High Energy Physics and Nuclear Physics

Vol. 13, No. 2

Electron Clouds in Axial Symmetrical Space-Charge Lenses

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The focusing properties of a space-charge lens depend on the size, shape and density distribution of its electron cloud. In this paper these parameters are discussed based on both numerical and analytical methods.

In space-charge lenses, there are electron clouds produced by cathode emission or gas ionization. They are confined by the electromagnetic field. The focusing properties of the lenses depend on the state of the electron clouds. This kind of electron cloud is considered by many people to be in an approximate equilibrium state[1]. For an axial symmetrical lens the canonical angular momentum of electrons is conservative. Taking the cylindrical coordinate (z,r,φ), the distribution function of the electrons, f[2], can be written as

$$f = n_0 \left(\frac{m}{2\pi k T} \right)^{3/2} \exp \left[- \frac{1}{kT} (H - \omega P) \right], \tag{1}$$

$$H = \frac{1}{2} m v^2 - eU, \quad P = m r v_\varphi - e r A_\varphi. \tag{2}$$

where H is the Hamiltonian function, P is the canonical angular momentum, e is the absolute value of the electron charge, m is the electron mass, T is the electron temperature, k is the Boltzmann's

Received on January 3, 1988

constant, v is the electron velocity, U is the electrical potential and $A\varphi$ is the angular component of the magnetic field vector potential, n_0 and ω are constants. Equation (1) becomes

$$f = n\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left\{-\frac{m}{2kT}\left[v_z^2 + v_r^2 + (v_\varphi - \omega r)^2\right]\right\}, \tag{3}$$

$$n = n_0 \exp\left[\frac{e}{kT}(\psi - \psi_0)\right], \tag{4}$$

$$\psi = U + \frac{m\omega^2 r^2}{2e} - \omega r A_\varphi. \tag{5}$$

where n is the electron density, ψ is the modified potential[3]. If we choose ψ_0 as the maximum of ψ, n_0 will be the maximum electron density, and ω is the apparent average angular velocity of the electrons.

In this case the poisson equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) + \frac{\partial^2 U}{\partial z^2} = \frac{en}{\varepsilon_0} \tag{6}$$

can be rewritten as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{\partial^2 \psi}{\partial z^2} = -\frac{en_0\gamma}{\varepsilon_0}\exp\left[\frac{e}{kT}(\psi - \psi_0)\right]. \tag{7}$$

where γ is the magnetic confinement factor

$$\gamma = \frac{2\varepsilon_0\omega}{en}\left(B_z - \frac{m\omega}{e}\right) - 1, \tag{8}$$

ϵ_0 is the dielectric constant in vacuum, and B_z is the axial magnetic induction.

To follow the numerical approach in Ref.[3], the electron density distribution can be attained from Equation (7) when n_0, T and ω are given (it is more convenient to use value γ_0 of γ for $n = n_0$ rather than ω). The electron clouds within two lenses, as well as the curves of $\gamma(z,0)$, $n(z,0)$ and the radius of the electron cloud $R(z)$ are shown in Fig.1 and Fig.2. Now let us briefly discuss two kinds of lenses.

1. Long lenses (see Fig.1). The parameters of the electron clouds inside long lenses can be considered independent of z. Then Equation (7) is rewritten as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = \frac{en_0}{\varepsilon_0}\left\{\exp\left[\frac{e}{kT}(\psi - \psi_0)\right] - \gamma_0 - 1\right\}. \tag{9}$$

where n_0, ψ_0 and γ_0 are values of n, ψ and γ along the axis. Note $\gamma_0 + 1 = (\gamma + 1)\exp[(e/kT)(\psi - \psi_0)]$. If $\psi - \psi_0 < 1$, $\exp[(e/kT)(\psi - \psi_0)] = 1 + (e/kT)(\psi - \psi_0)$, the solution of Equation (9) is

$$\psi = \psi_0 + \frac{kT\gamma_0}{e}\left[1 - I_0(r/\lambda_D)\right]. \tag{10}$$

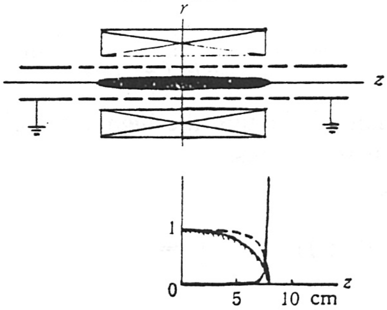

FIG.1 The electron cloud in a long space-charge lens for MeV ion beams. $n_0 = 10^{10}$ cm^{-3}, $kT = 20$ eV, $\omega = 10^9$ s^{-1}, $B_{max} = 0.09$ T, maximum voltage 2 kV. ⫼ R/R_{max}, --- n/n_0, —— γ.

where $\lambda_D = (\epsilon_0 kT/e^2 n_0)^{1/2}$ is the Debye length, I_0 is the zero-order modified Bessel function, and $I_0(\rho) \approx \sqrt{1/2\pi\rho}\exp\rho$ for $\rho >> 1$. Since on the boundary of the electron cloud $(e/kT)(\psi - \psi_0) \approx -1$. we get

$$R \approx -\lambda_D \ln \gamma_0. \tag{11}$$

Apparently $\gamma_0 > 0$ and R decreases as γ_0 increases. For practical lenses R should be larger than $4\lambda_D$, and $\gamma_0 < 0.02$. Moreover, R increases with the increasing of T and the decreasing of n_0.

