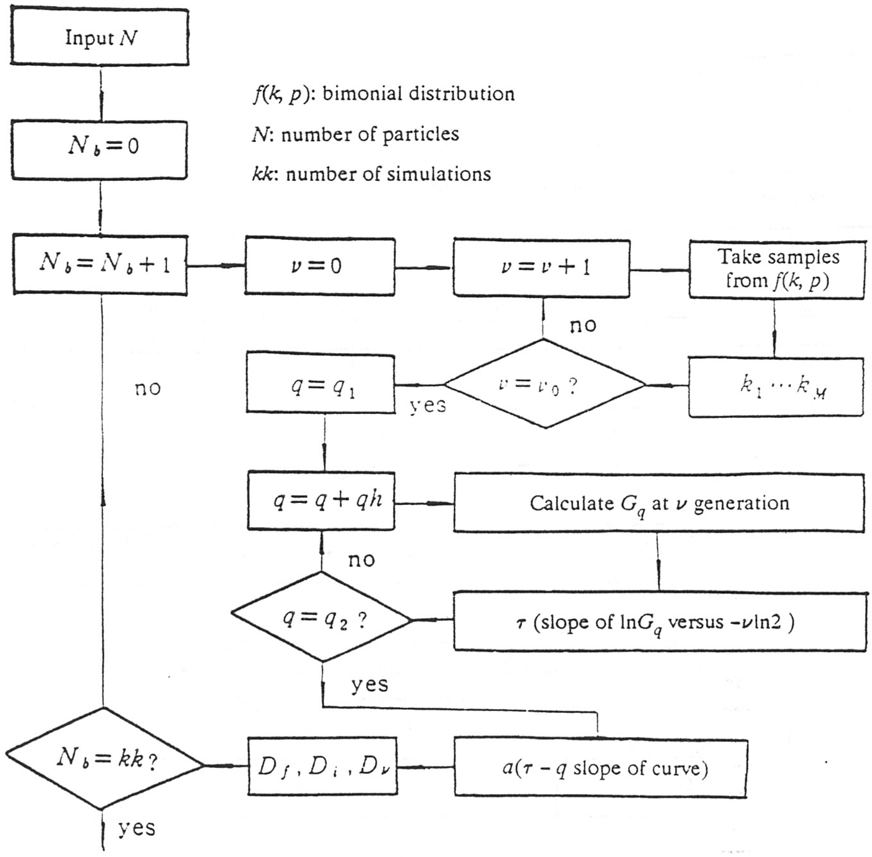

Fig. 2
Flow chart of MC simulation for
a pure statistical noise system.

for $i > 1$ but not for $i \leq 1$. In particular, using F-moments, one cannot obtain the most important two dimensions, i.e., the Hausdorff dimension d_0 and the information dimension d_1. Therefore, there exists a serious limitation of F-moments in investigating fractal property of density fluctuation.

In 1989, Hwa [6] introduced G-moments to deal with the multifractal properties of system. By using this method, one can obtain all kinds of dimensions, but unfortunately it cannot eliminate the statistical noise of the system.

It is necessary to investigate the influence of statistical noise on the fractal dimensions in order to find an effective method, in order to eliminate the statistical noise and analyze all kinds of dimensions. In this paper, we try to make an initial exploration in this field. We calculate three kinds of dimensions (Hausdorff dimension, information dimension and correlation dimension) event by event and investigate their distributions under different conditions. We discuss the influence of statistical noise on the fractal structure and propose a method to eliminate the statistical noise.

