Measurement of Complete Fusion Cross Sections of $^{12}\text{C} + ^{159}\text{Tb}$ and $^{12}\text{C} + ^{165}\text{Ho}$

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Complete fusion cross sections are measured for $^{12}\text{C} + ^{159}\text{Tb}$ and $^{12}\text{C} + ^{165}\text{Ho}$ reactions by measuring K-X rays of the evaporation residues with Si(Li) spectrometer. The half-lives of the evaporation residues and their yield distributions as a function of the incident energy are also obtained. The experimental values for the complete fusion cross sections are compared with the theoretical ones.

1. INTRODUCTION

The theoretical prediction shows that the complete fusion reaction is a very important reaction channel in the nuclear reactions induced by heavy ions at low energy. Therefore, many nuclear physicists have performed experiments to measure the complete fusion cross sections in heavy-ion-induced reactions, and carefully investigated the various complete fusion cross sections, thus providing a sound basis for developing the compound nucleus theory. The existing complete fusion theories [1,2] have been supported by many experimental results, especially at the incident energy well above the coulomb barrier. But for the bombardments of medium heavy nuclei by $^{12}\text{C}$, $^{14}\text{N}$, $^{16}\text{O}$ ions, particularly, at the incident energy near the coulomb barrier, the experimental information is rather poor because of the limitation of the method for measuring the compound nucleus residue. The complete fusion cross sections in this mass region, particularly for the incident energy near the coulomb barrier, are very important for studying the effects of the nuclear potential shape and the ratio of neutron to proton in the target nucleus on the

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complete fusion. Therefore, investigation of the complete fusion near the sub-coulomb barrier is a popular subject at recent time. The new methods of detecting the compound nucleus residues have also been developed. For instance, the X-ray measurement technique has been adopted [3]. In present work the measurement of the fusion cross sections for the reactions $^{12}\text{C} + ^{159}\text{Tb}$, $^{165}\text{Ho}$ at energy near the coulomb barrier has been performed. The fission cross section of the medium heavy nucleus bombarded by low energy $^{12}\text{C}$ ion is very small, especially for the target nucleus with the mass number below 150, it can be neglected. Thus, the cross section of the evaporation residues is essentially equal to the complete fusion cross section. Therefore, to measure the complete fusion cross section of these reactions it is necessary to directly measure the evaporation residues’ cross section for the compound nucleus with the mass number around 150. Because the incident energy is somewhat lower, it is difficult to use $\Delta E - E$ time-flight method and solid track detectors to measure the mass and charge of the residues. To obtain the cross sections of the evaporation residues in these reactions, the Si(Li) X-ray spectrometer was used to detect their characteristic X-rays in "off line" status. This method can protect them from being contaminated by the products of the coulomb excitation of the target nucleus.

2. EXPERIMENT

The experiment was performed at the 1.5 m cyclotron at the Institute of Modern Physics with carbon beam at six energies from 72.5 MeV down to 57 MeV. Aluminum foil was adopted to reduce the beam energy for getting right incident energies.

The isotope $^{159}\text{Tb}$ target was prepared by the vacuum evaporation, whereas the $^{165}\text{Ho}$ oxide target was prepared by the electrical deposition onto the aluminum foil of 524 $\mu$g/cm$^2$. Considering the demand for capturing the residues and measuring their K-X rays, the thicknesses of the target and the catcher foil were estimated to allow all the evaporation residues to penetrate through the target and then to be caught by the catcher foil, whereas the beam ions and the contamination products from some light elements were allowed to pass through the target and the catcher foil. So thicknesses of 450 $\mu$g/cm$^2$, 375 $\mu$g/cm$^2$ and 600 $\mu$g/cm$^2$ for $^{159}\text{Tb}$, $^{165}\text{Ho}$ targets and catcher foils were chosen, respectively.

In order to save the beam-time and reduce the data error a collimator of 6 mm in diameter was put approximately 1 mm ahead of the target. So the flux of the effective beam and the active area of the collected sample were limited. The catcher foil was placed behind the target about 2 mm away. The beam passing through the target and the catcher foil were collected in a Faraday cup.

