

Weinberg-Salam 模型中 W^\pm 矢量 中间玻色子的磁矩

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摘 要

本文讨论了 Weinberg-Salam 模型中 W^\pm 矢量中间玻色子 (VIB) 的磁矩问题。首先在规范对称性没有自发破缺的假定下, 利用规范不变性得出正规顶角母泛函的方程, 求出三线 VIB 正规顶角满足的 Ward-Takahashi 恒等式, 证明了它的张量结构形式与裸的 VIB 三线耦合项相同, 因而表明这时 W^\pm 介子不存在反常磁矩及反常电四极矩。对于有自发破缺的“实际”情况, 通过在标量场作用项中引入明显破缺项的办法, 对 VIB 的质量延拓问题完成整个讨论, 进而给出了上述结论在这时仍然成立的证明。

一、引 言

现今人们普遍相信 W-S 模型可能反映了真实世界中弱作用、电磁作用的基本规律, 因而对与此模型有关的各种问题进行了大量的研究。关于 W^\pm 介子的磁矩问题就是其一。这方面已有一些唯象分析^[1,2], 但尚未对 W-S 模型中 W^\pm 介子的磁矩值深入研究, 本文就想要完成这一任务。我们证明了在 W-S 模型中 W^\pm 介子磁矩为 e/M_W 、电四极矩相应为 e/M_W^2 , 换言之, 在规范理论模型中它的反常磁矩和反常电四极矩为零。具体步骤是: 第二节里先在没有自发破缺的理论中讨论。由 Green 函数母泛函的路径积分表达式出发, 通过规范不变性要求和 Legendre 变换得到正规顶角母泛函满足的方程。由此求出三线 VIB 正规顶角满足的 W-T 恒等式。再利用有关的传播函数和顶角部分的重态化手续, 证明了三线 VIB 正规顶角的张量结构与它的裸顶角的相同。表明高阶修正不会给它带来新的张量结构项。因而就证明了 W^\pm 介子不存在反常磁矩和反常电四极矩^[3]。第三节回到有自发破缺的“实际”情况讨论这问题。仿照有自发破缺的 σ 模型重态化时的办法^[4], 先在 \mathcal{L} 的标量场作用项中引进明显的对称性破缺项。通过这时的得到 Green 函数与正规顶角母泛函和第二节对称理论时的对应项之间关系式, 求出了存在破缺时正规顶角的母泛函方程。进而将它们对参量 μ^2 (标量场二次项系数) 作延拓到负值去, 然后再让 \mathcal{L} 中的外破缺项退回到零而完成对 W^\pm 介子质量的解析延拓问题。由于细节过于冗长,

本文 1982 年 10 月 18 日收到。

这里只给出了概要的证明,指出第二节的结论,在考虑到规范理论存在自发破缺时仍然成立. 文章末尾的附录给出了三线 VIB 正规顶角 W-T 恒等式的证明.

二、W-S 模型中三线 VIB 正规顶角函数

W-S 模型 \mathcal{L} 中有关三线 VIB 顶角部分为:

$$\mathcal{L}_{3W} = ig(\partial_\nu \vec{W}^\mu) \cdot \vec{W}_\mu \wedge \vec{W}^\nu \quad (2.1)$$

用 $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$, $W_\mu^3 = s_\theta A_\mu + c_\theta Z_\mu$ 代入 (s_θ, c_θ 是 $\sin \theta_W, \cos \theta_W$ 省写) 即

得 $WW\gamma$ 顶点的拉氏量:

$$\mathcal{L}_{WW\gamma} = ig s_\theta \{ W_\mu^- A^{\mu\nu} W_\nu^+ - A_\mu W^{\mu\nu+} W_\nu^- - W_\mu^+ W^{\mu\nu-} A_\nu \}$$

和 WWZ 顶点的拉氏量:

$$\mathcal{L}_{WWZ} = ig c_\theta \{ W_\mu^- Z^{\mu\nu} W_\nu^+ - Z_\mu W^{\mu\nu+} W_\nu^- - W_\mu^+ W^{\mu\nu-} Z_\nu \}$$

因此求 W^\pm 介子电磁顶角函数就化为讨论三线 VIB 顶角函数问题. 与图 (1) 给出的符号对应, (2.1) 式在动量空间为

$$V_{\alpha\beta\gamma}^{\mu\nu\rho} = g\varepsilon_{\alpha\beta\gamma} [g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \quad (2.2)$$

由此得出

$$k_\rho V_{\alpha\beta\gamma}^{\mu\nu\rho} = g\varepsilon_{\alpha\beta\gamma} [g^{\mu\nu}(p+q)^2 - (p+q)^\mu(p+q)^\nu - g^{\mu\nu}p^2 + p^\mu p^\nu] \quad (2.3)$$

下面先求出没有自发破缺时的三线 VIB 顶角函数满足的 W-T 恒等式, W-S 模型的 \mathcal{L} 由三部分组成:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_g + \mathcal{L}_{F-P} \quad (2.4)$$

其中 $\mathcal{L}_0 = \mathcal{L}_W + \mathcal{L}_B + \mathcal{L}_f + \mathcal{L}_\phi + \mathcal{L}_l$, 它们分别是:

$$\left\{ \begin{array}{l} \mathcal{L}_W = -\frac{1}{4} \vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu}; F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\varepsilon^{abc} W_\mu^b W_\nu^c \\ \mathcal{L}_B = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu}; G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \\ \mathcal{L}_f = -\bar{\psi} \phi \psi - \bar{l}_- \phi l_-, \phi = \frac{1}{2}(1+r_s) \begin{pmatrix} \nu_l \\ l \end{pmatrix}, l_- = \frac{1}{2}(1-r_s) l_- \\ D_\mu \phi = \left(\partial_\mu - ig \frac{\vec{\tau}}{2} \cdot W_\mu + \frac{i}{2} g^0 B_\mu \right) \phi, D_\mu l_- = (\partial_\mu - ig' B_\mu) l_- \\ \mathcal{L}_\phi = -(D_\mu \phi)^\dagger (D_\mu \phi) - \mu^2 \phi^\dagger \phi - \frac{1}{2} \lambda (\phi^\dagger \phi)^2 \\ D_\mu \phi = \left(\partial_\mu - ig \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu - \frac{i}{2} g' B_\mu \right) \phi \\ \mathcal{L}_l = -(\bar{\psi} \phi l_- + \bar{l}_- \phi^\dagger \psi). \end{array} \right. \quad (2.4a)$$