2. MODEL

Divide the observed rapidity region ΔY into M bins with size $\Delta Y/M$ and define G-moments as

$$G_q = \sum_{j=1}^{M}{}' P_j^q , \tag{3}$$

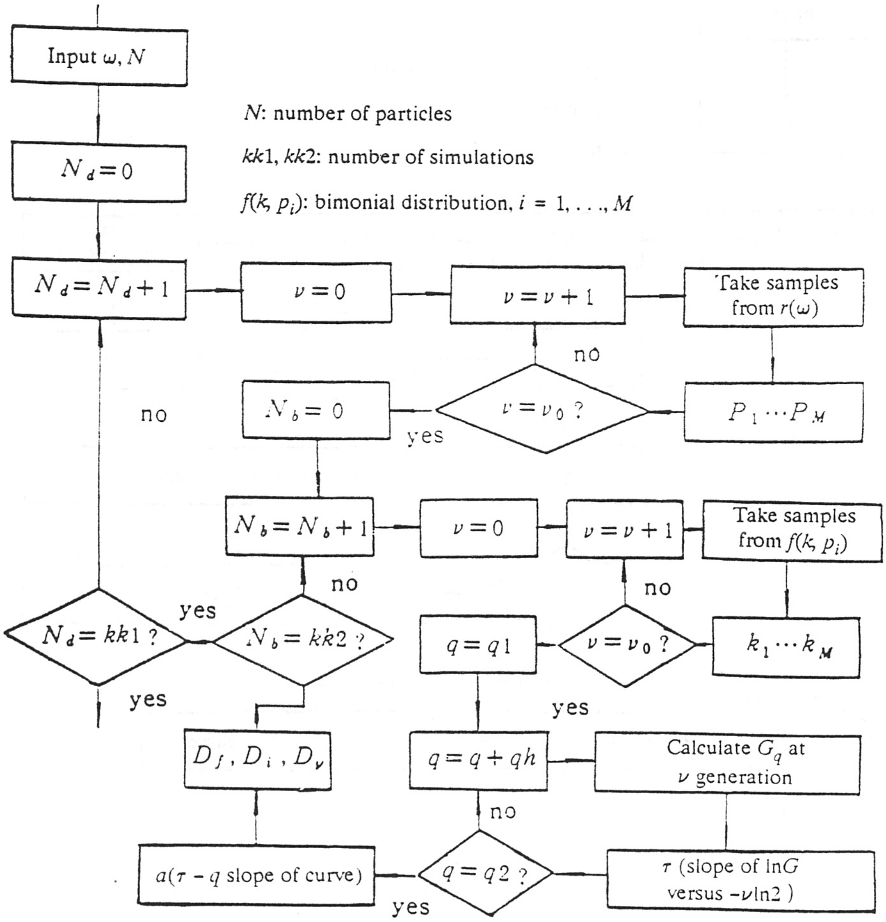

Fig. 3
Flow chart of MC simulation for a mixed system.

where

$$P_i = \frac{k_i}{N}, \tag{4}$$

q is a real number, the summation with prime is over all non-empty bins, N is the total multiplicity and k_j is the number of particles appeared in the jth bin. If a system has fractal structure, its G-moments might exhibit the following anomalous scale

$$G_q \propto \delta^{\tau(q)}, \tag{5}$$

with

$$\tau(q) = d_q(q-1), \tag{6}$$

where d_q denotes the generalized dimension, which corresponds to Hausdorff dimension (D_f), information dimension (D_i) or correlation dimension (D_ν) for q = 0, 1 or 2, respectively.

But P_j, defined in Eq.(4), possesses the meaning of probability only under the condition of $N \to \infty$. If N is finite, it does not correspond solely to dynamical probability but include also the

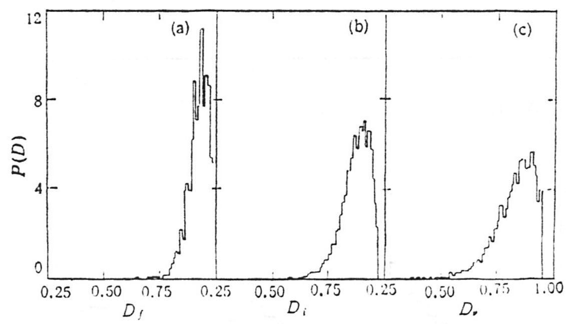

Fig. 4
Distributions of three kinds of dimensions for a pure dynamical system. (a) Hausdorff dimension; (b) information dimension; (c) correlative dimension.

statistical noise. Therefore, the fractal structure obtained from G-moments is not the real dynamical fractal structure and is discounted by the statistical fluctuation.