Before the experiment the number of neutrons evaporated from the compound nucleus in the reaction was estimated. Then the irradiation time was also estimated to obtain good statistics for activity measurement according to the known half-lives of evaporation residues. During the experiment the irradiation time was increased from 10 to 40 min and from 1 to 4 h for $^{159}\text{Tb}$ target and $^{165}\text{Ho}$ target respectively, while the beam energy was decreased from 72.5 MeV down to 57 MeV. The beam intensity was recorded as a function of time by a BM-96 multi-channel analyzer. Once the irradiation was stopped, the collected sample was moved as fast as possible from the chamber to the Si(Li) X-ray
FIG. 1 K-X ray spectra of the evaporation residue for reaction \( ^{12}\text{C} + ^{165}\text{Ho} \)

at \( E_{\text{beam}} = 71.5 \) MeV. The upper is the measured spectrum in 10 min after stopping beam irradiation, the lower in 378.3 min.

spectrometer (in 2–5 min) for measuring the K-X rays of the residues. The K-X ray energy spectra of the daughter of the evaporation residues were recorded by a M-20 multi-channel analyzer periodically. Here only the energy spectra of the products for the reaction \( ^{12}\text{C} + ^{165}\text{Ho} \) are presented in Fig. 1.

The efficiency of the X-ray spectrometer used in the experiment was calibrated with \(^{133}\text{Ba}\) and \(^{241}\text{Am}\) sources of known intensities. The efficiency was \(3.4 \times 10^{-4}\) at 56 keV.

3. DATA TREATMENT

The K-X ray energy spectra of the isotopes produced in \( ^{12}\text{C} + ^{165}\text{Ho} \) are given in Fig. 1. Only the data for the daughter nucleus Yb from \( ^{12}\text{C} + ^{159}\text{Tb} \) reaction and Hf from \( ^{12}\text{C} + ^{165}\text{Ho} \) were treated. Finally a series of Yb, Hf decay curves at different energy are obtained. As the K-X ray is the nuclear characteristic ray only related to atomic number \( Z \), each decay curve includes the contributions from several evaporation residue isotopes.
with different half-lives. The decay curves are fitted and decomposed in half-logarithm coordinate. Then the half-lives and the number of decay $N(0)$ at the time $t_0$ can be extracted, when the bombardment of the target is stopped. A typical decay curve and its decomposition are shown in Fig. 2. According to the known half-lives, we can determine which isotopes belong to the measured evaporation residues. In the figure 2n, 3n,... represent the emitted neutron numbers from compound nucleus $^{171}$Lu.

By decomposing the decay curves, both the half-lives and the counts at time $t = 0$ for various isotopes can be obtained.

The K-X rays come from two sources: the internal conversion and the electron capture of K-shell. This can be expressed by $N(0) \sim f(\epsilon_k, \lambda_k)$. Here $\epsilon_k$ and $\lambda_k$ represent the capture probability and the internal conversion probability of the K-shell electron, respectively.

$\epsilon_k$ can be derived from the decay theory [4]. $\lambda_k$ can be extracted from the data in isotopes chart [5].

According to the decay law of the radioactive isotope, the number of the decaying nucleus existing at the time in the figure. $t = 0$ is given by

$$N_0 = N(0) / (1 - e^{-\lambda t}).$$

where $\eta = \gamma \cdot \omega_k \cdot (\epsilon_k + \lambda_k)$, $\gamma$ is the detection efficiency of the X-ray spectrometer, $\omega_k$ is the fluorescence yield.

The cross section is

$$\sigma = N_0 / N_A \cdot \phi,$$

where $N_A$ is the number of target nucleus on unit area, $\phi$ is the equivalent beam-flux, considering the effect of the beam intensity variation with time on the product yield during the irradiation.
FIG. 3 Branch ratio of the residue isotope yield in the reactions $^{12}$C + $^{159}$Tb and $^{12}$C + $^{165}$Ho. 3n, 4n, 5n represent the numbers of the evaporated neutrons by the compound nucleus.

Then the corresponding evaporation residue cross section $\sigma (x,n)$ can be extracted from the number No of its isotope. Fig. 3 shows their relative cross sections.

Because the emission probability of the charged particle is small in this reaction, the sum of the cross sections of various evaporation residues is equal to the complete fusion cross section for either reaction $^{12}$C + $^{159}$Tb or reaction $^{12}$C + $^{165}$Ho.