\mathcal{L}_g 为规范固定项, \mathcal{L}_{F-P} 为 Faddeev-Popov 鬼项:

$$\mathcal{L}_g = \frac{-1}{2\xi} (\partial_\mu \vec{W}^\mu)^2 + \frac{-1}{2\xi} (\partial_\mu B^\mu)^2 \tag{2.5}$$

$$\mathcal{L}_{F-P} = \omega_a^+ [\delta_{\alpha\beta} \square + g \varepsilon_{\alpha\beta\gamma} \partial_\mu W^{\mu\gamma}] \omega_\beta \tag{2.6}$$

由于规范场 B_μ 与 $SU(2)$ 规范场 \vec{W}_μ 没量耦合, 而我们所讨论的问题也不涉及 B_μ 场, 为简便起见, 以下不再考虑只与 B_μ 场有关的量. 这样, 在 $SU(2)$ 群无穷小规范变换 $\vec{\theta}(x)$ 下:

$$\begin{cases} \phi \rightarrow \phi' = \left(1 - i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x)\right) \phi \\ \bar{\phi} \rightarrow \bar{\phi}' = \left(1 - i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x)\right) \bar{\phi} \\ \vec{W}_\mu \rightarrow \vec{W}'_\mu = \vec{W}_\mu + \vec{\theta}(x) \wedge \vec{W}_\mu - \frac{1}{g} \partial_\mu \vec{\theta}(x) \end{cases} \tag{2.7}$$

\mathcal{L}_0 为不变量, 而 \mathcal{L}_g 的变化为:

$$\begin{aligned} \mathcal{L}_g \rightarrow \mathcal{L}'_g &= \mathcal{L}_g - \frac{1}{\xi} (\partial_\mu \vec{W}^\mu) \cdot \partial_\nu (\vec{\theta} \wedge \vec{W}^\nu) \\ &+ \frac{1}{\xi} \frac{1}{g} (\partial_\mu \vec{W}^\mu) \cdot \square \vec{\theta}(x) \end{aligned} \tag{2.8}$$

其次考虑 Green 函数的路径积分表达式

$$W \equiv e^Z = \int \mathcal{D}(\omega) e^{i \int d^4x \omega_a^+ [\delta_{\alpha\beta} \square + i g \varepsilon_{\alpha\beta\gamma} \partial_\mu \frac{\delta}{\delta J_{\mu\gamma}^T}] \omega_\beta \cdot e^{Z_0}} \tag{2.9}$$

其中

$$e^{Z_0} = \int \mathcal{D}(W, \phi, \bar{\phi}) e^{i \int d^4x [\varphi_0 + \varphi_g + \vec{J}_\mu \cdot \vec{W}^\mu + \bar{\eta} \psi + \bar{\nu} \eta + \beta^+ \phi + \beta \phi^+]} \tag{2.10}$$

$\vec{J}, \eta, \bar{\eta}, \beta$ 和 β^+ 是分别与 $\vec{W}, \bar{\phi}, \psi, \phi^+$ 和 ϕ 耦合的外源. 要求 e^{Z_0} 在无穷小规范变换 (2.7)、(2.8) 式下为不变, 则由 (2.10) 式, 保留到 $\vec{\theta}(x)$ 线性项时得:

$$\begin{aligned} &\int \mathcal{D}(W, \phi, \bar{\phi}) \int d^4x \left\{ \frac{1}{\xi} g (\vec{W}^\nu \wedge \partial_\nu \partial_\mu \vec{W}^\mu) + \frac{1}{\xi} \square \partial_\mu \vec{W}^\mu + g \vec{W}^\mu \wedge \vec{J}_\mu + \partial_\mu \vec{J}^\mu \right. \\ &\quad \left. - i g \left[\bar{\eta} \frac{\vec{\tau}}{2} \phi - \eta \left(\gamma^0 \frac{\vec{\tau}}{2} \gamma^0 \right)^T \phi^{+T} + \beta \frac{+ \vec{\tau}}{2} \phi - \beta \frac{\vec{\tau}}{2} \phi^+ \right] \right\} \\ &\times e^{i \int d^4x [\varphi_0 + \varphi_g + \vec{J}_\mu \cdot \vec{W}^\mu + \bar{\eta} \psi + \bar{\nu} \eta + \beta^+ \phi + \beta \phi^+]} = 0 \end{aligned} \tag{2.11}$$

当然还得考虑到规范补偿项的贡献. 为此定义 F-P 鬼态的两点 Green 函数 $G_{\alpha\beta}(x, y; \delta / \delta J)$, 它满足方程:

$$\left[\square_x \delta_{\alpha\beta} + i g \varepsilon_{\alpha\gamma\beta} \partial_\mu \frac{\delta}{\delta J_{\mu\gamma}^T(x)} \right] G_{\beta\gamma} \left(x, y; \frac{\delta}{\delta J} \right) = \delta_{\alpha\gamma} \delta^{(4)}(x - y) \tag{2.12}$$

借助于这方程, (2.11) 式经部分积分后可改写为:

$$\begin{aligned} &\left\{ g \left[\bar{\eta} \frac{\tau_\alpha}{2} \frac{\delta}{\delta \bar{\eta}} - \eta \left(\gamma^0 \frac{\tau_\alpha}{2} \gamma^0 \right)^T \frac{\delta}{\delta \eta} + \beta^+ \frac{\tau_\alpha}{2} \frac{\delta}{\delta \beta^+} - \beta \frac{\tau_\alpha^T}{2} \frac{\delta}{\delta \beta} \right] \right. \\ &\quad \left. + J_{\mu,\beta} \left(\delta_{\alpha\beta} \partial^\mu - i g \varepsilon_{\alpha\beta\gamma} \frac{\delta}{\delta J_{\mu,\gamma}^T} \right) + \frac{i}{\xi} [G^{-1}]_{\alpha\beta} \partial_\mu \frac{\delta}{\delta J_{\mu,\beta}} \right\} e^{Z_0} = 0 \end{aligned} \tag{2.13}$$