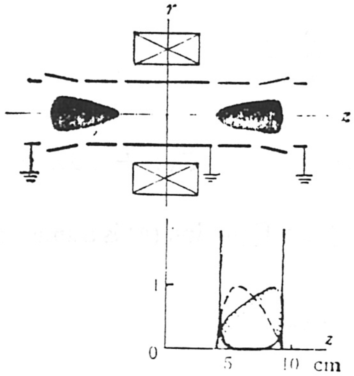

FIG.2 Two electron clouds in the space-charge lens reported in [4]. $n_0 = 10^9$ cm^{-3}, $\omega = 5 \times 10^8$ s^{-1}, $B_{max} = 0.057$ T, others are the same as Fig.1. ⫼ R/R_{max}, --- n/n_0, —— γ.

2. Short lenses (see Fig.2). The space-charge lens in Fig.2 was made in China[4]. Calculations reveal that it has two electron clouds located in the high potential regions of the two sides. The term $\partial^2\phi/\partial z^2$ in Equation (7) is not negligible, but in rough approximation, we take $R(z) \approx -\lambda_D \ln\gamma(z,0)$. Since in the electron cloud γ is smaller, we derive from Equation (8) that

$$n(z,0) \approx \frac{2\varepsilon_0\omega}{e} B_z(z,0), \quad R(z) \approx -\left(\frac{kT}{2e\omega B_z(z,0)}\right)^{1/2} \ln\gamma(z,0). \tag{12}$$

This indicates that the stronger the magnetic induction the higher the electron density, and the smaller the radius. It agrees with the calculated results.

Equation (8) can also be rewritten as

$$\omega^2 - \frac{e}{m} B_z\omega + \frac{1}{4}(1+\gamma)\left(\frac{e}{m} B_C\right)^2 = 0. \tag{13}$$

where $B_C = (2mn/\epsilon_0)^{1/2}$ is the critical magnetic induction. This equation has a real solution only if $B_z \geq \sqrt{1+\gamma}B_C$.

The collision between the electrons and the residual gas atoms is not considered in the above treatment, as the colliding electrons will diffuse outside and finally be lost on the electrodes. We will only study the long lens effects in which electrons would be replenished by ionization. Since the electron density in the central region of the electron cloud is uniform, from the continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}(nrv_r) = Zn, \tag{14}$$

we get

$$v_r = \frac{1}{2} Zr. \tag{15}$$

where Z is the ionization frequency. The radial fluid equation of electron motion is

$$v_r \frac{\partial v_r}{\partial r} = -\frac{e}{m} E_r - \frac{e}{m}\omega B_z r + \omega^2 r - \frac{kT}{mn}\frac{\partial n}{\partial r} - (\nu + Z)v_r. \tag{16}$$

where ν is the momentum transfer frequency for the electron-atom collisions. Substituting Equation (15) into (16) we get

$$\frac{\partial\phi}{\partial r} = \frac{kT}{en}\frac{\partial n}{\partial r}, \tag{17}$$

$$\phi = U + \frac{m\omega^2 r^2}{2e} - \omega r A_\varphi - \frac{m}{4e} Z(\nu + 1.5Z)r^2. \tag{18}$$

The only difference between Equation (18) and Equation (4) is a small collision term. Similarity we have

$$R = -\lambda_D \ln\gamma, \tag{19}$$

$$\gamma = \frac{2\varepsilon_0}{en}\left[\omega B_z - \frac{m\omega^2}{e} + \frac{m}{2e} Z(\nu + 1.5Z)\right] - 1. \tag{20}$$

From the above discussion we know that when considering the collision effect the equilibrium theory is also useful as an approximation so long as appropriate modifications are made. However the theory does not set the values of n_0, T and ω. To determine these parameters the electron generation and transport processes have to be studied.

REFERENCES

[1] A. I. Morozov and S. V. Lebedev, *Reviews of Plasma Physics*, Vol. **8**, Consultants Bureau, New York, 1980, 301.
[2] L. D. Landau and E. M. Lifshitz, Statistical Physics, Part 1, Pergamon Oxford, 1980, 11.
[3] Yu Qingchang, Atomic Energy Science and Technology, **21**(1987)666.
[4] Qiu Hong et al., High Energy Physics and Nuclear Physics, **7**(1983)583.