4. RESULTS

The K-X ray spectra of the evaporation residues for the reactions $^{12}$C + $^{159}$Tb and $^{12}$C + $^{165}$Ho were measured in this experiment. After analyzing the data, the half-lives for several products are obtained and are summarized as follows:

<table>
<thead>
<tr>
<th>reaction</th>
<th>compound nucleus</th>
<th>evaporation residue</th>
<th>half-lives</th>
<th>other works $^{1,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$C + $^{159}$Tb</td>
<td>$^{12}$C $^{121}$</td>
<td>$^{12}$C $^{124}$</td>
<td>2.6 min</td>
<td>2.12, 1.4, 2.6 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{12}$C $^{127}$</td>
<td>53 min</td>
<td>51.5 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{12}$C $^{144}$</td>
<td>6 min</td>
<td>5.3 min, 6.7 min</td>
</tr>
<tr>
<td>$^{12}$C + $^{165}$Ho</td>
<td>$^{12}$C $^{171}$</td>
<td>$^{12}$C $^{172}$</td>
<td>33 min</td>
<td>36.8 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{12}$C $^{173}$</td>
<td>3.03 hr</td>
<td>3.65 hr</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{12}$C $^{174}$</td>
<td>67 min</td>
<td>62.6 min</td>
</tr>
</tbody>
</table>

In the energy region above the barrier, the fusion cross section can be described by the following equation,

$$\sigma_{CF} = \pi R^2 \left( 1 - \frac{V_B}{E_{cm}} \right)$$
where $V_B$ is the fusion barrier, $R$ is the fusion radius, $\sigma_{cm}(E_{cm})$ is a linear function of $E_{cm}$ as shown in Fig. 4. According to the experimental data the barrier parameters are extracted

- $^{12}$C + $^{157}$Tb: $R = 10.74$ fm, $r_B = 1.39$ fm
- $V_B = 48.31$ MeV

- $^{12}$C + $^{165}$Ho: $R = 11.31$ fm, $r_B = 1.45$ fm
- $V_B = 50$ MeV

The experimental results are compared with the systematic values [8]. The systematic values of the two systems are

- $^{12}$C + $^{157}$Tb: $R = 10.91$ fm, $r_B = 1.42$ fm
- $V_B = 47.67$ MeV

- $^{12}$C + $^{165}$Ho: $R = 10.96$ fm, $r_B = 1.41$ fm
- $V_B = 48.84$ MeV

The calculated values are related to nuclear potential shape. The following values are given based on the proximity potential (see later statement)

- $^{12}$C + $^{157}$Tb $\gamma_B = 1.39$ fm, $V_B = 48.715$ MeV
- $^{12}$C + $^{165}$Ho $\gamma_B = 1.39$ fm, $V_B = 49.88$ MeV

From the relation between the complete fusion cross section and the critical angular momentum, we have

**FIG. 4** $\sigma_{cm}$ as a function of $1/E_{cm}$ for $^{12}$C + $^{157}$Tb reaction.

**FIG. 5** Excitation energy $E^*$ of the compound nucleus as a function of $I_{cr}/(I_{cr} + 1)$. 
Thus the critical angular momentum \( l_{cr} \) can be extracted. The obtained critical angular momentum as a function of excitation energy are shown in Fig. 5. According to the sharp cut-off model, there is an expression concerning the moment of inertia and \( E_{cm} \) for the incident energy above the coulomb barrier:

\[
\frac{L_{cr}(l_{cr} + 1)h^2}{2\mu R^2} = E_{cm} - V_b
\]

\[
= E^* - V_b - Q.
\]

Thus the moment of inertia, \( \mu R^2 \), can be extracted from the relation of \( l_{cr}(l_{cr} + 1) \) and \( E^* \):

\[
^{12}\text{C} + ^{199}\text{Tb} \quad \mu R^2 = 0.2296 \times 10^{-44} \text{g \cdot cm}^2, \quad R = 11.13 \text{fm}
\]

\[
^{12}\text{C} + ^{169}\text{Ho} \quad \mu R^2 = 0.2236 \times 10^{-44} \text{g \cdot cm}^2, \quad R = 10.99 \text{fm}
\]

The value of \( R \) is consistent with that extracted from \( \sigma_{CF} \sim E_{cm}^{-1} \), as both of them are obtained from the same equation.