再定义

$$\Delta \equiv \int \mathcal{D}(\omega) e^{i \int d^4x \omega_a^+ [G^{-1}]_{\alpha\beta} \omega_\beta} = e^{\text{Tr} \ln [G^{-1}]_{\alpha\beta}} \quad (2.14)$$

将 $\int d^4y \Delta G$ 作用到 (2.13) 式, 借助 (2.12) 式得出 Green 函数的母泛函 Z 所满足的方程:

$$\left\{ i \frac{1}{\xi} \frac{\delta}{\delta J_a^\mu(x)} + \int d^4y g \left[\bar{\eta} \frac{\tau_\beta}{2} \frac{\delta}{\delta \bar{\eta}} - \eta \left(\gamma^0 \frac{\tau_\beta}{2} \gamma^0 \right)^T \frac{\delta}{\delta \eta} + \beta^+ \frac{\tau_\beta}{2} \frac{\delta}{\delta \beta^+} - \beta \frac{\tau_\beta^T}{2} \frac{\delta}{\delta \beta} \right] \right. \\ \cdot G_{\beta\alpha} \left(y, x; \frac{\delta}{\delta J} \right) + \int d^4y J_{\mu,\tau}(y) \left[\partial^\mu \delta_{\beta\tau} - i g \varepsilon_{\tau\delta\beta} \frac{\delta}{\delta J_\delta^\mu(y)} \right] \\ \left. \cdot G_{\beta\alpha} \left(y, x; \frac{\delta}{\delta J} \right) \right\} e^Z = 0 \quad (2.15)$$

为了得到正规顶角的母泛函 Γ 满足的方程, 对有关的 Z 作 Legendre 变换:

$$\Gamma(\mathbf{W}, \boldsymbol{\psi}, \bar{\boldsymbol{\psi}}, \boldsymbol{\phi}, \boldsymbol{\phi}^+) = Z(J, \eta, \eta^+, \beta, \beta^+) - i \int d^4x [J_\mu \cdot \bar{\mathbf{W}}^\mu + \bar{\eta} \boldsymbol{\psi} + \bar{\boldsymbol{\psi}} \eta + \beta^+ \boldsymbol{\phi} + \boldsymbol{\phi}^+ \beta] \\ = 0 \quad (2.16)$$

其中 $\mathbf{W}, \boldsymbol{\psi}, \boldsymbol{\phi}$ 等经典场与外源 J, η, β 等的关系分别是:

$$\mathbf{W}_a^\mu = -i \frac{\delta Z}{\delta J_{\mu,a}}, \quad \bar{\boldsymbol{\psi}} = -i \frac{\delta Z}{\delta \eta}, \quad \boldsymbol{\psi} = -i \frac{\delta Z}{\delta \bar{\eta}}, \quad \boldsymbol{\phi}^+ = -i \frac{\delta Z}{\delta \beta}, \quad \boldsymbol{\phi} = -i \frac{\delta Z}{\delta \beta^+} \quad (2.17)$$

以及

$$J_a^\mu = i \frac{\delta \Gamma}{\delta \mathbf{W}_{\mu,a}}, \quad \bar{\eta} = i \frac{\delta \Gamma}{\delta \boldsymbol{\psi}}, \quad \eta = i \frac{\delta \Gamma}{\delta \bar{\boldsymbol{\psi}}}, \quad \beta = i \frac{\delta \Gamma}{\delta \boldsymbol{\phi}^+}, \quad \beta^+ = i \frac{\delta \Gamma}{\delta \boldsymbol{\phi}} \quad (2.18)$$

由于连通 Green 函数的母泛函 Z 是在外源存在时定义的, 在求泛函微商时要利用到类似于下面的公式

$$F \left(-i \frac{\delta}{\delta \eta} \right) e^{iZ} = e^{iZ} F \left(\mathbf{W} - i \frac{\delta}{\delta J} \right) \cdot 1 \quad (2.19)$$

为了消去 Γ 中与规范参数 ξ 有关的项, 可用 Γ_0 代替 Γ :

$$\Gamma_0 \equiv \Gamma - \frac{i}{2\xi} \int d^4y (\partial_\mu \bar{\mathbf{W}}^\mu)^2 \quad (2.20)$$

将 (2.16) 式代入 (2.15) 式并注意 (2.17)–(2.20) 各式, 得出与 ξ 参数无关的正规顶角母泛函 Γ_0 满足的方程:

$$\left\{ d^4y \left\{ i g \frac{\delta \Gamma_0}{\delta \boldsymbol{\psi}} \frac{\tau_\beta}{2} \left(\frac{\delta}{\delta \eta} + i \boldsymbol{\psi} \right) - \frac{\delta \Gamma_0}{\delta \bar{\boldsymbol{\psi}}} \left(\gamma^0 \frac{\tau_\beta}{2} \gamma^0 \right)^T \left(\frac{\delta}{\delta \eta} + i \bar{\boldsymbol{\psi}} \right)^T + \frac{\delta \Gamma_0}{\delta \boldsymbol{\phi}} \frac{\tau_\beta}{2} \left(\frac{\delta}{\delta \beta^+} + i \boldsymbol{\phi} \right) \right. \right. \\ \left. \left. - \frac{\delta \Gamma_0}{\delta \boldsymbol{\phi}^+} \frac{\tau_\beta^T}{2} \left(\frac{\delta}{\delta \beta} + i \boldsymbol{\phi}^+ \right) \right\} + \frac{\delta \Gamma_0}{\delta \mathbf{W}_{\mu,\tau}} \left[i \delta_{\beta\tau} \partial_y^\mu + g \varepsilon_{\tau\delta\beta} \left(\frac{\delta}{\delta J_{\mu,\delta}} + i \mathbf{W}_\delta^\mu(y) \right) \right] \right\} \\ \cdot G^{\beta\alpha} \left(y, x; \frac{\delta}{\delta J} + i \mathbf{W} \right) = 0 \quad (2.21)$$

在对上式作泛函微商时需要用到由鬼态泛函定义得的某些有关的正规顶角, 容易看出它们是正确的.