Let us adopt a simple model-α model, a random cascading model with self-similarity, to study the influence of statistical noise on dynamical fractal structure. The main idea of this model is that at the first level we divide the total rapidity space ΔY into λ sub-regions, then at the second level we divide each of the λ sub-regions again into next sub-regions and so forth. After ν levels we finally obtain M last sub-regions with size of δy

$$M = \frac{\Delta Y}{\delta y} = \lambda^{\nu}. \tag{7}$$

If $\lambda = 2$, the sub-regions at different levels are δ_1, δ_2; δ_{11}, δ_{12}, δ_{21}, δ_{22}, which correspond to probabilities

$$\begin{aligned} p_1 &= w_1, \qquad p_2 = 1 - w_1, \\ p_{11} &= w_{11} p_1, \; p_{12} = (1 - w_{11}) p_1; \\ p_{21} &= w_{21} p_2, \; p_2 = (1 - w_{21}) p_2. \end{aligned} \tag{8}$$

where w_j ($j = 1$; 11, 21) are dividing-probabilities of sub-regions. After ν levels of division, the probability of the mth bin (sub-region) is

$$P_m = w_1 w_2 \cdots w_{\nu}. \tag{9}$$

Assuming that α model describes the dynamical property of the system, we obtain the dynamical moments

$$\langle C_i \rangle = \frac{1}{M} \sum_m (M P_m)^i. \tag{10}$$

For $\delta \to 0$, its anomalous scales give the dynamical multifractal dimensions

$$\langle C_i \rangle \propto \delta^{-\phi_i}. \tag{11}$$

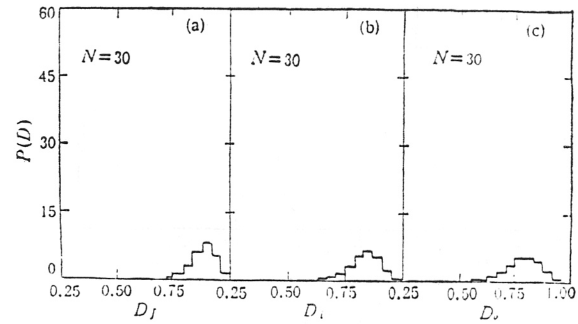

Fig. 5.1
Distributions of three kinds of dimensions for a pure statistical noise system (N = 30). (a) Hausdorff dimension; (b) information dimension; (c) correlative dimension.

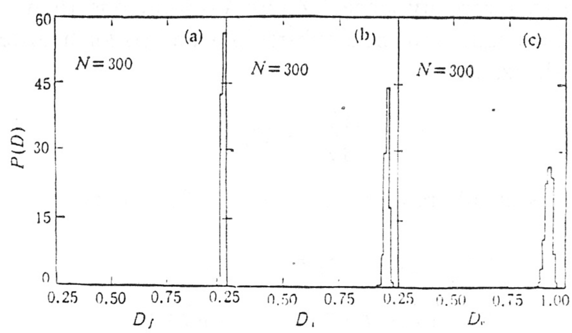

Fig. 5.2
Distributions of three kinds of dimensions for a pure statistical noise system (N = 300). (a) Hausdorff dimension; (b) information dimension; (c) correlative dimension.

Based on α model as the dynamics and by using binomial distribution to simulate the statistical fluctuation of particles, the G-moments and multifractal dimensions can be calculated, and compared with the pure dynamical fractal ones. Then the influence of statistical noise on fractal structure can be investigated.

3. MONTE CARLO SIMULATION

The flow diagrams of Monte Carlo simulation in three cases are given as follows:
(1) A system with dynamical fluctuation but without statistical noise (Fig. 1).
(2) A system with statistical noise but without dynamical fluctuation (Fig. 2).
(3) A system with both dynamical fluctuation and statistical noise (Fig. 3).