The experimental fusion cross sections are compared with the theoretical values for these two systems. The theoretical calculation depends on the classical equation

\[
\sigma_{CF} = \pi R^2 \left( 1 - \frac{V}{E_{cm}} \right)
\]

here the fusion barrier height \( V = V_N + V_C \) with \( V_N \) as the nuclear potential. The calculation is done in two forms of potentials: one is the proximity potential, the other is the unified potential [7].

The proximity potential is given by

\[
V_N = 4\pi \gamma \mathcal{R} \cdot b \cdot \phi(\zeta),
\]

where \( \gamma \) is the surface-energy constant, \( \gamma = (1 - Kf^2) \text{ fm}^{-2} \text{ MeV} \), \( I = (N - Z)/A, K = 1.7826 \), \( b = 1 \text{ fm} \). \( \mathcal{R} \) is the reduced radius,

\[
\mathcal{R} = C_1 C_2 / (C_1 + C_2)
\]

\[
C_1 = R_i - b/R_i,
\]

\[
R_i = 1.28 A_1^{1/3} - 0.76 + 0.8 A_i^{1/3} (\text{fm})
\]

\( \phi(\zeta) \) is the proximity force,

\[
\phi(\zeta) = \begin{cases} 
-\frac{1}{2} (\zeta - 2.45)^2 - 0.0852 (\zeta - 2.7885)^3 & \zeta < 1.2511 \\
-4.490 \exp(-\zeta/0.7755) & \zeta \geq 1.2511 
\end{cases}
\]

Here the proximity force is obtained by taking the statistical average value of an
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FIG. 6 Complete fusion cross section. The left is for $^{12}\text{C} + ^{159}\text{Tb}$, the right for $^{12}\text{C} + ^{165}\text{Ho}$. o indicates the experimental results, — calculated by proximity potential, —— calculated by unified potential.

abundance of data. It is different from the coefficient in the original equation. The calculated results are presented by the solid curve in Fig. 6.

The unified potential is given by

$$V_a = -D \left( F + \frac{S}{a} \right) \frac{R_{12}}{r} e^{-s/a}$$

the depth constant is

$$D = 4s^2 g(R_1/a)g(R_2/a) e^{-R_{12}^2} C_s'$$

where

$$g(x) = x \cosh(x) - x \sinh(x),$$

$$R_i = r_i A_i^{1/3}, \quad R_{12} = R_1 + R_2$$

For two separated nuclei,

$$C_s' = [C_s(1) \cdot C_s(2)]^{1/2}$$

$$C_s = a_s(1 - K_s r_0^2)$$ is the effective surface-energy constant;

$$l = (N - Z)/A, \text{ neutron-proton excess;}$$

$$a_s = 21.7 \text{ MeV}, \text{ the surface-energy constant;}$$

$$K_s = 3, \text{ the surface-asymmetry constant.}$$

$$F = 4 + \frac{R_{12}}{a} \frac{f(R_1/a)}{g(R_1/a)} - \frac{f(R_2/a)}{g(R_2/a)}$$

where $f(x) = x^2 \sinh(x), a$ is the range of the Yukawa folding function, $a = 0.65 \text{ fm}$. The calculated theoretical curves with the liquid drop radius parameter $r_0 = 1.20 \text{ fm}$ are in good agreement with the experimental results. The calculated results are presented by the dashed curves in Fig. 6.

In Fig. 6 the error bars include the contributions of the target thickness, the integral intensity of the beam, the detection efficiency, the deviation of the catcher foil position and
the K-X ray production probability. The uncertainty of the cross section is generally ±
(10%–20%) except at the lowest energy.

5. CONCLUSION

The cross sections of the evaporation residue of the reactions $^{12}$C + $^{159}$Tb and $^{165}$Ho
in the energy region of 56.4 to 72.25 MeV have been measured by means of measuring
the K-X rays and the results have been fitted with the simple models. It indicates that the
complete fusion cross section obtained in this way is reliable, in other words, the cross
sections of the complete fusion for the evaporation residues with the mass number of
about 150 can be determined properly by measuring their K-X rays. To obtain more precise
and more sufficient data, it is important to increase the beam intensity, to improve the
transport stability, and to reduce the deviation of the catcher foil position, especially at low
incident energies.

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