(i) 鬼态-鬼态-VIB (GGV) 正规顶角:

$$T_{\alpha\beta\tau}^\mu(x, y, z; \mathbf{W}, \boldsymbol{\psi}, \boldsymbol{\phi}) = - \frac{\delta}{\delta \mathbf{W}_{\mu,\tau}(z)} G_{\alpha\beta}^{-1}(x, y; \mathbf{W}, \boldsymbol{\psi}, \boldsymbol{\phi}) \quad (2.22)$$

(ii) 鬼态-鬼态-费米子-费米子 (GGFF) 正规顶角:

$$\begin{aligned} \int d^4\omega H_{\alpha\beta, mn}(x, y, z, \omega)\phi_n(\omega) &= -\frac{\delta}{\delta\bar{\phi}_m(z)} G_{\alpha\beta}^{-1} \\ \int d^4z\bar{\phi}_m(z)H_{\alpha\beta, mn}(x, y, z, \omega) &= -\frac{\delta}{\delta\phi_n(\omega)} G_{\alpha\beta}^{-1}. \end{aligned} \quad (2.23)$$

(iii) 鬼态-鬼态-标量场 (GG ϕ) 正规顶角:

$$T_{\alpha\beta}(x, y, z) = -\frac{\delta}{\delta\phi} G_{\alpha\beta}^{-1}, \quad T_{\alpha\beta}^+(x, y, z) = -\frac{\delta}{\delta\phi^+} G_{\alpha\beta}^{-1} \quad (2.24)$$

求泛函时还要用到类似下面的链导规则, 如

$$\begin{aligned} \frac{\delta}{\delta J_{\mu, \alpha}(y)} &= \int d^4x \left[\frac{\delta W_{\nu, \beta}(x)}{\delta J_{\mu, \alpha}(y)} \frac{\delta}{\delta W_{\nu, \beta}(x)} + \frac{\delta\bar{\phi}_m(x)}{\delta J_{\mu, \alpha}(y)} \frac{\delta}{\delta\bar{\phi}_m(x)} \right. \\ &\quad + \frac{\delta\phi_0(x)}{\delta J_{\mu, \alpha}(y)} \frac{\delta}{\delta\phi_0(x)} + \frac{\delta\phi(x)}{\delta J_{\mu, \alpha}(y)} \frac{\delta}{\delta\phi(x)} \\ &\quad \left. + \frac{\delta\phi^+(x)}{\delta J_{\mu, \alpha}(y)} \frac{\delta}{\delta\phi^+(x)} \right] \end{aligned} \quad (2.25)$$

以及 $\frac{\delta}{\delta\eta}$, $\frac{\delta}{\delta\bar{\eta}}$, $\frac{\delta}{\delta\beta}$ 和 $\frac{\delta}{\delta\beta^+}$ 的公式, 通过它们将 (2.21) 式中对 (J, η, β) 等的泛函微商表达成对 (W, ϕ, ϕ) 等的微商, 然后从右方乘以 $\int d^4x G_{\alpha\beta}^{-1}(x, z; \frac{\delta}{\delta J} + iW)$ 后, 可以得到

Γ_0 满足的冗长的方程式, 再对它求 W 的两次泛函微商并令所有外源等于零, 就得出三线 VIB 正规顶角函数满足的 W-T 恒等式, 细节见附录, 再把这恒等式变到动量空间, 最终得出我们所需要的公式^[4]:

$$\begin{aligned} k_\rho T_{\alpha\beta\gamma}^{\mu\nu\rho}(p, q, k)(1 + B(k^2)) &= -g[g^{\mu\nu}\epsilon_{\alpha\beta\gamma} - \bar{B}_{\alpha\beta\gamma}^{\mu\nu}(k, p)][\delta_{\beta\beta'} + \Pi_{\beta\beta'}((k+p)^2)] \\ &\quad \times [(p+k)^2 g^{\nu\nu'} - (p+k)^\nu(p+k)^{\nu'}] + g[g^{\mu\nu}\epsilon_{\alpha\beta\gamma} - \bar{B}_{\alpha\beta\gamma}^{\mu\nu}(k, p)] \\ &\quad \times [\delta_{\alpha\alpha'} + \Pi_{\alpha\alpha'}(p^2)][p^2 g^{\mu\mu'} - p^\mu p^{\mu'}] \end{aligned} \quad (2.26)$$

其中 $B(k^2)$ 与鬼态自能有关, 其传播子是:

$$G_{\alpha\beta}(k^2) = \frac{i\delta_{\alpha\beta}}{(k^2 + i\varepsilon)(1 + B(k^2))} \quad (2.27)$$

而 $\bar{B}_{\alpha\beta\gamma}^{\mu\nu}(k, p)$ 由图 2 定义, 附录中可以看出它们是如何得来的。

$\Pi_{\alpha\alpha'}(p^2)$ 是 VIB 的极化张量:

$$D_{\alpha\alpha'}^{\mu\mu'}(p) = -i \left[\frac{g^{\mu\mu'} - \frac{p^\mu p^{\mu'}}{p^2}}{(p^2 + i\varepsilon)} (\delta_{\alpha\alpha'} + \Pi_{\alpha\alpha'}(p^2))^{-1} + \xi \frac{p^\mu p^{\mu'}}{(p^2 + i\varepsilon)} \right] \quad (2.28)$$

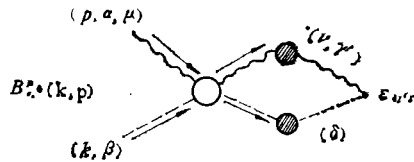


图 2 矢量 W-T 恒等式中的 \bar{B} 函数

注意到以下几点, 容易看出三线 VIB 正规顶角的 W-T 恒等式 (2.26) 与相应裸顶点表达式 (2.3) 的关系。

(1) 由 (2.26) 知三线 VIB 顶角部分的三重散度为零, 表明 $k \rightarrow 0$ 时张量 $k_\rho T_{\alpha\beta\rho}^{\mu\nu}$ 对 p 讲上有横向部分.