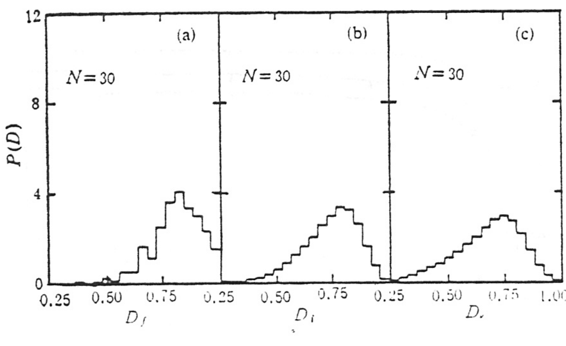

Fig. 6.1
Distributions of three kinds of dimensions for a mixed system (N = 30). (a) Hausdorff dimension; (b) information dimension; (c) correlative dimension.

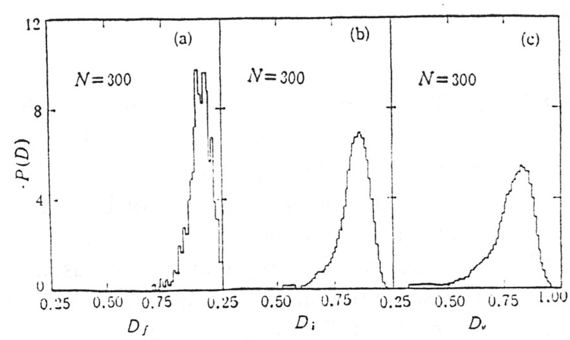

Fig. 6.2
Distributions of three kinds of dimensions for a mixed system (N = 300). (a) Hausdorff dimension; (b) information dimension; (c) correlative dimension.

Figs. 4-6 present the simulation results of the system in the above-mentioned three cases. In the calculation, w is taken to be two groups of values — (0, 1) and (0.4, 0.6), with corresponding probabilities of 0.1 and 0.9, respectively.

In piratical experiments, the system must be a mixed one, in which the statistical noise exists on top of the dominant dynamical probability. The simulation results show that the statistical noise makes the distribution of fractal dimension broader and its average value smaller; and that when multiplicity increases to N = 300, the influence of statistical noise can approximately be ignored.

The influence of statistical noise on different dimensions are shown further on the mean dimensions versus multiplicity plots, cf. Fig. 7. It can be seen that the mean dimensions of a mixed system approach the ones of the pure dynamical system when the multiplicity increases high enough. Assuming that the dynamical mechanism is the same for events with different multiplicities in the same type of collisions at the same energy, the curves of mean dimensions versus multiplicity can be

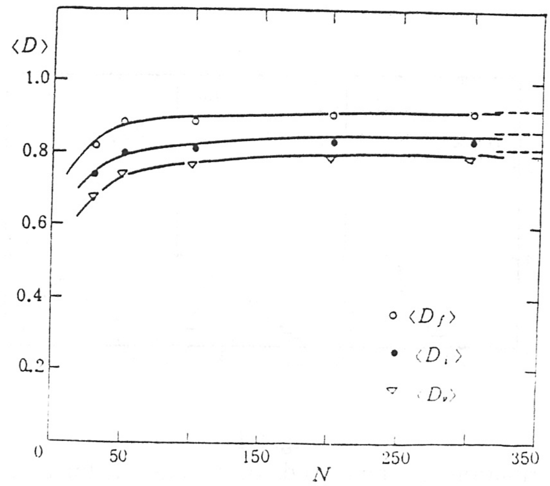

Fig. 7

Results of three kinds of mean dimensions versus multiplicity in a mixed system. Solid curves are from the fitting of $<D> = A - B\exp(-CN)$. Dash curves show the mean dimensions of three kinds of a pure dynamical system.

drawn from the experimental data. The asymptotes of these curves might be taken as the mean dimensions of the pure dynamics. Thus a possible method for eliminating statistical noise in investigating the multifractal properties of a system is obtained.

In a practical physical system, because of the existence of both dynamical fluctuation and statistical noise, how to erase statistical noise and how to analyze dynamical fluctuation and fractal properties of a system are still open problems. In this paper we have used Monte Carlo simulation to study the influence of statistical noise on fractal dimension distribution and try to give a method conductive to the solution of these problems. By using this method, the statistical noise might be erased under the condition that the dynamical production mechanism is the same for events with different multiplicities.

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