(2) $k \rightarrow 0$ 时, (2.27) 式中的 $B(0)$ 吸收到重态化常数之中, 即

$$\lim_{k \rightarrow 0} G_{\alpha\beta}(k^2) = \tilde{Z}_3^{-1} \frac{i\delta_{\alpha\beta}}{k^2 + i\varepsilon}, \quad \text{即} \quad (1 + B(0)) = \tilde{Z}_3, \quad (2.29)$$

(3) 从定义 $\bar{B}_{\alpha\beta\rho}^{\nu\mu}(k, p)$ 的图 2 看出, $\bar{B}_{\alpha\beta\rho}^{\nu\mu}(k, p)(k+p)_\nu$ 与 GVG 顶角 $\bar{T}_{\alpha\beta\rho}^{\nu\mu}(k, p)$ 之间只差裸 GVG 顶角 $\varepsilon_{\alpha\beta\rho}(p+k)^\mu$, 换言之,

$$\bar{B}_{\alpha\beta\rho}^{\nu\mu}(k, p)(k+p)_\nu - \varepsilon_{\alpha\beta\rho}(p+k)^\mu = g^{\mu\nu} \bar{T}_{\alpha\beta\rho, \nu}$$

而由 GVG 顶角部分重态化定义知, $k \rightarrow 0$ 时上式即 $g^{\mu\nu} \tilde{Z}_1 \varepsilon_{\alpha\beta\rho}(p+k)_\nu$, 因此

$$\lim_{k \rightarrow 0} (g^{\mu\nu} \varepsilon_{\alpha\beta\rho} - \bar{B}_{\alpha\beta\rho}^{\nu\mu}(k, p)) = \tilde{Z}_1 g^{\mu\nu} \varepsilon_{\alpha\beta\rho} \quad (2.30)$$

(4) 对称性不破缺时, $\Pi_{\alpha\alpha'}(p^2)$ 可表为 $\delta_{\alpha\alpha'} \Pi(p^2)$.

从以上几点得知, 当 $k \rightarrow 0$ 时, (2.26) 式化为

$$\begin{aligned} \tilde{Z}_3 k_\rho T_{\alpha\beta\rho}^{\mu\nu}(p_1 - (p+k), k) &= -g \tilde{Z}_1 \varepsilon_{\alpha\beta\rho} (1 + \Pi(p^2)) \\ &\quad \times \{(p+k)^2 g^{\mu\nu} - (p+k)^\mu (p+k)^\nu - p^2 g^{\mu\nu} + p^\mu p^\nu\} \end{aligned} \quad (2.31)$$

有了 (2.31) 式, 通过对 $T_{\alpha\beta\rho}^{\mu\nu}$ 的张量结构形式的分析, 易证如要求它在 $k \rightarrow 0$ 时对任何 p 值都满足 (2.31) 式, 则一定得出:

$$\lim_{k \rightarrow 0} T_{\alpha\beta\rho}^{\mu\nu}(p, -(p+k), k) = A(p^2) \varepsilon_{\alpha\beta\rho} [g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \quad (2.32)$$

这就达到了目的, 表明 W^\pm 介子这时的确不存在反常磁矩^[3]. 事实上这结论也与规范场重整化理论中得出的结果相适应. 例如当 $p, q = -(p+k)$ 均外线时, 极限 $k \rightarrow 0$ 按重态化要求应该有

$$\lim_{k \rightarrow 0} T_{\alpha\beta\rho}^{\mu\nu}(p, -(p+k), k) = Z_1 V_{\alpha\beta\rho}^{\mu\nu}$$

和

$$(1 + \pi(p^2)) \rightarrow Z_3$$

将它们代入 (2.31) 式时, 发现得到的正好是熟知的关系式 $\frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3}$.

三、存在自发对称性破缺时的讨论

下面简要地论证当存在对称性自发破缺时, 上面结论仍然成立. 这时由于 (2.4a) 式中 \mathcal{L}_ϕ 的 $\mu^2 < 0$ 导致 $\langle \phi \rangle_0 \neq 0$. 通过 Higgs 机制, 除光子外其余的 VIB 和费米子均获得了质量. 正如^[4] 中讨论 σ 模型时所强调的, 当存在自发破缺时不能直接对 μ^2 作延拓而将 $\mu^2 > 0$ 时得的对称理论结果搬到 $\mu^2 < 0$ 区域中去, 因为 $\mu^2 > 0$ 是正常相而 $\mu^2 < 0$ 是 Goldstone 相. 在 $\mu^2 = 0$ 处为相变点, 因此是奇异的. 这时要照^[4] 中指出的采用一种迂迴曲折的途径. 这仍然是从 $\mu^2 > 0$ 着手, 但首先在 \mathcal{L} 中引进明显的对称性破坏项:

$$\mathcal{L} = \mathcal{L}_s + c^+ \phi + c \phi^+ \quad (3.1)$$

\mathcal{L}_s 即 (2.4) 式, 代表对称的拉氏量. c 为复常数二重态, 当 $c \neq 0$ 时不论 μ^2 是否大于

零,都得出 $\langle \phi \rangle_0 \neq 0$. 这时允许我们对 μ^2 作延拓,从正值到负值去,因为在改动 μ^2 值时,始终在同一相中. 等到完成延拓后再让 c 退回到零去. 如图 3 所表示,为了从 A 点到 D 点,走的是一条 $A \rightarrow B \rightarrow B' \rightarrow D$ 的折线. 具体做法是将 $c \neq 0$ 时的路径积分用 $c = 0$ 时(对称理论的)相应式表达出来. 对于连通 Green 函数母泛函 Z , 照^[4]类似处理得出:

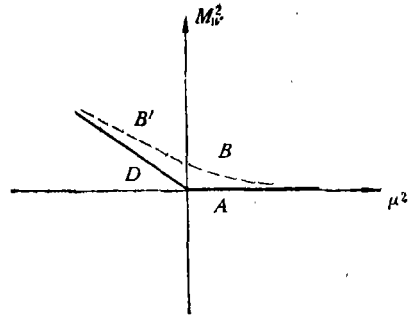


图 3 当存在(或不存在)外场 c 时 M_W^2 作为 μ^2 函数的行为图
 —— $c = 0$ - - - - $c \neq 0$

$$Z(\vec{J}, \eta, \bar{\eta}, \beta, \beta^+) = Z_s(\vec{J}, \eta, \bar{\eta}, \beta + c, \beta^+ + c^+) - Z_s(0, 0, 0, c, c^+) - i \int (V^+ \beta + \beta^+ V) d^4x \quad (3.2)$$

其中 $V = \langle \phi \rangle_0$, Z 的下标 s 表示对称理论时的量.

而由 $V = \left. \frac{\delta Z_s}{\delta \beta^+} \right|_{\beta^+ = c^+}$ 定出 V . 与 (2.36) 对应的正规顶角母泛函是

$$\Gamma(\vec{W}, \phi, \bar{\phi}, \phi', \phi'^+) = \Gamma_s(\vec{W}, \phi, \bar{\phi}, \phi, \phi^+) - \Gamma_s(0, 0, 0, V, V^+) + i \int (\phi'^+ c + c^+ \phi') d^4x \quad (3.3)$$

而与 (2.37) 对应的是

$$-c = \left. \frac{\delta \Gamma_s}{i \delta \phi'^+} \right|_{\phi'^+ = V^+} \quad (3.4)$$

与 (2.21) 式对应的方程是:

$$\left\{ d^4y \left\{ i g \left[\frac{\delta \Gamma_0}{\delta \phi} \frac{\tau_\beta}{2} \left(\frac{\delta}{\delta \eta} + i \phi \right) - \frac{\delta \Gamma_0}{\delta \bar{\phi}} \left(\gamma^0 \frac{\tau_\beta}{2} \gamma^0 \right)^T \left(\frac{\delta}{\delta \eta} + i \bar{\phi}^T \right) \right] + \left(\frac{\delta \Gamma_0}{\delta \phi'} - i c^+ \right) \frac{\tau_\beta}{2} \left(\frac{\delta}{\delta \beta^+} + i (\phi' + V) \right) - \left(\frac{\delta \Gamma_0}{\delta \phi'^+} - i c \right) \frac{\tau_\beta^T}{2} \left(\frac{\delta}{\delta \beta} + i (\phi'^+ + V^+) \right) \right\} + \frac{\delta \Gamma_0}{\delta W_{\mu, \gamma}} \left[i \delta_{\beta \gamma} \partial^\mu + g \epsilon_{\gamma \beta \beta} \left(\frac{\delta}{\delta J_{\mu, \delta}(y)} + i W_\delta^\alpha(y) \right) \right] \right\} \times G_{\beta \alpha}(y, x; \frac{\delta}{\delta J} + i W) = 0 \quad (3.5)$$

其中 Γ_0 与 (2.38) 式中 Γ 的关系也正好是 (2.20) 式.

由此可见,破缺理论时的正规顶角母泛函方程的改动仅仅是那些与 ϕ 的变分有关的项,其中 $\langle \phi \rangle_0 = V$ 的值从 (2.39) 式知道显然是与 c 和 μ^2 的数值有关. 允许我们讨论从 $\mu^2 > 0$ 到 $\mu^2 < 0$ 的延拓,然后这儿开始再让 $c = 0$ 得到了自发对称性破缺的一套方程式. 重复第二节所述各步骤,经过冗长推导并注意照 (2.39) 式有 $\left. \frac{\delta \Gamma_0}{\delta \phi'} \right|_{\phi' = 0} = 0$, 就可以证明这时也有类似 (2.26) 式结果. 但这时 VIB 等都具有质量. 同样可以得到 (2.32) 式. 因此在对称性自发破缺下,三线 VIB 正规顶角函数的张量结构仍然与裸顶角时一

致,表明在 W-S 模型这样的规范理论中 W^\pm 介子的反常磁矩和反常电四极矩均不存在。
作者感谢戴元本、邝宇平、易余萍同志有益的讨论。

附录 三线 VIB 正规顶角 W-T 恒等式证明

将 $\int d^4x G_{\alpha\beta}^{-1}(x, z; \frac{\delta}{\delta j} + iW)$ 左乘(2.21)式后,利用(2.25)式并注意(2.22)—(2.24)式后得:

$$\begin{aligned}
 & g \left[\bar{\Phi}(x) \frac{\tau_\alpha}{2} \frac{\delta \Gamma_0}{\delta \bar{\Phi}(x)} - \frac{\delta \Gamma_0}{\delta \Phi(x)} \left(\gamma^0 \frac{\tau_\alpha}{2} \gamma^0 \right) \Phi(x) \right] + g \left[\Phi^+(x) \frac{\tau_\alpha}{2} \frac{\delta \Gamma_0}{\delta \Phi^+(x)} - \frac{\delta \Gamma_0}{\delta \bar{\Phi}(x)} \frac{\tau_\alpha}{2} \Phi(x) \right] \\
 & - i \left[\partial_x^\mu \delta_{\alpha\beta} + g \varepsilon_{\alpha\beta\gamma} W_\gamma^\mu(x) \right] \frac{\delta \Gamma_0}{\delta W_\beta^\mu(x)} + ig \int d^4y d^4y' d^4z T_{\alpha\beta\gamma}^\mu(x, y, z) G_{\beta\beta'}(y, y') \\
 & \cdot \left[-i \frac{\delta W_\beta^\mu(z)}{\delta J_\beta^{\mu'}(y')} \varepsilon_{\beta'\gamma'\delta'} \frac{\delta \Gamma_0}{\delta W_{\delta'}^{\mu'}(y')} + \frac{\delta \Gamma_0}{\delta \Phi(y')} \frac{\tau_{\beta'}}{2} \frac{\delta W_\beta^\mu(z)}{\delta \bar{\eta}(y')} \right. \\
 & - \frac{\delta \Gamma_0}{\delta \bar{\Phi}(y')} \left(\gamma^0 \frac{\tau_{\beta'}}{2} \gamma^0 \right)^T \frac{\delta W_\beta^\mu(z)}{\delta \eta(y')} + \frac{\delta \Gamma_0}{\delta \Phi(y')} \frac{\tau_{\beta'}}{2} \frac{\delta W_\beta^\mu(z)}{\delta \beta^+(y)} - \frac{\delta \Gamma_0}{\delta \Phi^+(y')} \frac{\tau_{\beta'}}{2} \frac{\delta W_\beta^{\mu'}}{\delta \beta(y')} \left. \right] \\
 & + ig \int d^4y d^4y' d^4z d^4\omega H_{\alpha\beta\gamma\delta}(x, y, z, \omega) G_{\beta\beta'}(y, y') \left[-i \bar{\Phi}_m(y) \frac{\delta \Phi_n(\omega)}{\delta J_\beta^{\mu'}(y')} \varepsilon_{\gamma\delta\beta'} \frac{\delta \Gamma_0}{\delta W_\mu^{\gamma'}(y')} \right. \\
 & - i \frac{\delta \Phi_m(y)}{\delta J_\beta^{\mu'}(y')} \Phi_n(\omega) \varepsilon_{\gamma\delta\beta'} \frac{\delta \Gamma_0}{\delta W_\mu^{\gamma'}(y')} + \frac{\delta \Gamma_0}{\delta \Phi(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \bar{\Phi}_m(z)}{\delta \bar{\eta}(y')} \Phi_n(\omega) \\
 & - \frac{\delta \Gamma_0}{\delta \bar{\Phi}(y')} \left(\gamma^0 \frac{\tau_{\beta'}}{2} \gamma^0 \right)^T \frac{\delta \bar{\Phi}_m(z)}{\delta \eta(y')} \Phi_n(\omega) + \frac{\delta \Gamma_0}{\delta \Phi(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \Phi_n(\omega)}{\delta \bar{\eta}(y')} \Phi_m^T(z) \\
 & - \frac{\delta \Gamma_0}{\delta \bar{\Phi}(y')} \left(\gamma^0 \frac{\tau_{\beta'}}{2} \gamma^0 \right)^T \cdot \frac{\delta \Phi_n(\omega)}{\delta \eta(y')} \bar{\Phi}_m^T(z) + \frac{\delta \Gamma_0}{\delta \bar{\Phi}(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \bar{\Phi}_m(z)}{\delta \beta^+(y')} \Phi_n(\omega) \\
 & - \frac{\delta \Gamma_0}{\delta \Phi^+(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \bar{\Phi}_m(z)}{\delta \beta(y')} \Phi_n(\omega) + \frac{\delta \Gamma_0}{\delta \Phi(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \Phi_n(\omega)}{\delta \beta^+(y')} \bar{\Phi}_m^T(z) \\
 & - \frac{\delta \Gamma_0}{\delta \Phi^+(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \Phi_n(\omega)}{\delta \beta(y')} \bar{\Phi}_m^T(z) \left. \right] + ig \int d^4y d^4y' d^4z T_{\alpha\beta}(x, y, z) G_{\beta\beta'}(y, y') \\
 & \cdot \left[\frac{\delta \Gamma_0}{\delta \Phi(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \Phi}{\delta \bar{\eta}} - \frac{\delta \Gamma_0}{\delta \bar{\Phi}(y')} \left(\gamma^0 \frac{\tau_{\beta'}}{2} \gamma^0 \right)^T \frac{\delta \Phi}{\delta \eta} + \frac{\delta \Gamma_0}{\delta \Phi(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \Phi}{\delta \beta^+} \right. \\
 & - \frac{\delta \Gamma_0}{\delta \Phi^+(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \Phi}{\delta \beta} + \frac{\delta \Gamma_0}{\delta \Phi(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \Phi^+}{\delta \bar{\eta}} - \frac{\delta \Gamma_0}{\delta \bar{\Phi}(y')} \left(\gamma^0 \frac{\tau_{\beta'}}{2} \gamma^0 \right)^T \frac{\delta \Phi^+}{\delta \eta} \\
 & \left. + \frac{\delta \Gamma_0}{\delta \Phi(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \Phi^+}{\delta \beta^+} - \frac{\delta \Gamma_0}{\delta \Phi^+(y')} \frac{\tau_{\beta'}}{2} \frac{\delta \Phi}{\delta \beta} \right] = 0 \tag{A.1}
 \end{aligned}$$

为了得出三线 VIB 正规顶角的 W-T 恒等式。将 (A.1) 式对 W 求两次泛函微商。然后再令外源全等于零。由于(2.17)、(2.18)式,这时 (A.1) 中除第三、第四项外其余各项均为零,这样可以等效地将 (A.1) 对 W 的两次导数写为:

$$\begin{aligned}
 & \frac{\delta^2}{\delta W_\gamma^{\nu'}(u) \delta W_\beta^{\rho'}(v)} \left\{ -i \left[\partial_x^\mu \delta_{\alpha\beta} + g \varepsilon_{\alpha\beta\gamma} W_\gamma^\mu(x) \right] \frac{\delta \Gamma_0}{\delta W_\beta^\mu(x)} + ig \int d^4y d^4y' d^4z T_{\alpha\beta\gamma}^\mu(x, y, z) G_{\beta\beta'}(y, y') \right. \\
 & \cdot \left. \left[-i \frac{\delta W_\beta^\mu(z)}{\delta J_\beta^{\mu'}(y')} \varepsilon_{\beta'\gamma'\delta'} \frac{\delta \Gamma_0}{\delta W_{\delta'}^{\mu'}(y')} \right] \right\} = 0 \tag{A.2}
 \end{aligned}$$

上式左边第一、第二项结果显然是

$$\begin{aligned}
 & -i \left[\partial_x^\mu \frac{\delta^3 \Gamma_0}{\delta W_\alpha^\mu(x) \delta W_\gamma^{\nu'}(u) \delta W_\beta^{\rho'}(v)} + g \varepsilon_{\alpha\gamma\beta} \delta^{(4)}(x-u) \frac{\delta^2 \Gamma_0}{\delta W_\beta^\nu(x) \delta W_\beta^{\rho'}(v)} \right. \\
 & \left. + g \varepsilon_{\alpha\gamma\beta} \delta^{(4)}(x-v) \frac{\delta^2 \Gamma_0}{\delta W_\beta^\nu(x) \delta W_\gamma^{\nu'}(u)} \right] \tag{A.3}
 \end{aligned}$$

而(A.2)的第三项是(注意(2.17)、(2.18)以及(2.22)式):

$$\begin{aligned}
 & -ig \int d^4y d^4y' d^4z \left\{ T_{\alpha\beta\delta}^\mu(x, y, z) G_{\beta\beta'}(y, y') \frac{\delta^2 Z}{\delta J_\delta^\mu(y) \delta J_{\beta'}^\mu(y')} \varepsilon_{\beta'\delta''\delta'''} \frac{\delta^3 \Gamma_0}{\delta W_{\mu'}^{\delta'''}(y') \delta W_\eta^\nu(u) \delta W_\xi^\rho(v)} \right. \\
 & - \frac{\delta^2 G_{\alpha\beta}^{-1}(x, y)}{\delta W_\mu^\delta(x) \delta W_\nu^\eta(u)} G_{\beta\beta'}(y, y') \frac{\delta^2 Z}{\delta J_\delta^\mu(x) \delta J_{\beta'}^\mu(y')} \varepsilon_{\beta'\delta''\delta'''} \frac{\delta^3 \Gamma_0}{\delta W_{\mu'}^{\delta'''}(y') \delta W_\xi^\rho(v)} \\
 & \left. - \frac{\delta^2 G_{\alpha\beta}^{-1}(x, y)}{\delta W_\mu^\delta(x) \delta W_\xi^\rho(v)} G_{\beta\beta'}(y, y') \frac{\delta^2 Z}{\delta J_\delta^\mu(x) \delta J_{\beta'}^\mu(y')} \varepsilon_{\beta'\delta''\delta'''} \frac{\delta^3 \Gamma_0}{\delta W_{\mu'}^{\delta'''}(y') \delta W_\eta^\nu(u)} \right\} \quad (A.4)
 \end{aligned}$$

注意到 $G_{\beta\beta'}(y, y')$ 即鬼态的全传播子 $\Delta_{F, \beta\beta'}(y - y')$ 而 $\left. \frac{\delta^2 Z}{\delta J_\delta^\mu(x) \delta J_{\beta'}^\mu(y')} \right|_{J=0}$ 即 VIB 的全传播子 $\Delta_{F, \delta\delta'}^{\mu\mu'}(z - y')$ 后得知

(i) (A.4) 式中头一项可写为

$$\begin{aligned}
 & i \int d^4y' \left\{ g \int d^4y d^4z T_{\alpha\beta\delta}^\mu(x, y, z) \Delta_{F, \beta\beta'}(y - y') \Delta_{F, \delta\delta'}^{\mu\mu'}(y' - z) \varepsilon_{\beta'\delta''\delta'''} \right\} \frac{\delta^3 \Gamma_0}{\delta W_{\mu'}^{\delta'''}(y') \delta W_\eta^\nu(u) \delta W_\xi^\rho(v)} \\
 & = i \int d^4y' B_{\alpha\delta}^{\mu\mu'}(x, y') \frac{\delta^3 \Gamma_0}{\delta W_{\mu'}^{\delta'''}(y') \delta W_\eta^\nu(u) \delta W_\xi^\rho(v)}
 \end{aligned}$$

其中 $B_{\alpha\delta}^{\mu\mu'}(x, y')$ 代表上式左边{ }, 它对应图 A.1, 显然正好是鬼态自能图的散度, 并有 $B_{\alpha\delta}^{\mu\mu'}(x, y') = \partial_{\mu'}^\alpha B(x) \delta^{(\alpha)}(x - y') \delta_{\alpha\delta''}$. 因此上式化为

$$i \left[\partial_{\mu'}^\alpha B(x) \right] \frac{\delta^3 \Gamma_0}{\delta W_\alpha^{\mu'}(x) \delta W_\eta^\nu(u) \delta W_\xi^\rho(v)} \quad (A.5)$$

(ii) (A.4) 式中第二项可写为

$$\begin{aligned}
 & ig \int d^4y' \left\{ \int \frac{\delta^2 G_{\alpha\beta}^{-1}(x, y)}{\delta W_\mu^\delta(x) \delta W_\nu^\eta(u)} \Delta_{F, \beta\beta'}(y - y') \Delta_{F, \delta\delta'}^{\mu\mu'}(y' - z) \varepsilon_{\beta'\delta''\delta'''} d^4y d^4z \right\} \frac{\delta^3 \Gamma_0}{\delta W_{\mu'}^{\delta'''}(y') \delta W_\xi^\rho(v)} \\
 & = ig \int d^4y' B_{\alpha\eta}^{\nu\mu\mu'}(x, u, y') \frac{\delta^3 \Gamma_0}{\delta W_{\mu'}^{\delta'''}(y') \delta W_\xi^\rho(v)} \quad (A.6)
 \end{aligned}$$

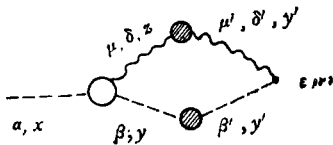


图 A.1 鬼态自能图的散度项

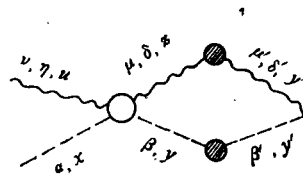


图 A.2 $B_{\alpha\eta}^{\nu\mu\mu'}(x, u, y')$

(A.4) 式第三项和它类似。其中右边 $B_{\alpha\eta}^{\nu\mu\mu'}(x, u, y')$ 代表左边{ }, 它对应图 A.2, 也即正文中的图 2, 将 (A.3)、(A.5)、(A.6) 和对应 (A.4) 中第三项的 (A.6) 型项都代到 (A.2) 式中, 经过移项合并, 化到动量空间后即得 (2.26) 式。

参 考 文 献

- [1] K. O. Mikaelian et al., *Phys. Rev.*, **D17**(1978), 750.
- [2] R. W. Brown et al., *Phys. Rev.*, **D19**(1979), 922.
- [3] 彭宏安, 裘忠平, 李家荣, 高能物理与核物理. (待发表)
- [4] C. Itzykson & J. Zuber, "Quantum Field Theory", E. J. Eichten & F. L. Feinberg, *Phys. Rev.*, **D10**(1974), 3254.

ON THE MAGNETIC MOMENT OF W^\pm BOSONS IN WEINBERG-SALAM MODEL

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Abstract

We discuss the magnetic moment of W^\pm bosons in W-S model. First we assume that there is no spontaneous symmetry broking, starting from the formalism of path integrals and using the gauge invariance requirement, we obtain the Ward-Takahashi identities for the 3-line VIB proper vertex functions and prove that its tensor structure is the same form as that of bare vertex couplings, which indicated the W^\pm bosons have neither anomalous magnetic moment nor electric quadrupole moment. For the realistic case of spontaneous symmetry breaking, we introduce in the Lagrangian an explicit symmetry breaking term. After discussing the problem of analytic continuation for mass of W^\pm bosons, we prove that the above mentioned conclusion is still valid here.